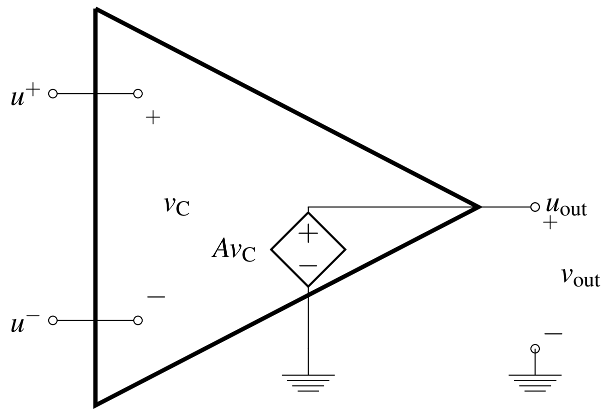
EECS 16A Designing Information Devices and Systems I
Spring 2019 Discussion 9A

1. Op-Amp Golden Rules

Here is an equivalent circuit of an op-amp (where we are assuming that $V_{SS} = -V_{DD}$) for reference:



(a) What are the currents flowing into the positive and negative terminals of the op-amp (i.e., what are I^+ and I^-)? What are some of the advantages of your answer with respect to using an op-amp in your circuit designs?

Answer:

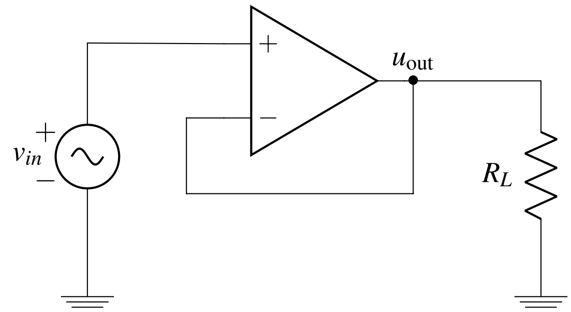
The u^+ and u^- terminals have no closed circuit connection between them, and therefore no current can flow into or out of them. This is very good because we can connect an op-amp to any other circuit, and the op-amp will not disturb that circuit in any way because it does not load the circuit (it is an open circuit).

(b) Suppose we add a resistor of value R_L between u_{out} and ground. What is the value of v_{out}? Does your answer depend on R_L? In other words, how does R_L affect Av_C? What are the implications of this with respect to using op-amps in circuit design?

Answer:

Notice that u_{out} is connected directly to a controlled/dependent voltage source, and therefore v_{out} will always have to be equal to Av_C regardless of what R_L is connected to the op-amp. This is very advantageous because it means that the output of the op-amp can be connected to any other circuit (except a voltage source), and we will always get the desired/expected voltage out of the op-amp.

For the rest of the problem, consider the following op-amp circuit in negative feedback:



(c) Assuming that this is an ideal op-amp, what is v_{out}?

Answer:

Recall for an ideal op-amp in negative feedback, we know from the Golden Rules that $u^+ = u^-$. In this case, $u^- = u_{\text{out}} = u^+$.

(d) Draw the equivalent circuit for this op-amp and calculate v_{out} in terms of A, v_{in}, and R_L for the circuit in negative feedback. Does v_{out} depend on R_L? What is v_{out} in the limit as $A \to \infty$?

Answer:

Notice that the op-amp can be modeled as a voltage-controlled voltage source. Thus, we have the following equation:

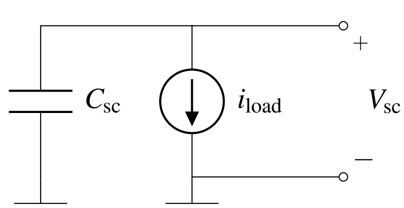
$$v_{\text{out}} = A(v_{in} - v_{\text{out}})$$
$$v_{\text{out}} + Av_{\text{out}} = Av_{in}$$
$$v_{\text{out}} = v_{in}\frac{A}{1+A}$$

Thus, as $A \to \infty$, $v_{\text{out}} \to v_{\text{in}}$. This is the same as what we get after applying the op-amp Golden Rules. Notice that output voltage does not depend on R. Thus, this circuit acts like a voltage source that provides the same voltage read at u^+ without drawing any current from the terminal at u^+. This is why the circuit is often referred to as a "unity gain buffer," "voltage follower," or just "buffer."

