

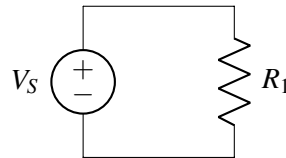
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EECS 16A    Designing Information Devices and Systems I    Discussion 6B  
Spring 2019

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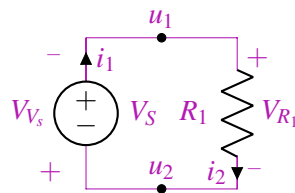
### 1. Revisiting A Simple Circuit

Let's revisit the simple circuit from the lecture. Suppose this time we don't assign a ground node. Let's work through the process again and see if we can still solve the system. Let  $R_1 = 10\Omega$  and  $V_s = 2\text{V}$



#### Answer:

Let's start by labeling all the nodes and all the branch currents:



Next, we write KCL equations for every node in the circuit. For node  $u_1$ :

$$i_1 - i_2 = 0$$

For node  $u_2$ :

$$-i_1 + i_2 = 0$$

Then we write element equations for the two elements in the circuit: For the resistor  $R_1$ :

$$u_1 - u_2 = V_{R_1}, V_{R_1} = i_2 R_1 \implies u_1 - u_2 = i_2 R_1$$

For the voltage source  $V_s$ :

$$u_2 - u_1 = V_s = -V_s$$

Now we have 4 equations for 4 unknowns. So we can setup the following matrix vector equation:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -10 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

Row reducing the matrix we see:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

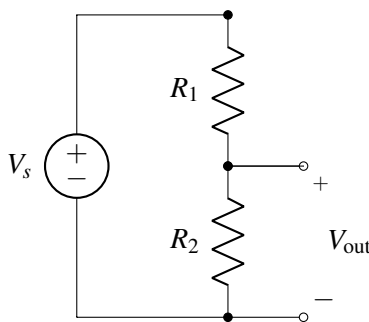
Notice that the above matrix has a non-trivial null space, specifically:

$$\text{span} \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Notice that this implies that we could add a constant to  $u_1$  and  $u_2$  and still get a solution. Therefore, the specific node potentials are unknown. However, we can calculate the difference between node potentials. This turns out to be generally true, and this is why we chose a ground node for every circuit to call 0.

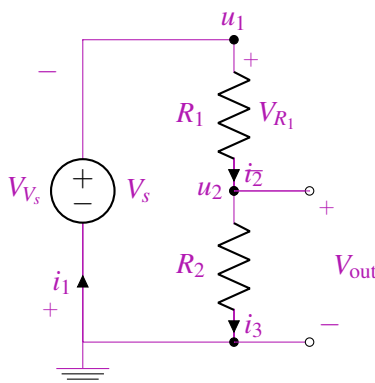
## 2. Voltage Divider

For the circuit below, find the voltage  $V_{out}$  in terms of the resistances  $R_1$ ,  $R_2$ , and  $V_s$ .



### Answer:

We start by selecting a ground node and labeling all the node potentials and branch currents.



Next we write KCL equations for all the nodes, except for the ground node:

$$i_1 - i_2 = 0$$

$$i_2 - i_3 = 0$$

$$i_3 - i_1 = 0$$

Notice the above equations imply that  $i_1 = i_2 = i_3$ . We will remove some of the variables and just set  $i_x = i_1 = i_2 = i_3$ .

Next we write element equations:

$$\begin{aligned} 0 - u_1 &= V_{V_s} = -V_s \\ u_1 - u_2 &= V_{R_1}, V_{R_1} = i_x R_1 \implies u_1 - u_2 = i_x R_1 \\ u_2 - 0 &= V_{R_2}, V_{R_2} = i_x R_2 \implies u_2 - u_0 = i_x R_2 \end{aligned}$$

We can again remove an extra variable by setting  $u_1$  to  $V_s$ . The above equations then become:

$$\begin{aligned} V_s - u_2 &= i_x R_1 \\ u_2 &= i_x R_2 \end{aligned}$$

Now we have two unknowns,  $i_x$  and  $u_2$ , and two equations. We can solve them directly for  $u_2$ . Notice that  $V_{\text{out}} = u_2 - 0 = u_2$

$$V_{\text{out}} = u_2 = \frac{R_2}{R_1 + R_2} V_s$$

### 3. Bio-Molecule Detector

One application for electronics that has gained a lot of attention over the past several years is in so-called “bio-molecule” detection. The idea is to build a system that detects the presence of specific molecules and/or cells (e.g. specific viruses, proteins, etc.) in a biological sample; if this detection can be performed automatically and using relatively low-cost components, it can have a dramatic impact on a number of areas such as medical diagnosis, drug development, DNA sequencing, etc.

In this problem, we’ll look at how some of the techniques we learned about in the touchscreen module can be applied to realize a hypothetical bio-molecule detector. (Real bio-molecule detection systems involve quite a bit more complexity than what we’ll include here, but in many designs the same basic principles apply.)

