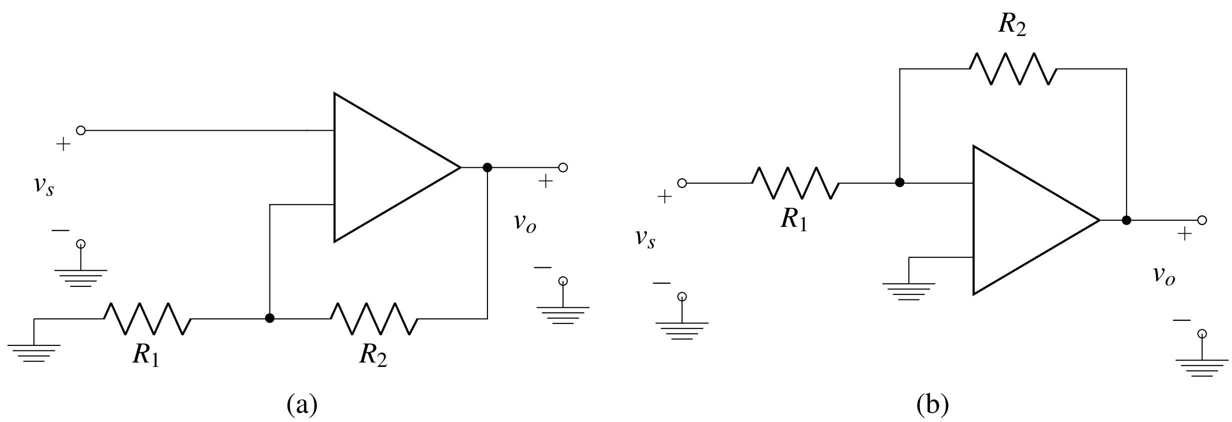
EECS 16A Designing Information Devices and Systems I
Spring 2019 Homework 10

You should plan to complete this homework by Thursday, April 11th. Everything in this homework is in scope for the midterm, but you do not need to turn anything in. There are no self-grades for this homework.

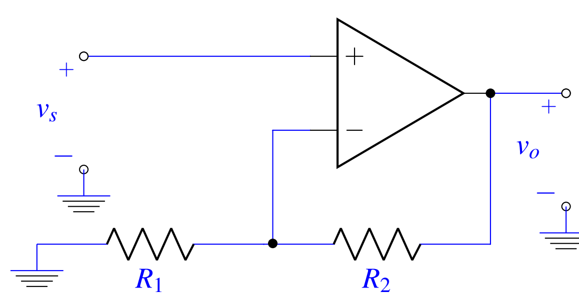
1. Basic Amplifier Building Blocks

The following amplifier stages are used often in many circuits and are well known as (a) the non-inverting amplifier and (b) the inverting amplifier.



(a) Label the input terminals of the op-amp labeled (a), so that it is in negative feedback. Then derive the voltage gain ($A_v = \frac{v_o}{v_s}$) of the non-inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.

Solution:



The $+, -$ should be labeled on the top and bottom of the op amp, respectively.

There are many ways to solve these circuits; here are some:

Method 1: The voltage at the positive input terminal is v_s, so by the Golden Rules, the op-amp will act such that the voltage at the negative input terminal also becomes v_s. Therefore, the voltage drop across R_1 is v_s, so there is a current of $i = \frac{v_s}{R_1}$ through resistor R_1. Since no current flows into the negative input terminal (by the Golden Rules), this current of i must flow through R_2 (by KCL at the inverting input). Thus, the voltage drop across R_2 is $V_2 = i \cdot R_2 = v_s\left(\frac{R_2}{R_1}\right)$. Therefore, v_o is v_s plus the voltage drop across R_2:

$$v_o = v_s + v_s\left(\frac{R_2}{R_1}\right) = v_s\left(\frac{R_1+R_2}{R_1}\right)$$

Method 2: Since there is no current flowing into the negative input terminal (by the Golden Rules), notice that the resistors R_1 and R_2 form a voltage divider between the output v_o and ground. The negative input terminal sees the output of this voltage divider:

$$u_- = v_o\left(\frac{R_1}{R_1+R_2}\right)$$

But $u_- = u_+ = v_s$ by the Golden Rules, so we have:

$$v_o\left(\frac{R_1}{R_1+R_2}\right) = v_s \implies v_o = v_s\left(\frac{R_1+R_2}{R_1}\right)$$

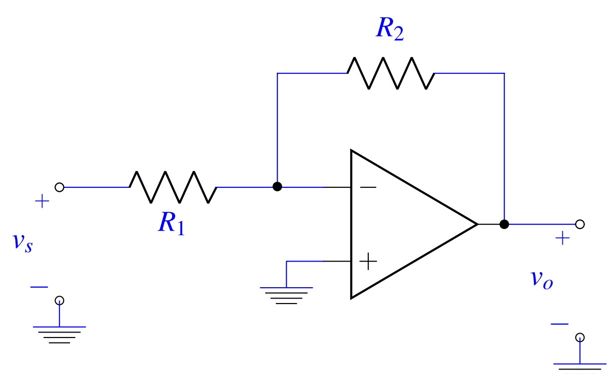
Therefore, the gain of this amplifier is:

$$A_v = \frac{v_o}{v_s} = \frac{R_1+R_2}{R_1}$$

This is called an *non-inverting amplifier* because the gain A_v is positive – it does not invert the input signal (in contrast to the amplifier in the next part of this problem).

(b) Label the input terminals of the op-amp labeled (b), so that it is negative feedback. Then derive the voltage gain ($A_v = \frac{v_o}{v_s}$) of the inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.