2. IoT4eva Revisited

After guiding them to make an intelligent selection for their super-capacitors, IoT4eva was so happy with your performance that you got a promotion! The good news is that you're getting paid more, but the "bad" news is that you have more responsibilities too. In particular, you are now responsible not only for selecting the super-capacitors used to power the device, but also for building the rest of the circuitry associated with the power supply.

Recall that the super-capacitor supplies voltage and can be represented by the following circuit:



In practice, many real circuits (especially sensors that are trying to detect very small signals) don't like to operate with supply voltages that vary substantially over time. Remember that the voltage on our super capacitor drops linearly over time. If we want to use these super capacitors for our device, we need to build another circuit that converts this changing voltage into a constant voltage. This circuit is powered by the super-capacitor and produces a constant voltage at its output, which is then used to supply power to rest of the device. These circuits are often referred to as "voltage regulators," and in this problem we'll explore how to build the simplest form of such a voltage regulator.

(a) To create such a voltage regulator, we first need to build a reference voltage. The basic idea of the voltage regulator is that we will use V_{sc} to power an op-amp, and the op-amp will convert this reference voltage into our desired output voltage.

Fortunately, someone else in the company has already built one of those and made it available to you – the Thévenin equivalent of this circuit is a voltage source whose value is $0.8\,\text{V}$ and a resistance of $1\,\text{k}\Omega$. (The internals of this voltage reference circuit aren't important for this problem. Also, as you should see shortly, this circuit by itself is not appropriate for supplying power to the rest of the device.)

Using this reference circuit, an op-amp, and resistors, design a circuit that is powered by the super-capacitor voltage V_{sc} (which for now you can assume is always high enough for the circuit to work) that produces a constant $1.2\,\text{V}$ supply voltage for the rest of the device. You can model the load from the rest of the device as a $10\,\text{mA}$ current source. Please be sure to choose specific values for any resistors you use in your circuit as well.

Hint: Remember that the op-amp itself needs to be supplied with power, and the only source of power we have available is the super-capacitor.

Answer:

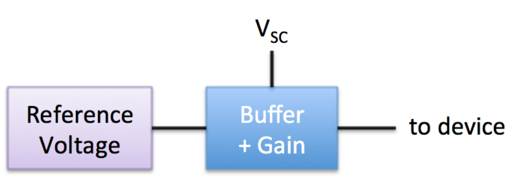
Let's practice the design method.

Step 1:

In this problem, the ultimate objective is to output a $1.2\,\text{V}$ node that is capable of driving the load modeled as a $10\,\text{mA}$ current source. We are also required to power the voltage regulator using the super-capacitor.

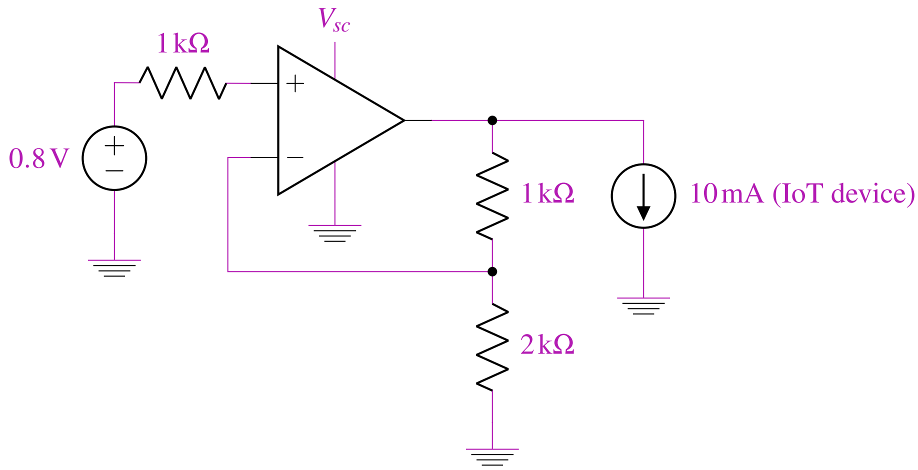
Step 2:

We are given a reference voltage source that we can use to power the IoT device. However, the voltage of the reference is not high enough and it has a high source resistance (in fact, if we run $0.8\,\text{mA}$ of current through the source resistance, the voltage drop across the resistor would be the same as the voltage source itself). The only other source of voltage is the super-capacitor, but the problem is that it has a variable voltage. Thus, we need to build a circuit that buffers (provides low output resistance) and amplifies the reference voltage which is powered by the super-capacitor.