As shown in Figure 1, the detector works by flowing a liquid that may or may not contain the biomolecules through a region in the device that has electrodes on the top and bottom of the liquid channel. The electrodes (E1/E2 in Figure 1) are chemically “functionalized” (using e.g. some appropriately designed antibodies), so that if the specific bio-molecule of interest is present in the fluid sample, one or more of the molecules will get physically trapped between the two electrodes (bottom right of Figure 1). After all of the fluid has been

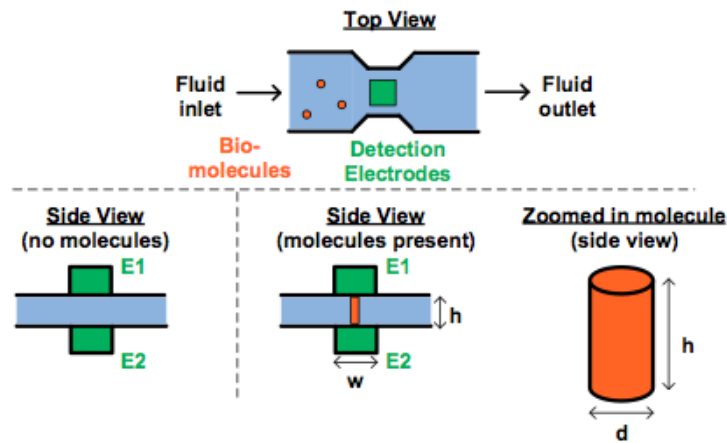


Figure 1: Bio-molecule detector.

cleared out of the device (i.e., so that if there are no bio-molecules present, there is only air in between the two electrodes E1/E2), we can then figure out whether or not one or more bio-molecules were trapped by measuring the resistance between the two electrodes.

- (a) Let's first assume that we want to detect the presence of a bio-molecule by measuring resistance. If no bio-molecule is present, what should be the resistance between E1/E2? As shown in Figure 1, if each bio-molecule is a cylinder with diameter  $d = 10\text{ nm}$ , height  $h = 100\text{ nm}$ , and has a resistivity  $\rho = 100\ \mu\Omega\text{ m}$ , what would be the resistance between E1 and E2 if only a single bio-molecule has been trapped? Note that you can assume that the trapped molecule is exactly vertically oriented when it is trapped – i.e., the top and bottom faces of the molecule are both aligned with surfaces of the electrodes.

**Answer:**

If no bio-molecule is present between E1/E2, we are left with just air between the electrodes, so for all intents and purposes, the resistance between them should be *infinite*.

If there is a bio-molecule trapped between E1/E2, the resistance  $R$  should be:

$$R_{\text{molecule}} = \frac{\rho \cdot l}{A} = \frac{100 \times 10^{-6} \Omega\text{ m} \cdot 100 \times 10^{-9} \text{ m}}{\pi \cdot \left(\frac{1}{2} \cdot 10 \times 10^{-9} \text{ m}\right)^2} = 127.3 \text{ k}\Omega$$

- (b) Using the same numbers for  $d$ ,  $h$ , and  $\rho$  as part (a), as a function of the number of trapped bio-molecules  $N_{\text{molecules}}$ , what is the resistance between E1 and E2? (Note that you can assume that  $N_{\text{molecules}}$  is small enough that all of the molecules fit within the electrode area and that all of the molecules are still trapped in an exactly vertical orientation.)

**Answer:**

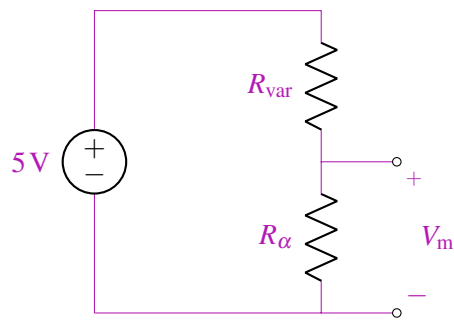
By having many bio-molecules in the detector, the cross-sectional area becomes the sum of the areas of all of the molecules. Therefore, the net resistance between E1/E2 is:

$$R = \frac{127.3 \text{ k}\Omega}{N_{\text{molecules}}}$$

- (c) Given your answers to parts (a) and (b), design a circuit that will output a voltage greater than 2.5 V if more than 5 molecules are trapped.

**Answer:**

Design idea: use the relationship of the resistors in a voltage divider and the respective drops across them.



We use a voltage divider with a 5 V source and resistors  $R_\alpha$  and  $R_{\text{var}}$  in series.  $R_{\text{var}}$  is the resistance across the molecules, and  $R_\alpha$  is used to compare against  $R_{\text{var}}$ . We measure across  $R_\alpha$ . We want the measured voltage  $V_m > 2.5 \text{ V}$  when we have more than 5 molecules, so we want:

$$R_\alpha > R_{\text{var}}(6)$$

$$R_\alpha > \frac{127.3 \text{ k}\Omega}{6}$$

When we have 5 or less molecules, we want  $V_m < 2.5 \text{ V}$  Therefore,

$$R_\alpha < R_{\text{var}}(5)$$

$$R_\alpha < \frac{127.3 \text{ k}\Omega}{5}$$

We can then set  $R_\alpha$  to  $\frac{127.3 \text{ k}\Omega}{5.5} = 23.15 \text{ k}\Omega$ .