# EECS 16A Designing Information Devices and Systems I
# Spring 2019 Homework 5

## This homework is due March 1, 2019, at 23:59.
## Self-grades are due March 5, 2019, at 23:59.

**Submission Format**
Your homework submission should consist of **one** file.

- `hw5.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit the file to the appropriate assignment on Gradescope.

**1. The Dynamics of Romeo and Juliet's Love Affair**

In this problem, we will study a discrete-time model of the dynamics of Romeo and Juliet's love affair—adapted from Steven H. Strogatz's original paper, *Love Affairs and Differential Equations*, Mathematics Magazine, 61(1), p.35, 1988, which describes a continuous-time model.

Let $R[n]$ denote Romeo's feelings about Juliet on day $n$, and let $J[n]$ denote Juliet's feelings about Romeo on day $n$. The sign of $R[n]$ (or $J[n]$) indicates like or dislike. For example, if $R[n] > 0$, it means Romeo likes Juliet. On the other hand, $R[n] < 0$ indicates that Romeo dislikes Juliet. $R[n] = 0$ indicates that Romeo has a neutral stance towards Juliet.

The magnitude (i.e. absolute value) of $R[n]$ (or $J[n]$) represents the intensity of that feeling. For example, a larger $|R[n]|$ means that Romeo has a stronger emotion towards Juliet (love if $R[n] > 0$ or hatred if $R[n] < 0$). Similar interpretations hold for $J[n]$.

We model the dynamics of Romeo and Juliet's relationship using the following linear system:

$$R[n+1] = a R[n] + b J[n], \quad n = 0, 1, 2, \ldots$$

and

$$J[n+1] = c R[n] + d J[n], \quad n = 0, 1, 2, \ldots,$$

which we can rewrite as

$$\vec{s}[n+1] = \mathbf{A}\vec{s}[n],$$

where $\vec{s}[n] = \begin{bmatrix} R[n] \\ J[n] \end{bmatrix}$ denotes the state vector and $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the state transition matrix for our dynamic system model.

The selection of the parameters $a, b, c, d$ results in different dynamic scenarios. The fate of Romeo and Juliet's relationship depends on these model parameters (i.e. $a, b, c, d$) in the state transition matrix and the initial state ($\vec{s}[0]$). In this problem, we'll explore some of these possibilities.

 

(a) Consider the case where $a+b=c+d$ in the state-transition matrix

$$\mathbf{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Show that

$$\vec{v}_1=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

is an eigenvector of $\mathbf{A}$, and determine its corresponding eigenvalue $\lambda_1$. Show that

$$\vec{v}_2=\begin{bmatrix} b \\ -c \end{bmatrix}$$

is an eigenvector of $\mathbf{A}$, and determine its corresponding eigenvalue $\lambda_2$. Now, express the first and second eigenvalues and their eigenspaces in terms of the parameters $a, b, c,$ and $d$.

**Hint:** You could use the characteristic polynomial approach to find the eigenvalues and eigenvectors. You may find it easier to use the following approach instead:

- First find $\lambda_1$ by showing $\vec{v}_1=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector of $\mathbf{A}$.
- Then find $\lambda_2$ by showing $\vec{v}_2=\begin{bmatrix} b \\ -c \end{bmatrix}$ is an eigenvector of $\mathbf{A}$.

For parts (b) - (d), consider the following state-transition matrix:

$$\mathbf{A}=\begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

(b) Determine the eigenpairs (i.e. $(\lambda_1, \vec{v}_1)$ and $(\lambda_2, \vec{v}_2)$) for this system. Note that this matrix is a special case of the matrix explored in part (a), so you can use results from that part to help you.

(c) Determine all of the *steady states* of the system. That is, find the set of points such that if Romeo and Juliet start at, or enter, any of those points, their states will stay in place forever: $\{\vec{s}_* \mid \mathbf{A}\vec{s}_* = \vec{s}_*\}$.

(d) Suppose Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

Now suppose we have the following state-transition matrix:

$$\mathbf{A}=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Use this state-transition matrix for parts (e) - (g).

(e) Determine the eigenpairs (i.e. $(\lambda_1, \vec{v}_1)$ and $(\lambda_2, \vec{v}_2)$) for this system. Note that this matrix is a special case of the matrix explored in part (a), so you can use results from that part to help you.

(f) Suppose Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

(g) Now suppose that Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

Finally, we consider the case where we have the following state-transition matrix:

$$\mathbf{A} = \begin{bmatrix}1 & -2\\-2 & 1\end{bmatrix}$$

Use this state-transition matrix for parts (h) - (j).

(h) Determine the eigenpairs (i.e. $(\lambda_1, \vec{v}_1)$ and $(\lambda_2, \vec{v}_2)$) for this system. Note that this matrix is a special case of the matrix explored in part (a), so you can use results from that part to help you.

(i) Suppose Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix}1\\-1\end{bmatrix}\right\}$. What happens to their relationship over time if $R[0] > 0$ and $J[0] < 0$? What about if $R[0] < 0$ and $J[0] > 0$? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

(j) Now suppose that Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

**2. Homework Process and Study Group**

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

**3. Midterm Problem 3**

Redo Midterm Problem 3.

(a)

(b)

(c)

**4. Midterm Problem 4**

Redo Midterm Problem 4.

(a)

(b)

**5. Midterm Problem 5**

Redo Midterm Problem 5.

(a)

(b)

(c)

(d)

**6. Midterm Problem 6**

Redo Midterm Problem 6.

  

(a)

(b)

(c)

(d)

**7. Midterm Problem 7**

Redo Midterm Problem 7.

(a)

(b)

(c)

**8. Midterm Problem 8**

Redo Midterm Problem 8.

(a)

(b)

(c)