EECS 16A Designing Information Devices and Systems I
Spring 2019 Discussion 6A

1 Circuit Analysis Algorithm

- **Step 1:** Pick a junction and label it as $u = 0$ ("ground"), meaning that we will measure all of the voltages in the rest of the circuit relative to this point.

- **Step 2:** Label all remaining junctions as some "u_i", representing the potential at each junction relative to the zero junction/ground.

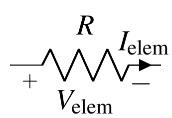
- **Step 3:** Label the current through every element in the circuit "i_n". Every element in the circuit that was listed above should have a current label, including ideal wires. The direction of the arrow indicates which direction of current flow you are considering to be positive. At this stage of the algorithm, you can pick the direction of all of the current arrows *arbitrarily* - as long as you are consistent with this choice and follow the rules described in the rest of this algorithm, the math will work out correctly.

  Note that we only label the current once for each element – for example, we can label i_3 as the current leaving the resistor *or* we can label it as the the current entering the resistor. These are equivalent because KCL also holds within the element itself – i.e., the current that enters an element must be equal to the current that exits that same element.

- **Step 4:** Add $+/-$ labels on each element, following **Passive Sign Convention** (discussed below). These labels will indicate the direction with which voltage will be measured across that element.

Passive sign convention

The **passive sign convention** dictates that positive current should *enter* the positive terminal and *exit* the negative terminal of an element. Below is an example for a resistor:



As long as this convention is followed consistently, it does not matter which direction you arbitrarily assigned each element current to; the voltage referencing will work out to determine the correct final sign. When we discuss *power* later in the module, you will see why we call this convention "passive."

- **Step 5:** Set up the relationship $\mathbf{A}\vec{x} = \vec{b}$, where $\vec{x}$ is comprised of the unknown circuit variables we want to solve for (currents and node potentials – that is, the i's and u's). $\mathbf{A}$ will be an $n \times n$ matrix where n is equal to the number of unknown variables.

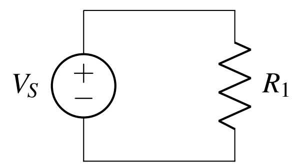
- **Step 6:** Use KCL to fill in as many **Linearly Indpendent** rows of $\mathbf{A}$ and $\vec{b}$ as possible. You should get a KCL equation for every junction including the ground junction. However, one of these junctions will give us a linearly dependent equation. Generally, we do not write a KCL equation for the ground junction.

- **Step 7:** Use the IV relationships of each of the elements to fill in the remaining equations (rows of **A** and values of $\vec{b}$).

  At this point the analysis procedure is effectively complete - all that's left to do is solve the system of linear equations (by applying Gaussian Elimination, inverting **A**, etc.) to find the values for the u's and i's.

1. A Simple Circuit

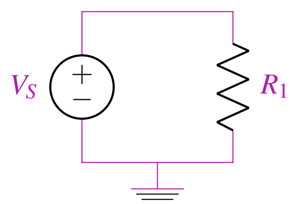
For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a ground node. Does your choice of ground matter?

Answer:

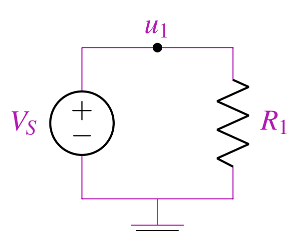
There are two nodes in this circuit and thus two choices for the ground node. The choice of ground does not matter. We will use the ground node shown below:



(b) With your choice of ground, label the node potentials for every node in the circuit.

Answer:

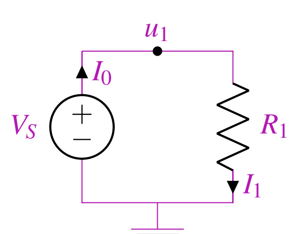
Since this circuit only has two nodes, there will only be one additional node potential.



(c) Label all of the branch currents. Does the direction you pick matter?

Answer:

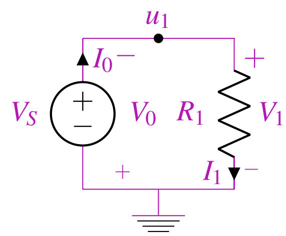
When labeling the currents through branches, the direction you pick does not matter.



(d) Draw the $+/-$ labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal of every element.



(e) Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents. What are the dimensions of the matrix $\mathbf{A}$?

Answer:

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

$\mathbf{A}$ will be a 3×3 matrix since there are three unknowns in the circuit, the two currents I_0 and I_1 and the one potential u_1.

(f) Use KCL to find as many equations as you can for the matrix.

