EECS 16A Designing Information Devices and Systems I
Spring 2019 Homework 11

This homework is due April 19, 2019, at 23:59.
Self-grades are due April 23, 2019, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw11.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.

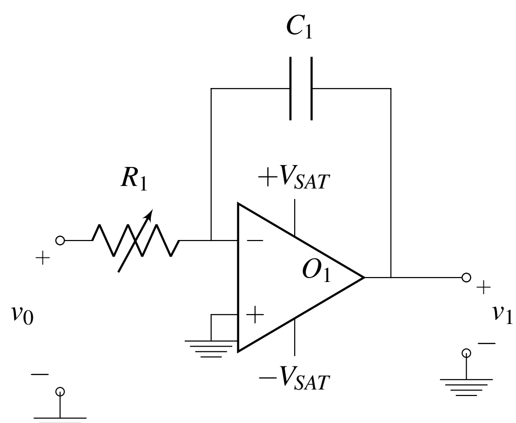
  If you do not attach a PDF "printout" of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

Submit the file to the appropriate assignment on Gradescope.

1. Jumpbot

In this problem, you will be designing circuits allowing a robot named Jumpbot to execute a set of commands that will be described below. Specifically, the output voltages produced by your circuits are interpreted by Jumpbot as setting its vertical position in meters in free space (both positive and negative values will be used). You will be generating an oscillating triangular waveform with a controllable time period.

(a) One of the circuit blocks you will use to generate the triangular waveform is the integrator. An integrator integrates the input signal. For the circuit given below express v_1 in terms of R_1, C_1, and v_0. You may assume the capacitor C_1 has $0V$ across it at time $t = 0$.
Hint: You will have to apply KCL, and the current flowing through a capacitor is given by $I = C\frac{dV}{dt}$.

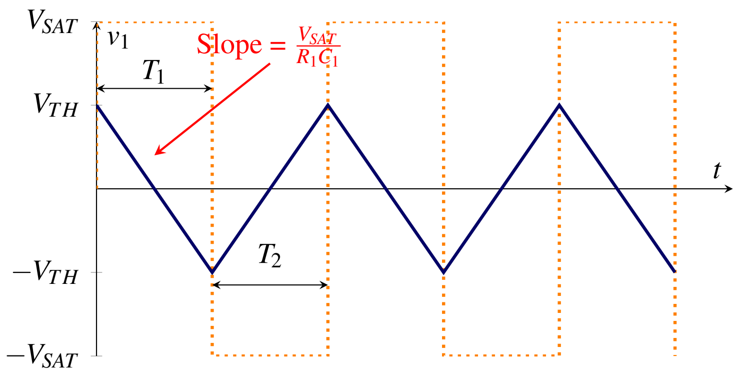


Solution:

Let's write the KCL equation at V^- assuming all currents are leaving.

$$\begin{aligned} i_{R_1} &= -i_{C_1} \\ i_{C_1} &= C_1 \frac{d(0 - v_1(t))}{dt} \\ \frac{0 - v_0}{R_1} &= C_1 \frac{d(v_1(t) - 0)}{dt} \\ -\frac{v_0}{R_1 C_1} &= \frac{dv_1(t)}{dt} \\ v_1(t) &= -\frac{1}{R_1 C_1} \int_0^t v_0 d\tau \end{aligned}$$

(b) Suppose for a specific v_0, shown by the dotted orange line below, v_1 looks as shown by the blue line. Derive an expression for T_1 and T_2 as a function R_1, C_1, V_{TH}, and V_{SAT}.



Solution:

Note that the indicated slope is the magnitude of the slope.

For T_1, using the waveform of v_1:

$$\begin{aligned} \frac{-V_{TH} - V_{TH}}{T_1} &= -\frac{V_{SAT}}{R_1 C_1} \\ T_1 &= R_1 C_1 \frac{2V_{TH}}{V_{SAT}} \end{aligned}$$

