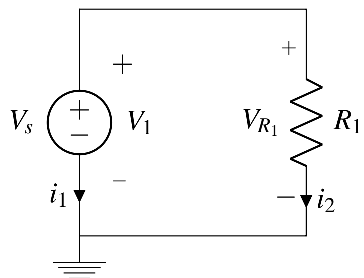
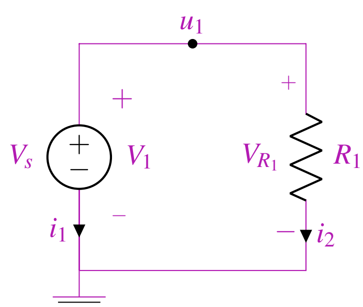
EECS 16A Designing Information Devices and Systems I
Spring 2019 Discussion 7A

1. Passive Sign Convention and Power

(a) Suppose we have the following circuit and label the currents as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.



Answer: We'll start by solving the circuit for the unknown node potentials and currents.



The KCL equation for the one node in this circuit is:

$$i_1 + i_2 = 0$$

The Element equations for the two elements in this circuit are:

$$u_1 - 0 = V_1 = V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = -1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

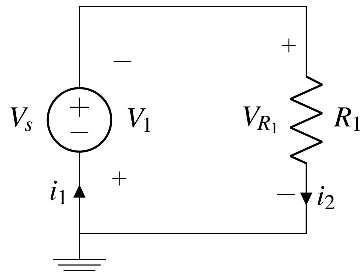
$$P_{R_1} = IV = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

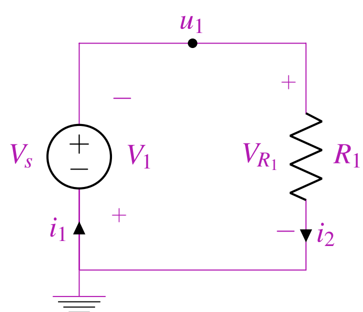
$$P_{V_s} = IV = i_1 V_1 = i_1 V_s = -1\,\mathrm{A} \cdot 5\,\mathrm{V} = -5\,\mathrm{W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is adding power to the circuit.

(b) Suppose we change the label of the currents in the circuit to be as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\mathrm{V}$ and let $R_1 = 5\,\Omega$.



Answer: We'll solve the circuit the same way as last time.



The KCL equation for the one node in this circuit is:

$$-i_1 + i_2 = 0$$

The Element equations for the two elements in this circuit are:

$$0 - u_1 = V_1 = -V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\mathrm{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\mathrm{V}$$
$$i_1 = 1\,\mathrm{A}$$
$$i_2 = 1\,\mathrm{A}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = IV = i_2 V_{R_1} = 1\,\mathrm{A} \cdot 5\,\mathrm{V} = 5\,\mathrm{W}$$

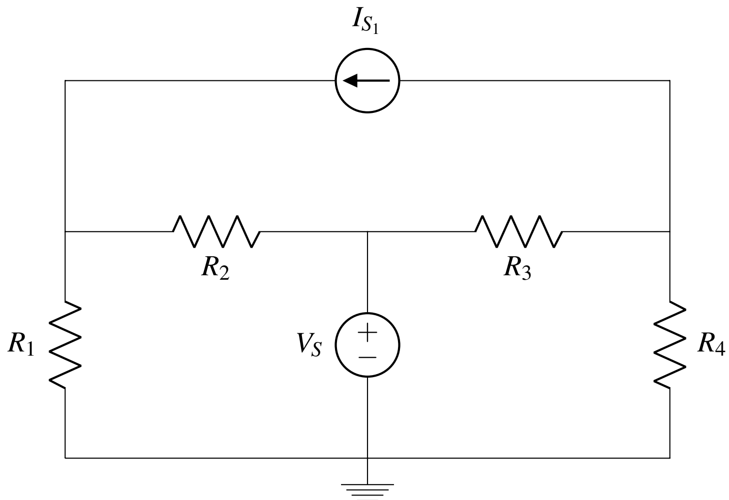
Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = IV = i_1 V_1 = i_1(-V_s) = 1\,\mathrm{A} \cdot -5\,\mathrm{V} = -5\,\mathrm{W}$$

Notice here that the circuit has the same power dissipated by all the elements. This is because with both labeling of currents, we followed the passive sign convention.