Solution:



The $+,-$ should be labelled on the bottom and top of the op amp, respectively.

Here is one way to solve for the gain:

Since the potential at the positive input terminal is $u_+ = 0$, the op-amp will act such that the potential at the negative input terminal is $u_- = 0$ as well (by the Golden Rules). Now, by KCL at the node with potential u_-:

$$\frac{v_s - 0}{R_1} + \frac{v_o - 0}{R_2} = 0$$

Solving this yields:

$$v_o = -\left(\frac{R_2}{R_1}\right)v_s$$

Thus, the voltage gain of this amplifier circuit is:

$$A_v = \frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

This is called an *inverting amplifier* because the voltage gain A_v is *negative*, meaning it "inverts" its input signal.

2. Cool For The Summer

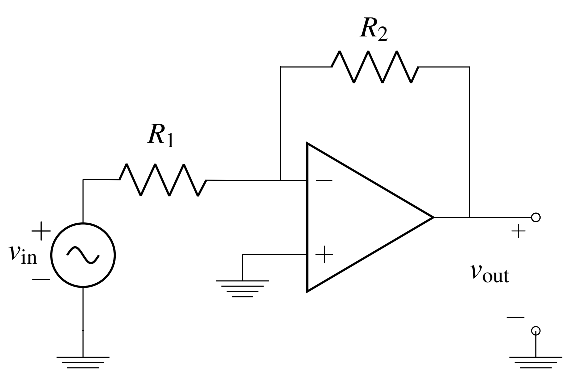
You and a friend want to make a box that helps control an air conditioning unit using both your inputs. You both have individual dials where you can set a control voltage: input of 0 means that you want to leave the temperature as it is. Negative voltage input would mean that you want to reduce the temperature. (It's hot, so we will assume that you never want to increase the temperature – so, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds to positive voltages. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off. You also need a system that sums up both you and your friend's control inputs.

Therefore, you need a box that is **an inverting summer** – *it outputs a weighted sum of two voltages where the weights are both negative*. The sum is weighted because each of you has your own subjective sense of how much to turn the dial down, so you need to compensate for this.

This problem walks you through designing this inverting summer using an op-amp.

(a) As a first step, derive v_{out} in terms of R_2, R_1, v_{in}.



Solution: First, we need to check that the amplifier is in negative feedback. In other words, if the negative input terminal is moved upward, the feedback needs to move it back downward. Going around the loop:

- We move the negative input of the op amp upward
- The output of the amplifier moves downward
- The negative input moves downward with it

The important thing here is that the result of the initial stimulus needs to go in the opposite direction of the initial stimulus! Thus, we've confirmed that the amplifier is in negative feedback.

Second, we perform KCL.

$$\frac{v_{in} - u_-}{R_1} + \frac{v_{out} - u_-}{R_2} = 0$$

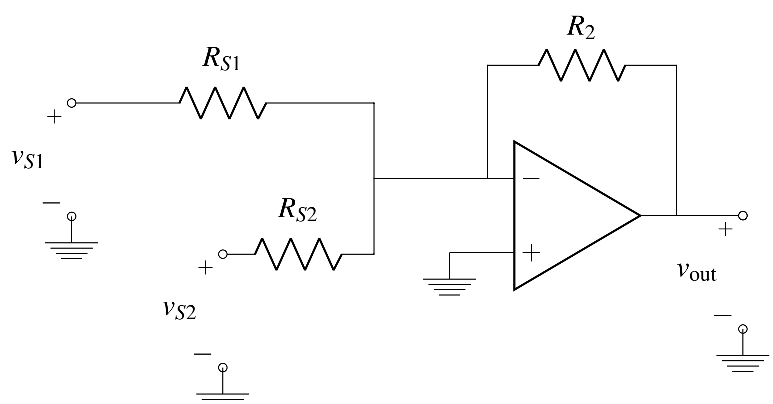
Since we're in negative feedback, we can apply the golden rules. From those, we know the voltages at the negative and positive input terminals of the amplifier—u_- and u_+, respectively—are held at the same voltage. In other words, $u_+ = u_- = 0$V.

$$\frac{v_{\text{in}}}{R_1} + \frac{v_{\text{out}}}{R_2} = 0$$

$$v_{\text{out}} = v_{\text{in}}\left(-\frac{R_2}{R_1}\right)$$

The general inverting amplifier shown above has a voltage gain $v_{\text{out}} = -\frac{R_2}{R_1}v_{\text{in}}$.

(b) Now we will add a second input to this circuit as shown below. Find v_{out} in terms of v_{S1}, v_{S2}, R_{S1}, R_{S2} and R_2.



Solution:

Method 1: Superposition

We can find the overall voltage gain of this amplifier using superposition. When v_{S1} is on, we can ignore R_{S2}. From the Golden Rules, we know that the voltage at the − terminal of the op-amp must be equal to the voltage at the + terminal. Thus, the voltage across R_{S2} is 0 V. Now apply the equation from part (a) $v_{\text{out}} = -\frac{R_2}{R_{S1}}v_{S1}$. Similarly, when v_{S2} is on, we get $v_{\text{out}} = -\frac{R_2}{R_{S2}}v_{S2}$. Combining the two equations, we get $v_{\text{out}} = -R_2\left(\frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}}\right)$.