Step 3:

Now that we have a high-level block diagram of the circuit, we can think about how to implement it. We need some form of buffer, so we will definitely need an op-amp. Moreover, we also know that the output of the op-amp must be directly connected to the device for it to act as a buffer. We also know that the circuit can only be powered by the super-capacitor, so we power the op-amp using V_{sc}. The other functionality we have not dealt with is the gain. We need a gain of $\frac{1.2\,\text{V}}{0.8\,\text{V}} = 1.5$, and we have seen that we can implement this using a non-inverting amplifier. Since the gain of a non-inverting amplifier is $\frac{R_2+R_1}{R_1}$, we can set any value to R_1 and R_2 such that this ratio is 1.5. For example, we can use $R_1 = 2\,\mathrm{k}\Omega$ and $R_2 = 1\,\mathrm{k}\Omega$. Note that the source resistance of the reference voltage doesn't play a role here since there is no current flowing into the inputs of the op-amp. The circuit is shown below.



(b) Now that we've built the voltage regulator and we know that we want its output voltage to stay fixed at 1.2 V, what is the minimum voltage we need on our super capacitors $V_{sc,min}$ to ensure that the regulator can indeed produce a fixed 1.2 V output?

Answer:

The op-amp will not be able to produce 1.2 V at its output if $V_{sc} < 1.2\,\mathrm{V}$, so $V_{sc,min} = 1.2\,\mathrm{V}$.

(c) One of the most important things to evaluate about a voltage regulator is its **efficiency**, or the power dissipated by the load circuits (in this case, the rest of the IoT4eva device) divided by the total amount of power delivered by the power supply. Continuing to model the rest of the IoT4eva device as a 10 mA current source, how much power is dissipated by the 10 mA current source?

Also, assuming that all of the IoT4eva's 10 mA current flows through the super-capacitor, how much power is delivered by the super-capacitor? In this case, what is the overall efficiency of our design?

Answer:

The 10 mA curent source is supplied with 1.2 V, so

$$P_{\text{device}} = 1.2\,\mathrm{V} \cdot 10\,\mathrm{mA} = 12\,\mathrm{mW}$$

Our reference voltage does not output any current since the current into the input of an op-amp in negative feedback is 0 A, so the power associated with that source is 0 W. It is important to note that the op-amp by itself cannot generate any power. Any current flowing to the 1.2 V output node has to come from the op-amp, and any current that flows out of the op-amp must come from the super-capacitor. The op-amp is not supplying any power, it just dissipates a part of the power it receives and passes on the rest to the output. The total power supplied by the super-capacitor is the product of the output current of the op-amp with the voltage of the super-capacitor. This output current is the sum of the current source and the current flowing through the negative feedback resistors to ground.

$$I_{\text{op-amp}} = 10\,\mathrm{mA} + \frac{1.2\,\mathrm{V}}{3\,\mathrm{k}\Omega} = 10.4\,\mathrm{mA}$$

Power from super-capacitor is thus $V_{sc} \cdot 10.4\,\mathrm{mA}$. So, the efficiency is

$$\frac{1.2\,\mathrm{V} \cdot 10\,\mathrm{mA}}{V_{sc} \cdot 10.4\,\mathrm{mA}} = \frac{12}{10.4 V_{sc}}$$

Note: The efficiency might differ depending on your choice of resistor values.
Notice that V_{sc} shows up in the denominator. Thus, if we increase the super-capacitor voltage, our efficiency drops. This might be counterintuitive at first, but the reason this happens is that the op-amp is configured in a way that forces the output to a certain voltage. That means there is some voltage drop that happens inside the op-amp itself, and this voltage drop is wasted since it does not get delivered to the IoT device.
Even though the efficiency of this circuit is quite low, this circuit is actually used in real life quite often. This is because the circuit is small (in PCB area) and has nice properties in terms of isolating different components in a circuit from each other.