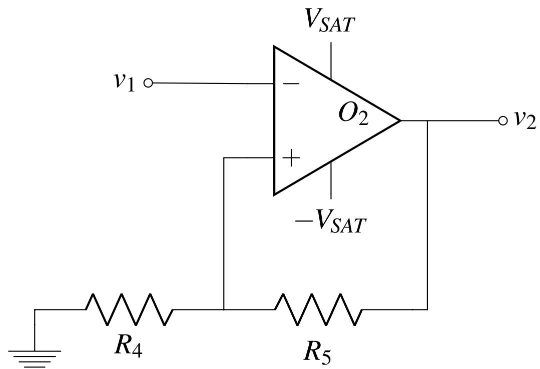
Similarly for T_2, using the waveform of v_1:

$$\begin{aligned} \frac{V_{TH} - (-V_{TH})}{T_2} &= \frac{V_{SAT}}{R_1 C_1} \\ T_2 &= R_1 C_1 \frac{2V_{TH}}{V_{SAT}} \end{aligned}$$

(c) We have a circuit that generates a triangle wave from a square wave. However, we need to create the initial signal (v_0) that helped us to create the triangular waveform (v_1). For the circuit below, draw the waveform (v_2) if we use v_1 from part (b) as the input. Now, draw the waveform (v_2) if we use $-v_1$. Which v_2 (v_1 as input or $-v_1$ as input) matches v_0 from part (b)?

$$\begin{aligned} +V_{TH} &= \frac{R_4}{R_4 + R_5} V_{SAT} \\ -V_{TH} &= \frac{R_4}{R_4 + R_5} (-V_{SAT}) \end{aligned}$$

Hint: read section 9.8.1 of the book "Electronics" by Prof. Ali Niknejad, you can find it on the Resources section on the class website.

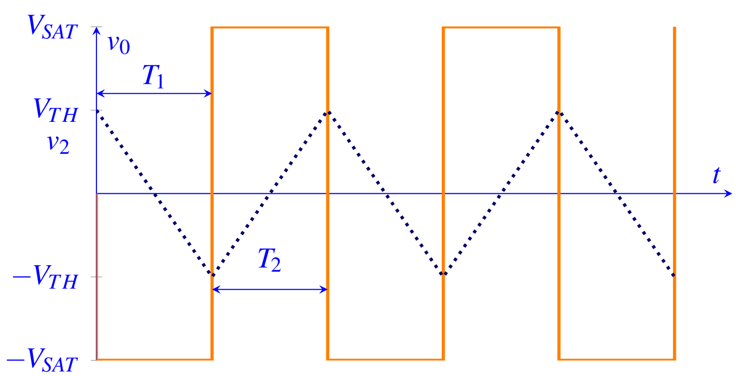


Solution:

The circuit above is not in negative feedback. However, it does not act quite like a comparator either because it has two different thresholds for when the output of the opamp flips. The opamp is then in one of two states, with it's output at V_{SAT} and with it's output at $-V_{SAT}$. Since the opamp itself is in one of two states, regardless of the input, we have four cases to analyze, enumerated below.

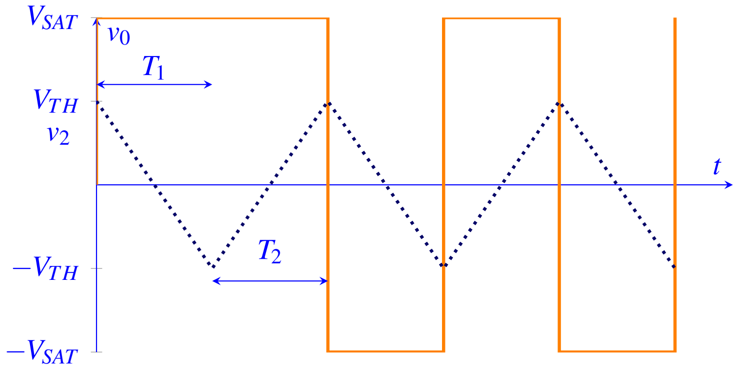
Case 1: Assume $v_1 = v_1$, and that at time $t = 0$, the output of the opamp is at $-V_{SAT}$.

Since the output is at $-V_{SAT}$, we know $V^+ = -V_{TH}$. Eventually, v_1 will fall slightly below $-V_{TH}$ and the output of the opamp will flip to $+V_{SAT}$. This is shown in the figure below.

