EECS 16A Designing Information Devices and Systems I
Spring 2019 Discussion 14A

1. Mechanical Gram-Schmidt (Fall 2016 Final)

(a) Use Gram-Schmidt to find an orthonormal basis for the following three vectors.

$$\vec{v}_1 = \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} \sqrt{2} \\ 0 \\ -\sqrt{2} \end{bmatrix}$$

(b) Express $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ as vectors in the basis you found in part (a).

2. Cactus Care (30 points) (Spring 2018 final)

You want to monitor the light received by your cactus over the course of the day, when you aren't around. You design a transmitter that sends the following periodic code of length $N = 5$:

$$\vec{c} = \begin{bmatrix} 1 & -3 & 2 & 1 & 2 \end{bmatrix}^T$$

You can encode information about the light by multiplying the code with the light intensity (l). With your cell phone, you received $\vec{r}$, which is a shifted version of the code $\vec{c}l$.

(a) (4 points) Write a matrix A such that

$$A\vec{y} = \vec{r}$$

where $\vec{r}$ is the received signal (length 5) and $\vec{y}$ is a vector of all zeros except one entry which contains the light intensity l. For example, if $\vec{r} = \begin{bmatrix} 2l & l & -3l & 2l & l \end{bmatrix}^T$ i.e. $\vec{c}l$ is shifted by 1, $\vec{y} = \begin{bmatrix} 0 & l & 0 & 0 & 0 \end{bmatrix}^T$. If shifted by 3, $\vec{y} = \begin{bmatrix} 0 & 0 & 0 & l & 0 \end{bmatrix}^T$.

(b) (6 points) This semester your learned several techniques for solving linear systems of equations. For each of the following techniques, could you use it to solve the matrix equation from Part A? Justify your answer in 1-2 sentences. Assume there is no noise.

Gaussian Elimination ◯ yes ◯ no
Explain:

Least Squares ◯ yes ◯ no
Explain:

Orthogonal Matching Pursuit ◯ yes ◯ no
Explain:

You set up another light detector in the lab to see if it's better. The two light detectors has a transmitter with a different **periodic** codes: $\vec{c}_1$ and $\vec{c}_2$.

$$\vec{c}_1 = \begin{bmatrix} 1 & -3 & 2 & 1 & 2 \end{bmatrix}^T$$

$$\vec{c}_2 = \begin{bmatrix}3 & 1 & 2 & -2 & -1\end{bmatrix}^T$$

As before, the codes are multiplied by the light intensities at each location, l_1 and l_2, and your cell phone receives the sum of shifted codes, each weighted by the light at that location.

(c) (5 points) Write a new matrix A such that

$$A\vec{y} = \vec{r}$$

where $\vec{r}$ is the received signal (length 5) and $\vec{y}$ is a vector of all zeros except two entries which contain l_1 and l_2.
Hint: The positions of l_1 and l_2 in the vector $\vec{y}$ will depend on the unknown shifts in $\vec{c}_1$ and $\vec{c}_2$, respectively. For example, $\vec{y} = \begin{bmatrix}0 & l_1 & 0 & 0 & 0 & 0 & 0 & l_2 & 0 & 0\end{bmatrix}^T$, if $\vec{c}_1$ and $\vec{c}_2$ are shifted by 1 and 2 respectively.

(d) (6 points) For each of the following techniques, could you use it to solve the matrix equation from Part D, with two different light sensors? Justify your answer in 1-2 sentences. Assume there is no noise.
Gaussian Elimination ◯ yes ◯ no
Explain:

Least Squares ◯ yes ◯ no
Explain:

Orthogonal Matching Pursuit ◯ yes ◯ no
Explain:

(e) (3 points) In order to judge if your codes are "good", you want to calculate the autocorrelations and cross-correlation of your codes. Professor Waller helps you calculate the following:

$$\text{autocorrelation of } \vec{c}_1 = \begin{bmatrix}19 & -3 & \mathbf{??} & -2 & -3\end{bmatrix}^T$$

$$\text{autocorrelation of } \vec{c}_2 = \begin{bmatrix}19 & 0 & -5 & -5 & 0\end{bmatrix}^T$$

$$\text{cross-correlaton of } \vec{c}_1 \text{ with } \vec{c}_2 = \begin{bmatrix}0 & -10 & 12 & 11 & -4\end{bmatrix}^T$$

Finish the set by calculating the unknown term in the autocorrelation of c_1.

(f) (6 points) Consider the following set of codes (c_3 and c_4).

$$\vec{c}_3 = \begin{bmatrix}1 & -2 & -3 & 2 & 1\end{bmatrix}^T \qquad \vec{c}_4 = \begin{bmatrix}1 & 1 & 2 & -2 & -3\end{bmatrix}^T$$

$$\text{autocorr. of } \vec{c}_3 = \begin{bmatrix}19 & 1 & -10 & -10 & 1\end{bmatrix}^T$$

$$\text{autocorr. of } \vec{c}_4 = \begin{bmatrix}19 & 2 & -11 & -11 & 2\end{bmatrix}^T$$

$$\text{cross-correlation of } \vec{c}_3 \text{ with } \vec{c}_4 = \begin{bmatrix}-14 & -16 & 5 & 18 & -2\end{bmatrix}^T$$

If you use OMP to solve for the light intensities, which set of codes (c_1,c_2 OR c_3,c_4) is more robust to noise in the received signal? Justify your answer. For the set of codes that is worse, what mistake will is most likely to happen during the OMP algorithm in the presence of noise?

◯ c_1, c_2 are more robust ◯ c_3, c_4 are more robust