Method 2: KCL without superposition

The following analysis is also correct and arrives at the same conclusion. According to the golden rules, $u_- = u_+ = 0$V, so we can write a single equation and solve:

$$\frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} + \frac{v_{\text{out}}}{R_2} = 0$$

$$v_{\text{out}} = -v_{S1}\left(\frac{R_2}{R_{S1}}\right) - v_{S2}\left(\frac{R_2}{R_{S2}}\right)$$

(c) Let's suppose that you want $v_{\text{out}} = -\left(\frac{1}{4}v_{S1} + 2v_{S2}\right)$ where v_{S1} and v_{S2} represent the input voltages from you and your friend. Select resistor values such that the circuit implements this desired relationship.

Solution: Using the configuration from the previous part, the conditions which need to be satisfied are:

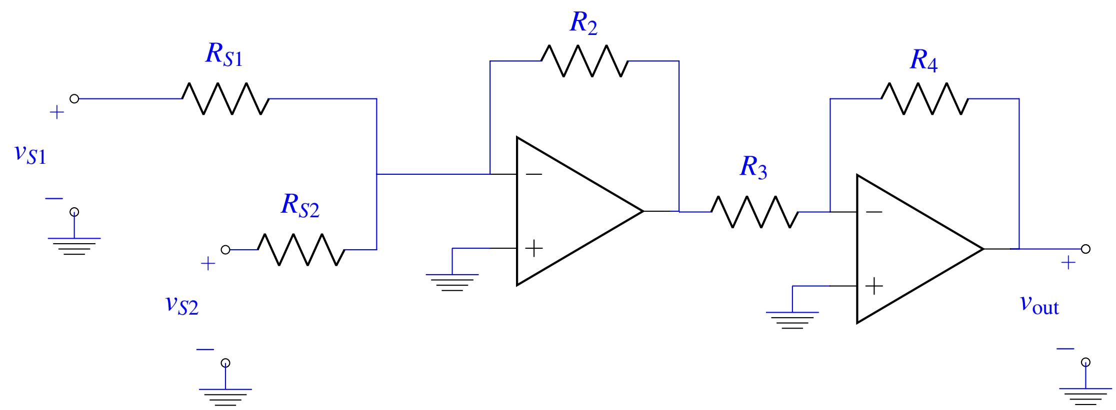
- $\frac{R_2}{R_{S1}} = \frac{1}{4}$

- $\frac{R_2}{R_{S2}} = 2$

One possible set of values is $R_2 = 2\,\text{k}\Omega$, $R_{S1} = 8\,\text{k}\Omega$, and $R_{S2} = 1\,\text{k}\Omega$, but any combination of resistors which satisfies the conditions listed above are valid solutions.

(d) Now suppose that you have a new AC unit that you want to use with your control inputs v_{S1} and v_{S2}. This unit, however, functions opposite to the previous unit; it responds to negative voltages. The higher the magnitude of the negative voltage, the stronger the AC runs.
Now design a circuit that *outputs a weighted sum of two control input voltages where both weights are positive*. Specifically, add another op-amp based circuit to your circuit in part (b), so that you invert the output of the circuit from part (b).

Solution:



Here, we add another inverting op-amp stage with a voltage gain of 1, and we can pick any equal-valued resistors for R_3 and R_4.

3. Island Karaoke Machine

You're stuck on a desert island and everyone is bored out of their minds. Fortunately, you have your EE16A lab kit with op-amps, wires, resistors, and your handy breadboard. You decide to build a karaoke machine. You recover one speaker from the crash remains and use your iPhone as your source. You know that many songs put instruments on either the "left" or the "right" channel, but the vocals are usually present on both channels with equal strength.

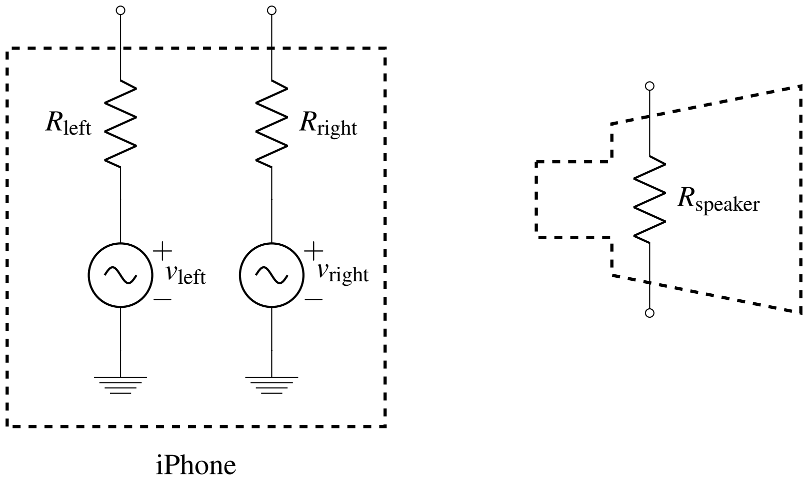
The Thevenin equivalent model of the iPhone audio jack and speakers is shown below. We assume that the audio signals v_{left} and v_{right} have equivalent source resistance of the left/right audio channels of $R_{\text{left}} = R_{\text{right}} = 3\,\Omega$. The speaker has an equivalent resistance of $R_{\text{speaker}} = 4\,\Omega$.

For this problem, we'll assume that the vocals are present on both left and right channels, but the instruments are only present on the right channel, i.e.