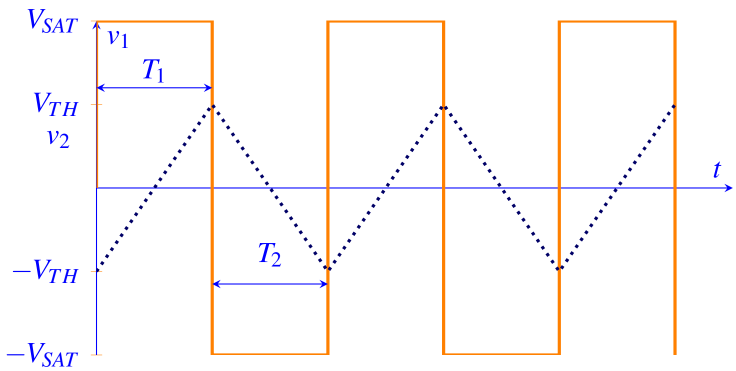


Case 2: Assume $v_1 = v_1$, and that at time $t = 0$, the output of the opamp is at V_{SAT}.

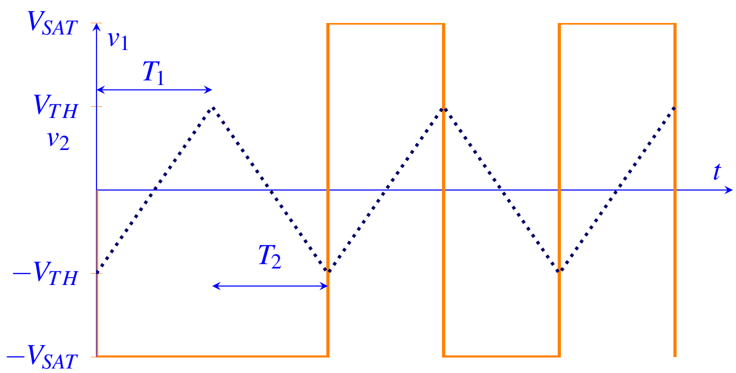
Since the output is at V_{SAT}, we know $V^+ = V_{TH}$. Eventually, v_1 will rise above V_{TH} and the output of the opamp will flip to $-V_{SAT}$. However this will happen on the next cycle. This is shown in the figure below.



Case 3: Assume $v_1 = -v_1$, and that at time $t = 0$, the output of the opamp is at V_{SAT}.
Since the output is at V_{SAT}, we know $V^+ = V_{TH}$. Eventually, v_1 will rise above V_{TH} and the output of the opamp will flip to $-V_{SAT}$. This is shown in the figure below.



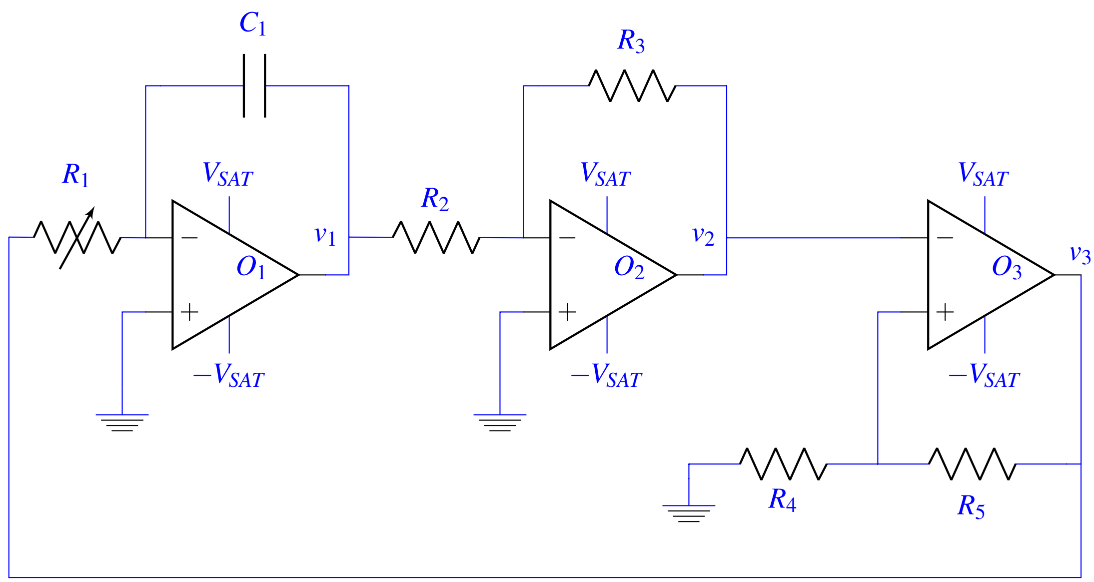
Case 4: Assume $v_1 = -v_1$, and that at time $t = 0$, the output of the opamp is at $-V_{SAT}$.
Since the output is at V_{SAT}, we know $V^+ = V_{TH}$. Eventually, v_1 will rise above V_{TH} and the output of the opamp will flip to $-V_{SAT}$. Once again, this flip will happen after a cycle though. This is shown in the figure below.



