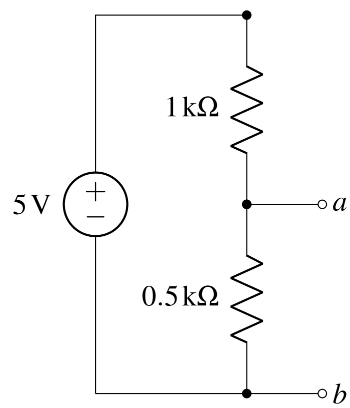
EECS 16A Designing Information Devices and Systems I
Spring 2019

Discussion 7B

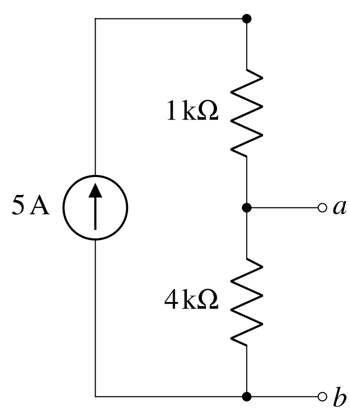
1. Equivalence

Find the Thévenin and Norton equivalents across terminals *a* and *b* for the circuits given below.

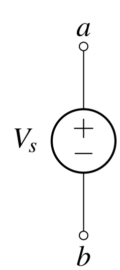
(a)



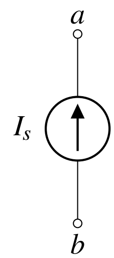
(b)



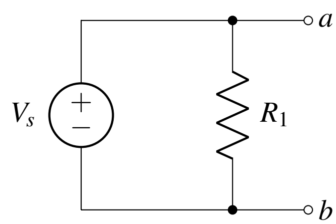
(c)



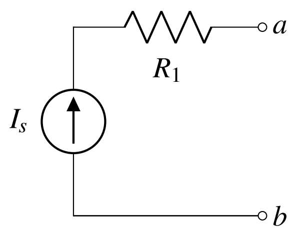
(d)



(e) (Practice)

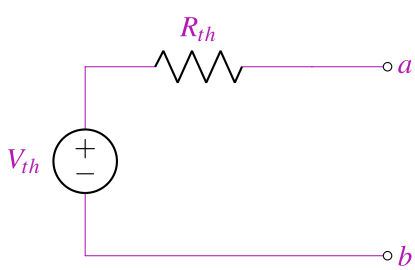


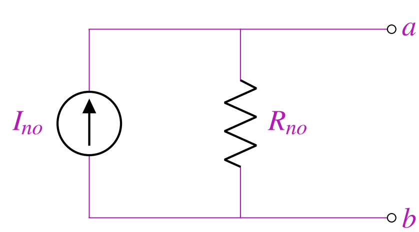
(f) (Practice)



Answer:

The general Thévenin and Norton equivalents are shown below:





(a)

$$V_{th} = 1.67\,\text{V}, I_{no} = 5\,\text{mA}, R_{th} = R_{no} = 333\,\Omega$$

(b)

$$V_{th} = 20\,000\,\text{V}, I_{no} = 5\,\text{A}, R_{th} = R_{no} = 4000\,\Omega$$

(c) A Norton equivalent of a voltage source is not necessary, since a voltage source is a basic element. The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$.

(d) A Thévenin equivalent of a voltage source is not necessary because a current source is a basic element and cannot be represented as a voltage source. The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$.

(e) The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$. Notice that adding a parallel resistor does not change the Thévenin equivalent. As before, since the circuit is effectively a voltage source, a Norton equivalent is not required.

(f) The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$. Notice that adding a series resistor does not change the Norton equivalent. With a similar argument as before, the Thévenin equivalent for the source is not required, as it is a current source.

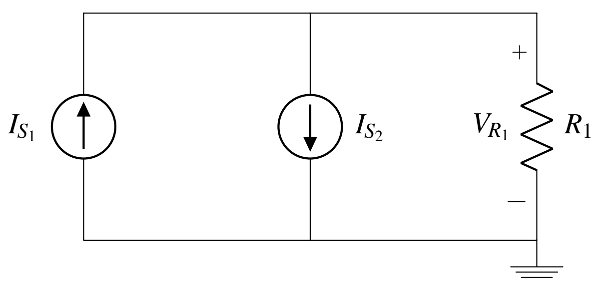
2. Super-power

For the following circuits:

i. Use the superposition theorem to solve for the voltages across the resistors.

ii. For parts (a) and (b) only, find the power dissipated/generated by all components. Is power conserved?

(a)



Answer:

i. While we could apply the algorithm we have learned in class, let's see if there's a way to find the answer quicker than before. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know $i_{R_1} = I_{S_1} - I_{S_2}$. Applying Ohm's Law we find:

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

We could also solve this using superposition. Turning on I_{S_1} gives $V_{R_1} = I_{S_1}R_1$. Turning on I_{S_2} gives $V_{R_1} = -I_{S_2}R_1$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = (I_{S_1} - I_{S_2})^2 R_1$$

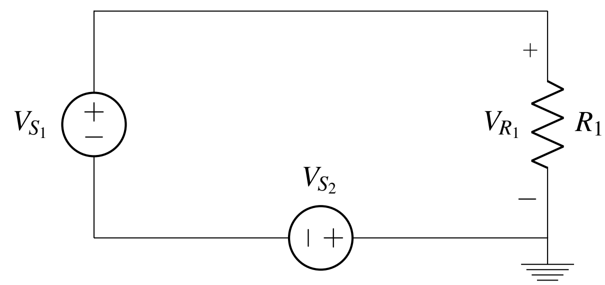
$$P_{I_{S_1}} = -I_{S_1}V_{R_1} = -(I_{S_1} - I_{S_2})I_{S_1}R_1$$

$$P_{I_{S_2}} = I_{S_2}V_{R_1} = (I_{S_1} - I_{S_2})I_{S_2}R_1$$

$$P_{R_1} + P_{I_{S_1}} + P_{I_{S_2}} = (I_{S_1} - I_{S_2})^2 R_1 - (I_{S_1} - I_{S_2})I_{S_1}R_1 + (I_{S_1} - I_{S_2})I_{S_2}R_1 = 0$$

Power is conserved.