Answer:

KCL gives us one equation for the node at the top, namely that $I_0 - I_1 = 0$. Thus, so far our matrix is as follows:

$$\begin{bmatrix} 1 & -1 & 0 \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ ? \\ ? \end{bmatrix}$$

(g) Use IV relations to find the remaining the equations for the matrix.

Answer:

We know that the difference in potentials across the voltage source must be the voltage on the voltage source, i.e.

$$-V_0 = V_S \tag{1}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$V_1 = I_1 R_1 \tag{2}$$

Writing the equations for node potentials we have:

$$\begin{aligned} 0 - u_1 &= V_0 \\ u_1 - 0 &= V_1 \end{aligned} \tag{3}$$

Substituting expressions from Equations (1) and (2) into Equation (3), we have:

$$\begin{aligned} -u_1 = -V_S &\implies u_1 = V_S \\ u_1 = I_1 R_1 &\implies -I_1 R_1 + u_1 = 0 \end{aligned} \tag{4}$$

Our matrix is then:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -R_1 & 1 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ V_S \\ 0 \end{bmatrix}$$

(h) Solve the system of equations if $V_S = 5\,\text{V}$ and $R_1 = 5\,\Omega$.

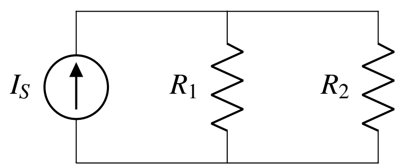
Answer:

By plugging the given values into the system of equations, we get:

$$\begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

2. A Slightly More Complicated Circuit

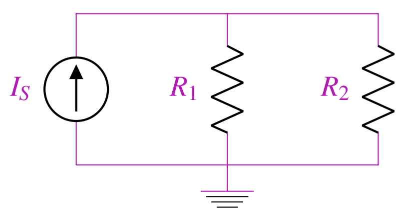
For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a ground node. Does your choice of ground matter?

Answer:

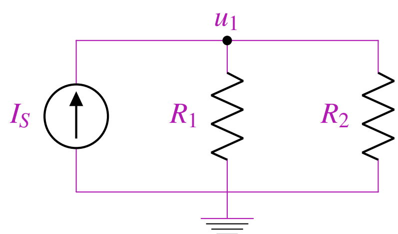
There are two nodes in this circuit and thus two choices for the ground node. The choice of ground does not matter. We will use the ground node shown below:



(b) With your choice of ground, label the node potentials for every node in the circuit.

Answer:

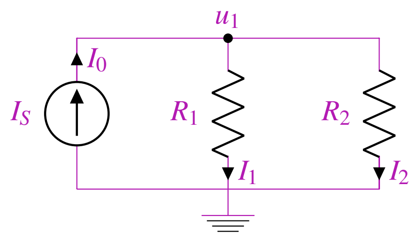
Since this circuit only has two nodes, there will only be one additional node potential.



(c) Label all of the branch currents. Does the direction you pick matter?

Answer:

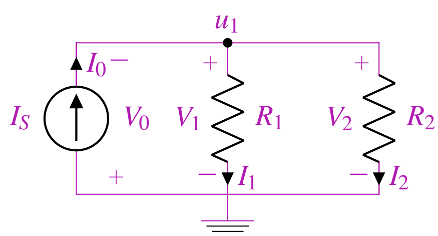
When labeling the currents through branches, the direction you pick does not matter.



(d) Draw the $+/-$ labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal of every element.



(e) Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents. What are the dimensions of the matrix $\mathbf{A}$?

Answer:

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix}$$

$\mathbf{A}$ will be a 4×4 matrix since there are four unknowns in the circuit, the currents I_0, I_1, and I_2 and the one potential u_1.

(f) Use KCL to find as many equations as you can for the matrix.

Answer:

KCL gives us one equation for the node at the top, namely that $I_0 - I_1 - I_2 = 0$. Thus, so far our matrix is as follows:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ ? \\ ? \\ ? \end{bmatrix}$$

(g) Use IV relations to find the remaining the equations for the matrix.

Answer: We know that the current through the current source must be the value of the current source, i.e.

$$I_0 = I_S \tag{5}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$\begin{aligned} V_1 &= I_1 R_1 \\ V_2 &= I_2 R_2 \end{aligned} \tag{6}$$

Writing the equations for node potentials we have:

$$\begin{aligned} 0 - u_1 &= V_0 \\ u_1 - 0 &= V_1 \\ u_1 - 0 &= V_2 \end{aligned} \tag{7}$$

Using Equation (5) and substituting expressions from Equation (6) into Equation (7), we have:

$$\begin{aligned} I_0 &= I_S \\ u_1 = I_1 R_1 \implies -I_1 R_1 + u_1 &= 0 \\ u_1 = I_2 R_2 \implies -I_2 R_2 + u_1 &= 0 \end{aligned} \tag{8}$$

Our matrix is then:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -R_1 & 0 & 1 \\ 0 & 0 & -R_2 & 1 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ I_S \\ 0 \\ 0 \end{bmatrix}$$

(h) Solve the system of equations if $I_S = 5\,\text{A}$, $R_1 = 5\,\Omega$, and $R_2 = 10\,\Omega$.

