

EECS 16A Designing Information Devices and Systems I

Spring 2019 Discussion 1B

1. Finding The Bright Cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoulish fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers, but they don't know any linear algebra – and they need your help.

Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):

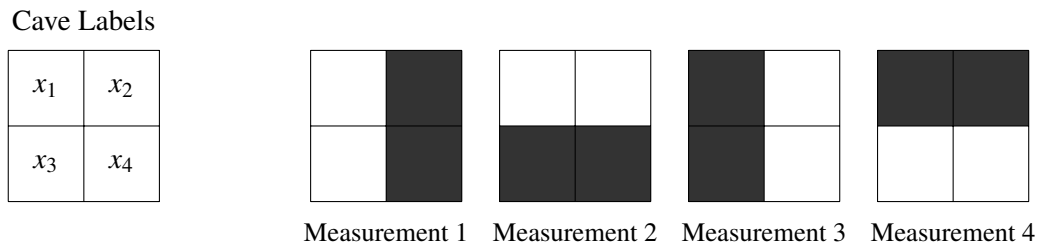


Figure 1: Four image masks.

- Let \vec{x} be the four-element vector that represents the magnitude of light emanating from the four cave entrances. Write a matrix \mathbf{K} that performs the masking process in Figure 1 on the vector \vec{x} , such that $\mathbf{K}\vec{x}$ is the result of the four measurements.
- Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not?
- Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However, her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:

50%	100%
100%	50%

Does this additional measurement give them enough information to solve the problem? Why or why not?

2. Proofs

- (a) Suppose for some non-zero vector \vec{x} , $\mathbf{A}\vec{x} = \vec{0}$. Prove that the columns of \mathbf{A} are linearly dependent.
- (b) Suppose there exist two unique vectors \vec{x}_1 and \vec{x}_2 that both satisfy $\mathbf{A}\vec{x} = \vec{b}$, that is, $\mathbf{A}\vec{x}_1 = \vec{b}$ and $\mathbf{A}\vec{x}_2 = \vec{b}$. Prove that the columns of \mathbf{A} are linearly dependent.
- (c) Suppose there exists a matrix \mathbf{A} whose columns are linearly dependent. Prove that if there exists a solution to $\mathbf{A}\vec{x} = \vec{b}$, then there are infinitely many solutions.