(b)



Answer:

i. Once again, we could apply the circuit analysis algorithm or find the answer directly. Notice the circuit only has one loop, so we can use KVL to find the voltage across the resistor.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

We could also solve with superposition. Turning on V_{S_1} gives $V_{R_1} = V_{S_1}$. Turning on V_{S_2} gives $V_{R_1} = -V_{S_2}$. The overall voltage is then the sum.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = \frac{(V_{S_1} - V_{S_2})^2}{R_1}$$

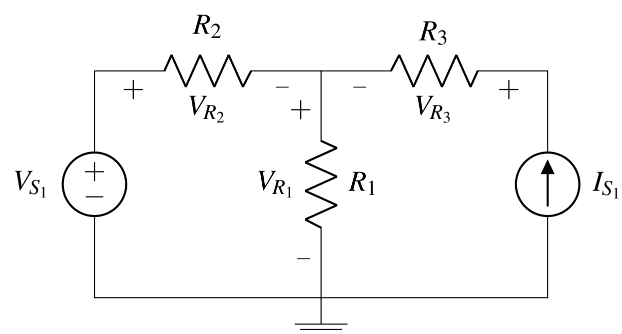
$$P_{V_{S_1}} = -I_{R_1} V_{S_1} = -\frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1}$$

$$P_{V_{S_2}} = I_{R_1} V_{S_2} = \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1}$$

$$P_{R_1} + P_{V_{S_1}} + P_{V_{S_2}} = \frac{(V_{S_1} - V_{S_2})^2}{R_1} - \frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1} + \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1} = 0$$

Power is conserved.

(c)



Answer: Turning on only V_{S_1}, we have the following voltages across the resistors:

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1}$$

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_{S_1}$$

$$V_{R_3} = 0$$

Then turning only I_{S_1}, we have the following voltages:

$$V_{R_1} = \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

$$V_{R_2} = -\frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

$$V_{R_3} = I_{S_1} R_3$$

Using superposition we can sum up the contributions from both V_{S_1} and I_{S_1} to get:

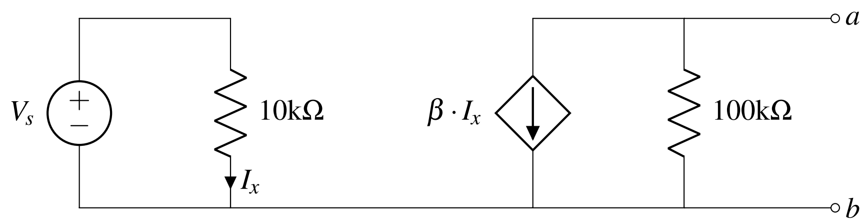
$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1} + \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

$$V_{R_2} = V_{S_1} - V_{R_1} = \frac{R_2}{R_1 + R_2} V_{S_1} - \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

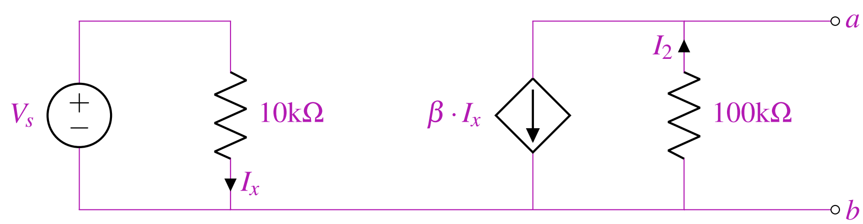
$$V_{R_3} = I_{S_1} R_3$$

3. Equivalence

Find the Thévenin equivalent of the following circuit across the terminals a and b (in terms of V_s and β). Note that the current source is dependent on the current I_x.



Answer:



We start by calculating the open circuit voltage V_{th}. To calculate the open circuit, we start on the left, and work our way to the right. We begin by calculating I_x.

$$I_x = \frac{V_s}{10\,\text{k}\Omega}$$

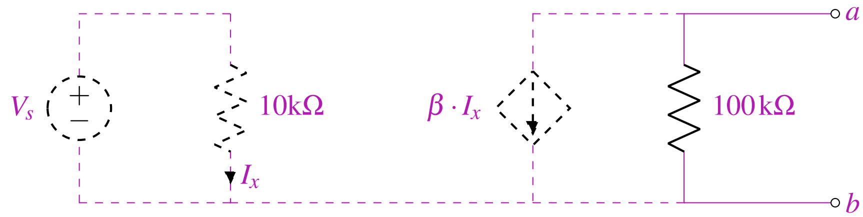
Now, knowing I_x, we can find the current through the dependent source.

$$I_2 = \beta \frac{V_s}{10\,\text{k}\Omega}$$

Knowing I_2, we can find the votlage across the resistor:

$$V_{ab} = -I_2 \cdot 100\,\text{k}\Omega = -\beta 100\,\text{k}\Omega \frac{V_s}{10\,\text{k}\Omega} = -10\beta V_s$$

Next, we need to turn off the voltage source. There's no current through the resistor, so the current source is not sourcing any current. Then, a model of the circuit is shown below:



Thus the Thévenin resistance is $100\,\text{k}\Omega$. i