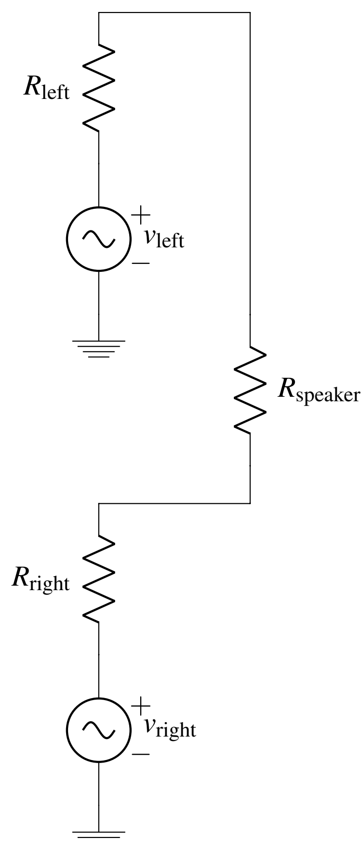
$$v_{\text{left}} = v_{\text{vocals}}$$
$$v_{\text{right}} = v_{\text{vocals}} + v_{\text{instrument}},$$

where the voltage source v_{vocals} can have values anywhere in the range of $\pm 120\,\text{mV}$ and $v_{\text{instrument}}$ can have values anywhere in the range of $\pm 50\,\text{mV}$.

What is the goal of a karaoke machine? The ultimate goal is to *remove* the vocals from the audio output. We're going to do this by first building a circuit that takes the left and right audio outputs of the smartphone and then calculates its difference. Let's see what happens.

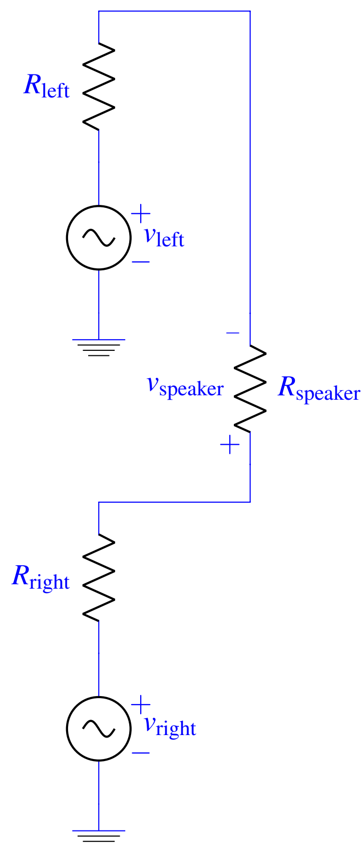


(a) One of your island survivors suggests the following circuit to do this. Calculate the voltage across the speaker as a function of v_{vocals} and $v_{\text{instruments}}$.
Does the voltage across the speaker depend on v_{vocals}? What do you think the islanders will hear – vocals, instruments, or both?



Solution:

Let's mark the voltage across the speaker, v_{speaker}, from bottom to top as in the figure:



We can apply the principle of superposition to solve for v_{speaker}. First, we solve for the voltage across the speaker when only v_{left} is on. Let's call this $v_{\text{speaker,left}}$. Notice that the circuit becomes a voltage divider. Therefore, we get

$$-v_{\text{speaker,left}} = \frac{v_{\text{left}} R_{\text{speaker}}}{R_{\text{speaker}} + R_{\text{left}} + R_{\text{right}}} = \frac{4v_{\text{vocals}}}{10} = 0.4v_{\text{vocals}},$$

giving

$$v_{\text{speaker,left}} = -0.4v_{\text{vocals}}.$$

Similarly, we solve for the voltage across the speaker when only v_{right} is on. Let's call this $v_{\text{speaker,right}}$. Again, notice that the circuit becomes a voltage divider. Therefore, we get

$$v_{\text{speaker,right}} = \frac{v_{\text{right}} R_{\text{speaker}}}{R_{\text{speaker}} + R_{\text{left}} + R_{\text{right}}} = \frac{4(v_{\text{vocals}} + v_{\text{instrument}})}{10} = 0.4(v_{\text{vocals}} + v_{\text{instrument}}).$$

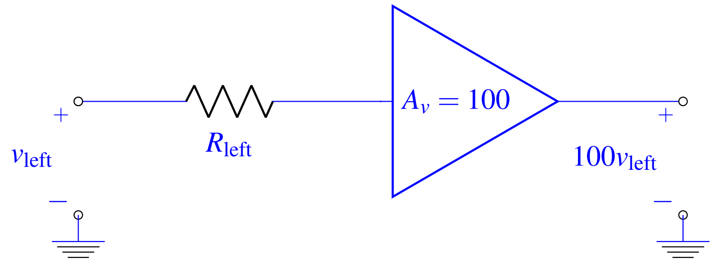
Superposition tells us that $v_{\text{speaker}} = v_{\text{speaker,left}} + v_{\text{speaker,right}} = 0.4v_{\text{instrument}} = 0.4 \cdot 50\,\text{mV} = 20\,\text{mV}$. What did you notice? The vocals got canceled out! The islanders will only hear the instruments, just as they wanted.

(b) We need to boost the sound level to get the party going. We can do this by *amplifying* both v_{left} and v_{right}. Keep in mind that we could use inverting or non-inverting amplifiers.
Let's assume, just for this part, that we have already implemented circuits that amplify v_{left} and v_{right} by some factor A_v (Consider $A_v = 100$ for this part). We now have two voltages, v_{Gl} and v_{Gr} that are $A_v \cdot v_{\text{left}}$ and $A_v \cdot v_{\text{right}}$ respectively. Use v_{Gl} and v_{Gr} to get $A_v \cdot v_{\text{instrument}}$ across R_{speaker}.
Solution:

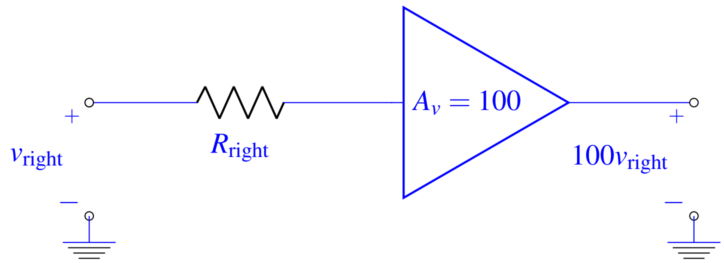
Note: In the following figures, we use a symbolic representation of the amplifier with gain $A_v = 100$. We will add in the corresponding amplifier circuit in part c.