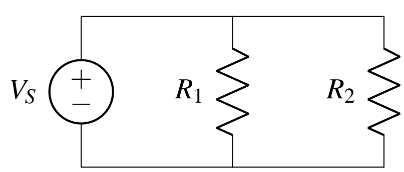
Answer:

By plugging in the values into the system of equations, we get:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 5 \\ 3.33 \\ 1.67 \\ 16.67 \end{bmatrix}$$

3. (PRACTICE) Another Circuit

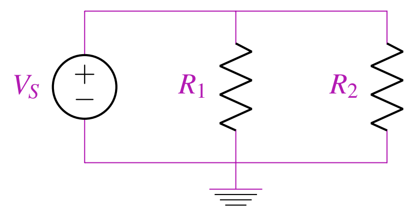
For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a ground node. Does your choice of ground matter?

Answer:

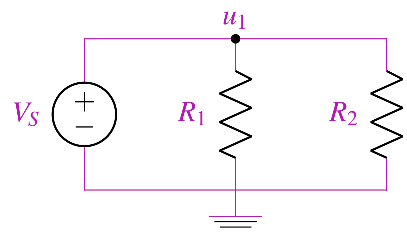
There are two nodes in this circuit and thus two choices for the ground node. The choice of ground does not matter. We will use the ground node shown below:



(b) With your choice of ground, label the node potentials for every node in the circuit.

Answer:

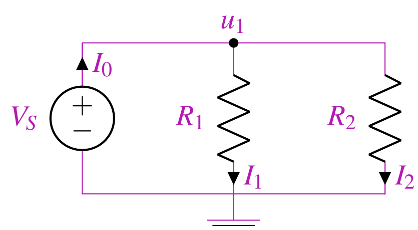
Since this circuit only has two nodes, there will only be one additional node potential.



(c) Label all the branch currents. Does the direction you pick matter?

Answer:

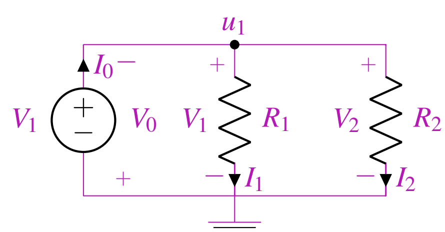
When labeling the currents through branches, the direction you pick does not matter.



(d) Draw the $+/-$ labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal of every element.



(e) Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents. What are the dimensions of the matrix $\mathbf{A}$?

Answer:

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix}$$

A will be a 4×4 matrix since there are four unknowns in the circuit, the currents I_0, I_1, and I_2 and the one potential u_1.

(f) Use KCL to find as many equations as you can for the matrix.

Answer:

KCL gives us one equation for the node at the top, namely that $I_0 - I_1 - I_2 = 0$. Thus, so far our matrix is as follows:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ ? \\ ? \\ ? \end{bmatrix}$$

(g) Use IV relations to find the remaining equations for the matrix.

Answer: We know that the difference in potentials across the voltage source must be the voltage on the voltage source, i.e.

$$-V_0 = V_S. \tag{9}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$\begin{aligned} V_1 &= I_1 R_1 \\ V_2 &= I_2 R_2 \end{aligned} \tag{10}$$

Writing the equations for node potentials we have:

$$\begin{aligned} 0 - u_1 &= V_0 \\ u_1 - 0 &= V_1 \\ u_1 - 0 &= V_2 \end{aligned} \tag{11}$$

Substituting expressions from Equations (9) and (10) into Equation (11), we have:

$$\begin{aligned} -u_1 = -V_S &\implies u_1 = V_S \\ u_1 = I_1 R_1 &\implies -I_1 R_1 + u_1 = 0 \\ u_1 = I_2 R_2 &\implies -I_2 R_2 + u_2 = 0 \end{aligned} \tag{12}$$

Our matrix is then:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -R_1 & 0 & 1 \\ 0 & 0 & -R_2 & 1 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ V_S \\ 0 \\ 0 \end{bmatrix}$$

(h) Solve the system of equations if $V_S = 5\,\text{V}$, $R_1 = 5\,\Omega$, and $R_2 = 10\,\Omega$.

Answer:

By plugging in the values into the system of equations, we get:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1 \\ 0.5 \\ 5 \end{bmatrix}$$