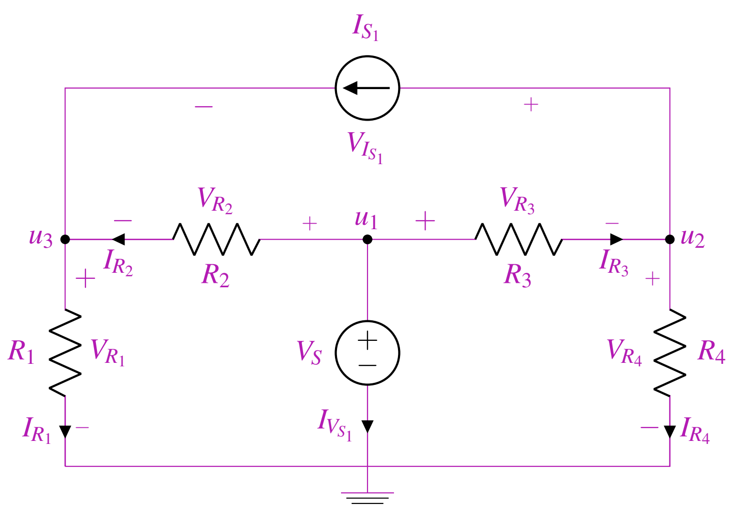
2. Circuit Analysis

Setup the matrix to solve for the voltages across and the currents flowing through each component.



Answer:

We first label all potentials, currents, and voltages.



We will use simplifications for the sources. Since we know V_s, and the voltage source is connected between u_1 and ground, we know $u_1 - 0 = V_s \implies u_1 = V_s$. Similarly, we don't need to add variables for the current sources when writing KCL equations.

Let's start by writing KCL equations for the nodes u_2 and u_3:

$$-I_{R_3} + I_{S_1} + I_{R_4} = 0$$
$$-I_{R_2} - I_{S_1} + I_{R_1} = 0$$

Now let's write element equations, we've already included the sources, so we only need to write equations for the resistors:

$$u_3 - 0 = I_{R_1} R_1$$

$$V_s - u_3 = I_{R_2} R_2$$
$$u_2 - 0 = I_{R_4} R_4$$
$$V_s - u_2 = I_{R_3} R_3$$

Notice we have 6 equations for 6 unknowns. We can setup the matrix:

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & -R_1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -R_2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -R_4 \\ -1 & 0 & 0 & 0 & -R_3 & 0 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ I_{R_1} \\ I_{R_2} \\ I_{R_3} \\ I_{R_4} \end{bmatrix} = \begin{bmatrix} I_{S_1} \\ -I_{S_1} \\ 0 \\ -V_s \\ 0 \\ -V_s \end{bmatrix}$$

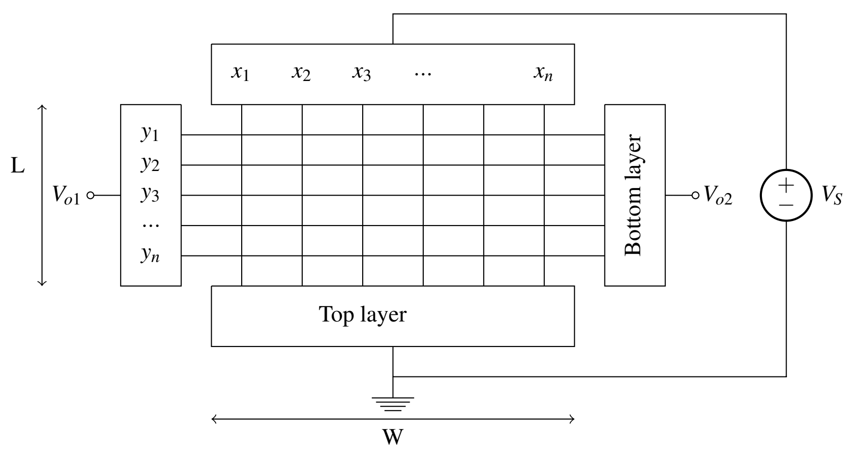
3. Resist the Touch

In this question, we will be re-examining the 2-dimensional resistive touchscreen previously discussed in both lecture and lab. The general touch screen is shown in Figure 1 (a). The touchscreen has length L and width W and is composed of a rigid bottom layer and a flexible upper layer. The strips of a single layer are all connected by an ideal conducting plate on each side. The upper left corner is position $(1,1)$.