We have three components of the circuit we want to build that we already know:

- The part of the circuit that amplifies v_{left} to $100v_{\text{left}}$, which we can draw as below:

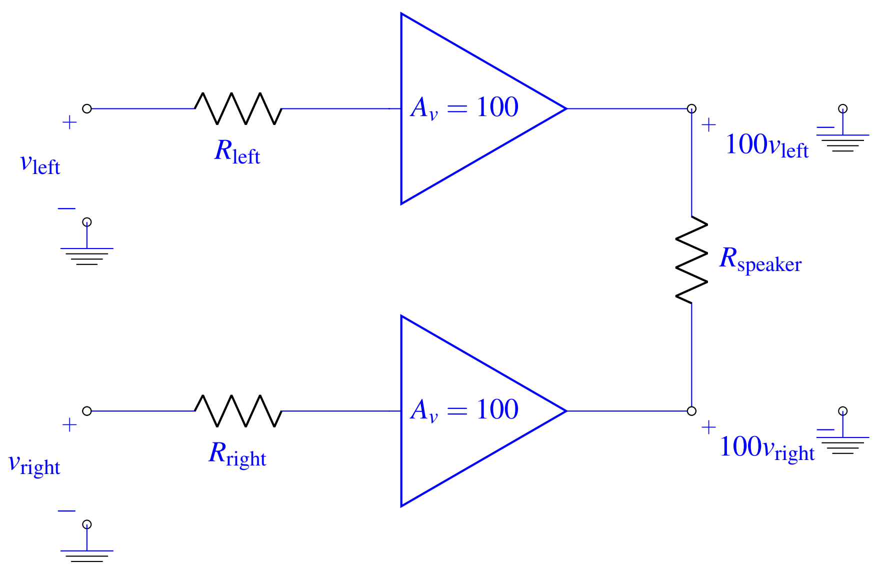


- The part of the circuit that amplifies v_{right} to $100v_{\text{right}}$, which we can draw as below:



- The speaker

If we want to take the difference of the two amplified outputs across the speaker, all we need to do is connect the output terminals of the first two components to the terminals of the speaker as shown below:



You can see this solution taking inspiration from part (a). Why do we get exactly $100(v_{\text{right}} - v_{\text{left}})$ across the speaker? Why does the voltage not divide as before?

To answer this, look back at the circuit for the non-inverting amplifier. If we solve for the Thevenin output resistance of this circuit, we will find that it is zero. Furthermore, the Thevenin voltage will be $100v_{\text{left}}$ (or $100v_{\text{right}}$). This implies that, no matter what R_{left} or R_{right} is, we are going to only see $100v_{\text{left}}$ or $100v_{\text{right}}$ at the output.

(c) Now, you want $\pm 2\,\text{V}$ across the speaker to get the party going. Using the scheme in part (b), design a circuit that takes in v_{left} and v_{right} and outputs an amplified version of $v_{\text{instrument}}$ across the speaker with the range of $\pm 2\,\text{V}$. You need to design both amplifiers with the right gain A_v to achieve this.

You can use up to two op-amps, and each of them can be inverting or non-inverting.

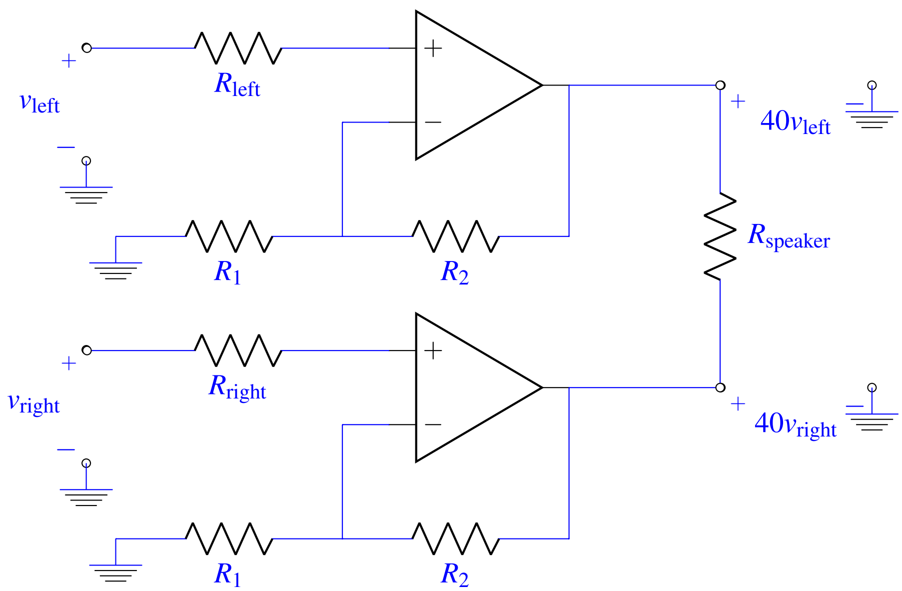
Solution:

Feed the non-ideal voltage source $\{v_{\text{left}}, R_{\text{left}}\}$ into a non-inverting amplifier with gain A_v and the non-ideal voltage $\{v_{\text{right}}, R_{\text{right}}\}$ into another non inverting amplifier with gain A_v. (We have a different gain from the previous part, which we need to determine.) Then connect the two outputs across R_{speaker} as shown in the previous part.

In this circuit, we will get $v_{\text{speaker}} = A_v \cdot v_{\text{instrument}}$. Since $v_{\text{instrument}}$ has a range of $\pm 50\,\text{mV}$, v_{speaker} will have a range of $\pm 50\,\text{mV} \cdot A_v = \pm 0.05 \cdot A_v \text{V}$. Now we need $\pm 0.05 \cdot A_v \text{V} = \pm 2$, i.e. $A_v = 40$.

Therefore, we want to design a non-inverting amplifier with voltage gain of 40.

We can use the circuit schematic from part (b), but now, we just need to design the non-inverting amplifier with op-amps to have gain 40. We get the equivalent circuit below:

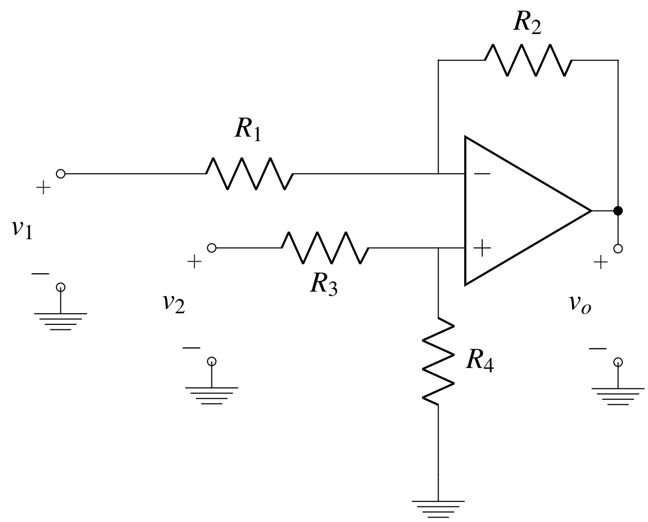


Now, we need to find R_1 and R_2.

$$A_v = 1 + \frac{R_2}{R_1}$$

Therefore, we can then choose any R_1 and R_2 such that $\frac{R_2}{R_1} = 39$. Note that there are multiple ways of choosing them. One such choice is $R_1 = 1\,\text{k}\Omega$ and $R_2 = 39\,\text{k}\Omega$, for instance.

(d) The trouble with the approach in part (c) is that multiple op-amps are required. Let's say you only have one op-amp with you. What would you do? One night in your dreams, you have an inspiration. Why not combine the inverting and non-inverting amplifier into one, as shown below!



If we set $v_2 = 0\,\text{V}$, what is the output v_o in terms of v_1? (This is the inverting path.)

Solution:

If we set $v_2 = 0\,\text{V}$, we would get $u_+ = 0\,\text{V}$. Applying the Golden Rules, we will get $u_- = u_+ = 0\,\text{V}$. Writing KCL at the $-$ terminal of the op-amp, we get

$$\frac{v_1 - 0}{R_1} = \frac{0 - v_{o,1}}{R_2},$$

which gives

$$v_{o,1} = \frac{-v_1 R_2}{R_1}.$$

(e) If we set $v_1 = 0\,\text{V}$, what is the output v_o in terms of v_2? (This is the non-inverting path.)

Solution:

If we set $v_1 = 0\,\text{V}$, we would get $u_+ = \frac{v_2 R_4}{R_3 + R_4} = u_-$. Writing KCL at the $-$ terminal gives

$$\frac{0 - u_-}{R_1} = \frac{u_- - v_{o,2}}{R_2},$$

which gives

$$v_{o,2} = u_-\left(1 + \frac{R_2}{R_1}\right) = v_2\left(\frac{R_4}{R_3 + R_4}\right)\left(1 + \frac{R_2}{R_1}\right).$$

(f) Now, determine v_o in terms of v_1 and v_2. (*Hint:* Use superposition.) Choose values for R_1, R_2, R_3 and R_4, such that the speaker has $\pm 2\,\text{V}$ across it.

Solution:

By the principle of superposition,

$$v_o = v_{o,1} + v_{o,2}.$$

If we set $v_1 = v_{\text{left}}$ and $v_2 = v_{\text{right}}$, we'd ideally want $v_o = -40v_1 + 40v_2$. We can choose R_1, R_2, R_3 and R_4, so that this happens.

How do we do this? Let's do this in steps. First, note that, looking for the expression for $v_{o,1}$, we'll want $\frac{R_2}{R_1} = 40$. Therefore, we can choose any values of R_2 and R_1, such that this happens. One such choice is $R_1 = 1\,\text{k}\Omega$ and $R_2 = 40\,\text{k}\Omega$. Then, plug that into the expression of $v_{o,2}$, and the condition we now want is

$$\left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right) = 40,$$

which gives us

$$\frac{R_4}{R_3+R_4} = \frac{40}{41}.$$

Thus, we need to choose R_3 and R_4. As before, we can choose these values in many ways. One such choice is $R_4 = 40\,\text{k}\Omega$ and $R_3 = 1\,\text{k}\Omega$.

Note: Keep in mind that, for this problem, we actually assumed that $v_1 = v_{\text{left}}$ and $v_2 = v_{\text{right}}$, which would mean that we are ideally connecting v_{left} and v_{right} as inputs. However, in reality, we're actually connecting the outputs from the iPhone as inputs. This means that R_{left} and R_{right} will also actually affect the output.