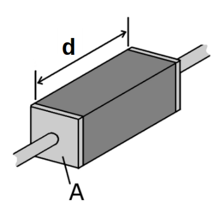
The top layer has N vertical strips denoted by $x_1, x_2, ..., x_N$. These vertical strips all have cross sectional area A, and resistivity ρ_x.

The bottom layer has N horizontal strips denoted by $y_1, y_2, ..., y_N$. These horizontal strips all have cross sectional area A as well, and resistivity ρ_y.

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally. Also assume that all resistive strips are rectangular as shown by Figure 1 (b).



(a) 2-D Resistive Touch Screen



(b) 3D Model of a Single Resistive Strip

Figure 1:

(a) (3 points) Figure 1(b) shows a model for a single resistive strip. Find the equivalent resistance R_x for the vertical strips and R_y for the horizontal strips, as a function of the screen dimensions W and L, the respective resistivities, and the cross-sectional area A.

Answer: The equation for resistance is $R = \frac{\rho l}{A}$

Therefore, $R_x = \frac{\rho_x L}{A}$.

For the bottom, $R_y = \frac{\rho_y W}{A}$.

(b) (5 points) Consider a 2×2 example for the touchscreen circuit.

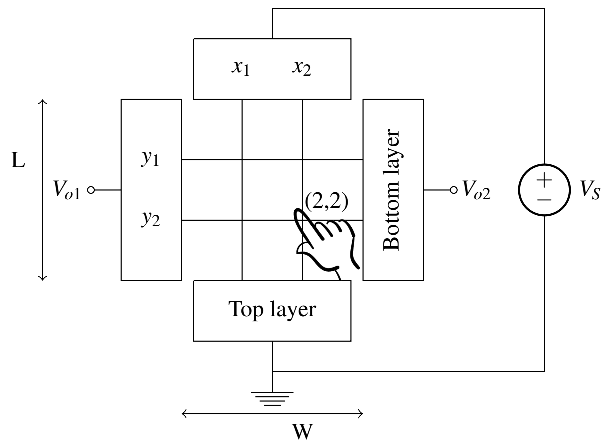


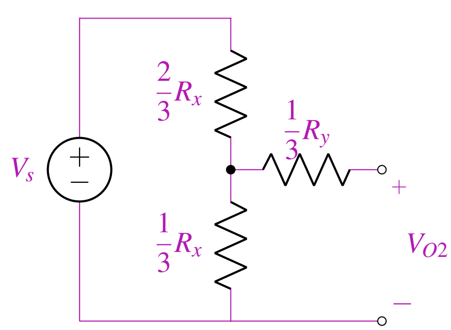
Figure 2: 2×2 Case of the Resistive Touchscreen

Given that $V_s = 3\,\text{V}$, $R_x = 2000\,\Omega$, and $R_y = 2000\,\Omega$, draw the equivalent circuit for when the point $(2,2)$ is pressed and solve for the voltage at terminal V_{O2} with respect to ground.

Answer:

Since all of the resistive strips are equally spaced, the resistor above point $(2,2)$ on strip x_2 becomes $\frac{2}{3}R_x$ and the resistor below point $(2,2)$ on strip x_2 becomes $\frac{1}{3}R_x$.

The bottom layer resistors, although they must be drawn in the equivalent circuit, do not affect the voltage at V_{O2} as they are open circuits.



Observing that the resistive strips form a voltage divider, we can determine V_{O2} using the voltage divider equation.

Therefore, $V_{O2} = V_{(2,2)} = V_s \dfrac{\frac{1}{3}R_x}{\frac{1}{3}R_x + \frac{2}{3}R_x} = \dfrac{1}{3}V_s = 1\text{V}.$

(c) (8 points) Suppose a touch occurs at coordinates (i, j) in Figure 1(a). Find an expression for V_{O2} as a function of V_s, N, i, and j. The upper left corner is the coordinate $(1,1)$ and the upper right coordinate is $(N,1)$.

Answer:

$$
\begin{aligned}
V_{O2} &= \frac{\frac{N+1-j}{N+1}R_x}{R_x}Vs \\
&= \frac{N+1-j}{N+1}V_s
\end{aligned}
$$