With this effect, we will actually get

$$v_o = -\frac{v_1 R_2}{R_{1,eq}} + v_2\left(\frac{R_4}{R_{3,eq}+R_4}\right)\left(1+\frac{R_2}{R_{1,eq}}\right),$$

where $R_{1,eq} = R_1 + R_{\text{left}}$ and $R_{3,eq} = R_3 + R_{\text{right}}$.

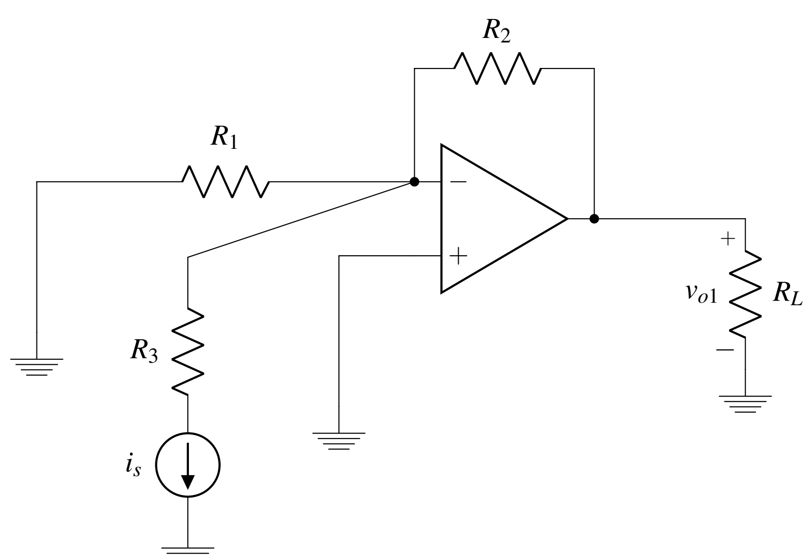
Therefore, we can just *fold in* the effect of R_{left} and R_{right} into these. For instance, we want to set $R_{3,eq} = 1\,\text{k}\Omega$. Now, we can actually make $R_3 = R_{3,eq} - 3\,\Omega = 997\,\Omega$ and $R_1 = R_{1,eq} - 3\,\Omega = 997\,\Omega$.

Give yourself full credit even if you didn't notice this, but keep this in mind!

Bonus: Can you now see why we wanted to keep R_1 and R_3 in the order of $\text{k}\Omega$ or larger?

4. Amplifier with Multiple Inputs

(a) Use the Golden Rules to find v_{o1} for the circuit below.



Solution:

Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is 0. The voltage drop across R_1

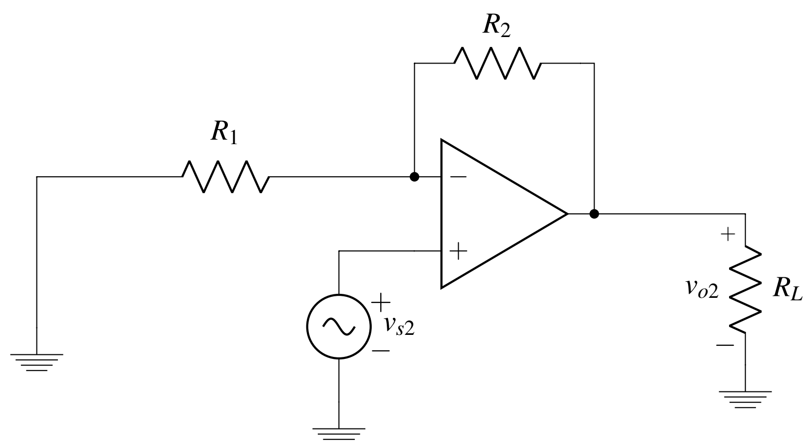
is 0 and no current flows through it. In addition, no current flows into the op-amp from the negative terminal due to its infinite input resistance (the negative terminal is connected to an "open" circuit). By KCL at the negative terminal of the op-amp, this means that the current going through R_3 and R_2 is i_s. Taking the positive terminal of R_2 to be on the right, the voltage drop across R_2 is v_{o1}. By Ohm's law, we conclude:

$$\frac{v_{o1}}{R_2} = i_s$$

Rearranging we get:

$$v_{o1} = i_s \cdot R_2$$

(b) Use the Golden Rules to find v_{o2} for the circuit below.



Solution:

Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $V^- = v_{s2}$. In addition, since no current can enter into the negative terminal of the op-amp, R_1 and R_2 are in series. This means that the voltage at the negative terminal of the op-amp can be expressed in terms of v_{o2} using the voltage divider formula:

$$v^- = v_{o2}\left(\frac{R_1}{R_1 + R_2}\right)$$

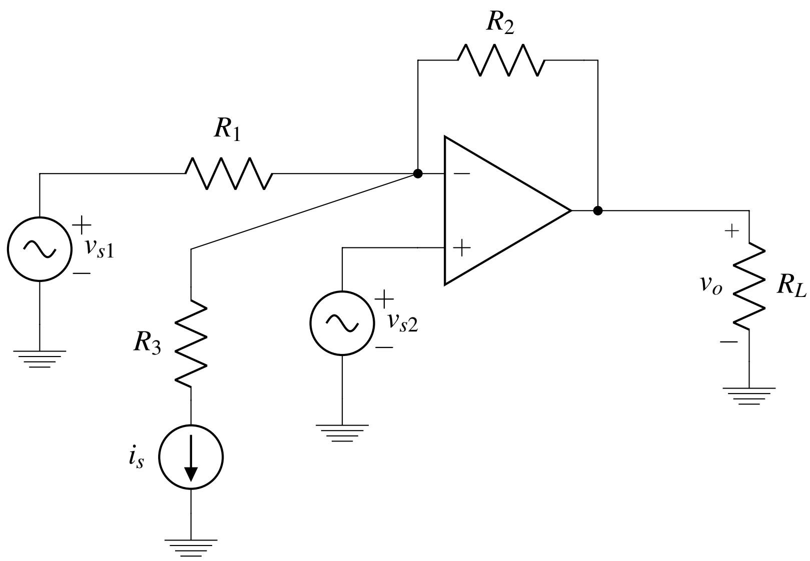
We also know that $v^- = v_{s2}$ and conclude:

$$v_{s2} = v_{o2}\left(\frac{R_1}{R_1 + R_2}\right)$$

After rearranging, we have:

$$v_{o2} = v_{s2}\left(\frac{R_2}{R_1} + 1\right)$$

(c) Use the Golden Rules to find the output voltage v_o for the circuit shown below.



Solution:

Applying the Golden Rules we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $v^- = v_{s2}$. Then we write a KCL equation at the node connected to the minus terminal of the op-amp (recalling that no current flows into or out of the op-amp's terminals). All currents are defined as flowing out of the node:

$$i_{R_1} + i_{R_2} + i_{R_3} = 0$$

Because of the independent current source, we know:

$$i_{R_3} = i_s$$

By Ohm's law, we know:

$$i_{R_1} = \frac{v^- - v_{s1}}{R_1}$$

and

$$i_{R_2} = \frac{v^- - v_o}{R_2}$$

Then, substituting back into the original KCL equation, we have:

$$\frac{v^- - v_{s1}}{R_1} + \frac{v^- - v_o}{R_2} + i_s = 0$$

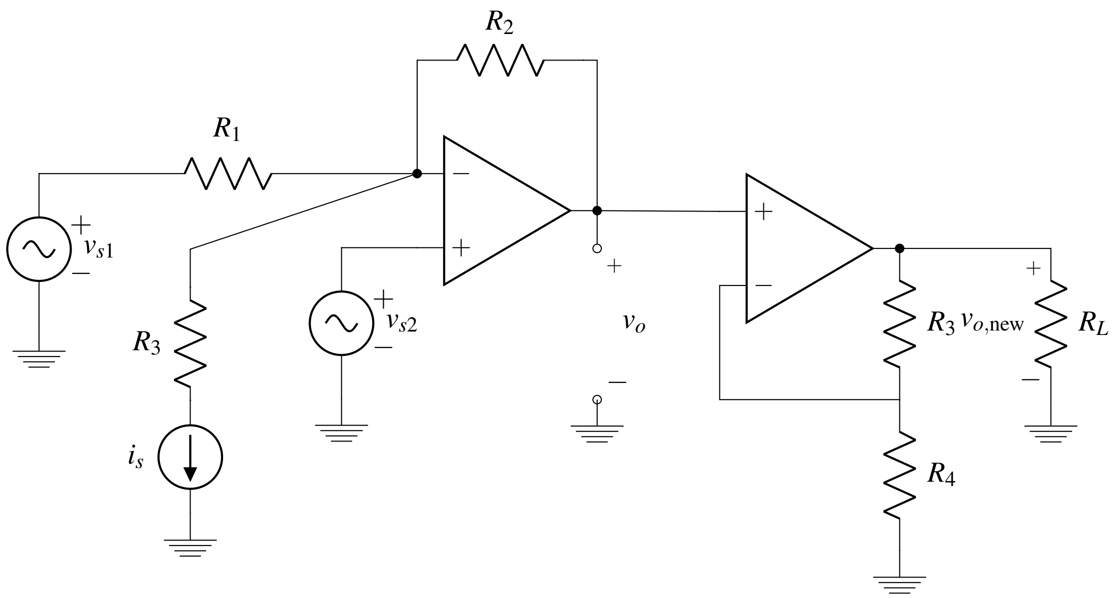
and substituting $v^- = v_{s2}$, we have:

$$\frac{v_{s2} - v_{s1}}{R_1} + \frac{v_{s2} - v_o}{R_2} + i_s = 0$$

which we rearrange to find v_o, giving:

$$v_o = v_{s2}\left(1 + \frac{R_2}{R_1}\right) + i_s \cdot R_2 - \left(\frac{R_2}{R_1}\right) v_{s1}$$

(d) Now add a second stage as shown below. What is $v_{o,\text{new}}$? Does v_o change between part (c) and this part? Does the voltage $v_{o,\text{new}}$ depend on R_L?



Solution:

Adding the second stage does not change the voltages in the first stage. This is because the circuit connected to the positive and negative terminals of the first stage op-amp "sees" an open circuit/infinite input resistance in the op-amp.

Call the output voltage of the first stage v_{o1}. Then it remains unchanged from part (c).

$$v_{o1} = -\left(\frac{R_2}{R_1}\right) v_{s1} + i_s \cdot R_2 + v_{s2}\left(\frac{R_2+R_1}{R_1}\right)$$

By the Golden Rules, the negative terminal of the second op-amp must have the same voltage as the plus terminal, which is v_{o1}. No current can flow into the negative terminal, so R_3 and R_4 are in series and have the same current, so we know:

$$\frac{v_{o1}}{R_4} = \frac{v_o - v_{o1}}{R_3}$$

Therefore:

$$v_o = \left(\frac{R_3+R_4}{R_4}\right) v_{o1} = \frac{R_3+R_4}{R_4}\left(-\frac{R_2}{R_1} \cdot v_{s1} + i_s \cdot R_2 + v_{s2} \cdot \frac{R_2+R_1}{R_1}\right)$$

5. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.