If $-v_1$ is used as the input, the output matches v_0 from part (a), if we initially hold the output of the opamp to V_{SAT}.
Because of the recursive nature of this circuit we were required to analyze the circuit in all four possibilities. You were not expected to analyze the circuit in all the unique states. In the real world, people take great care to make sure to start the circuit into a "known" state, where the outputs and inputs are set so that correct operation occurs.

(d) Now let's put it all together. The circuit from part (a) generates a triangle wave (v_1) from a square wave (v_0). The circuit above takes an input triangle wave and creates a square wave. Connect the two circuits together so that the circuit keeps generating a triangle wave on it's own. You will use the circuits from part (a) and part (c), in addition you can use any opamps and resistors.

Solution:



We've added an inverting amplifier, O2, with a gain of 1, so that v_2 has the polarity that we want as we saw in part (c).

(e) In your circuit, if $\pm V_{SAT} = \pm 10\,\text{V}$, $C_1 = 0.01\,\text{mF}$, and $R_4 = 10\,\text{k}\Omega$, find the values for R_1 and R_5, so that the jumpbot jumps with 10 V peak-to-peak amplitude ($\pm V_{TH} = \pm 5\,\text{V}$) with 1 kHz frequency (period = 1 / frequency).

Solution:

10 V peak-to-peak amplitude means $\pm V_{TH} = \pm 5\,\text{V}$. Therefore,

$$V_{TH} = \frac{R_4}{R_4 + R_5} V_{SAT}$$
$$5\,\text{V} = \frac{10\,\text{k}\Omega}{10\,\text{k}\Omega + R_5} 10\,\text{V}$$
$$R_5 = 10\,\text{k}\Omega$$

To find R_1, we can use the relationship derived in part (b). However, here $T_1 = 0.5\,\text{ms}$ because $T_1 + T_2 = T = 1\,\text{ms}$ (oscillation frequency 1 kHz).

$$T_1 = R_1 C_1 \frac{2V_{TH}}{V_{SAT}}$$
$$0.5\,\text{ms} = R_1 \cdot 0.01\,\text{mF} \cdot \frac{2 \cdot 5\,\text{V}}{10\,\text{V}}$$
$$R_1 = 50\,\Omega$$

2. From FA18 Final: A Tool to Help Compute All the Fun You're Having

Starting in the 1950-1960's, the world began a series of missions to get to the moon. Back then though, computers took up entire rooms and could never fit on a spaceship! They needed a better way of computing values on the fly using what they did have: analog circuits.

In class you have seen circuits that can amplify, add, subtract and even integrate voltages, but we're missing a key ingredient to make computational circuits: **multiplication**. Although making a multiplier circuit is not as straightforward as we would like it to be, we can definitely use our now fully-developed EE16A skills to make this a reality.

You may find the following formulas useful throughout the problem:

$$\ln(e^a) = a \qquad \ln(a) + \ln(b) = \ln(ab)$$
$$e^{\ln(a)} = a \qquad \ln(a) - \ln(b) = \ln(\tfrac{a}{b})$$

(a) To start off, your TA Nick suggests that you first draw a block diagram that would do what you want, and then worry about how to implement it later. He starts you off with the following incomplete block diagram and blocks.
You are allowed to use any amount of the following logarithmic, exponential and summer blocks:

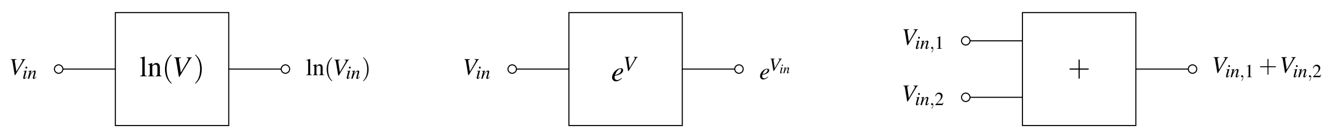
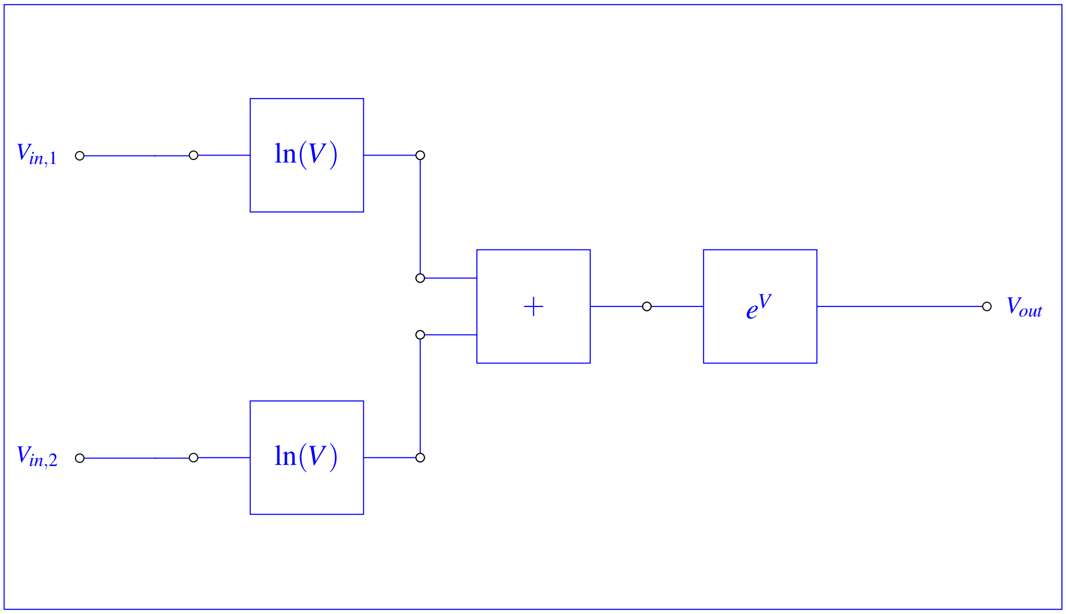


Figure 1: Function blocks you can use

Finish the **block diagram** so that the output is equal to $V_{out} = V_{in,1}V_{in,2}$. **Also provide a mathematical justification for why your block diagram works for full credit!**

Solution: We want to use the hints at the beginning of the problem. In order to get two voltages multiplied together, we need to get them both inside the same logarithm, then cancel the logarithm by using an exponential block. See the diagram below:



the voltages going into the summer are:

$$\ln(V_{in,1})$$

and,

$$\ln(V_{in,2})$$

The output of the summer is therefore:

$$\ln(V_{in,1}) + \ln(V_{in,2})$$

Using the fact that $\ln(a) + \ln(b) = \ln(ab)$ simplifies this expression into

$$\ln(V_{in,1}V_{in,2})$$

And lastly, passing through the e block gives:

$$e^{\ln(V_{in,1}V_{in,2})}$$

Using the fact that $e^{\ln(a)} = a$ gives:

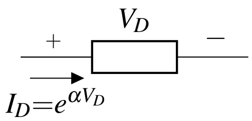
$$V_{in,1}V_{in,2}$$

as desired.

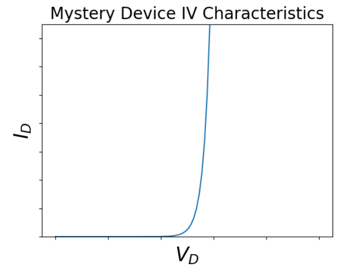
We made a multiplier circuit! Go us!

While you were making your block diagram, your friend Vlad was watching over your shoulder. With his near-infinite knowledge of circuits, he predicts what you are trying to do and gives you a "magic device" that he claims will help you out.

This device is shown below:



This device has the following IV characteristics:



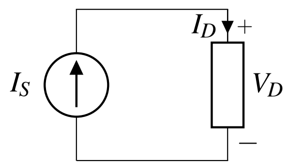
Numerically, this can be stated as:

$$I_D = e^{\alpha V_D}$$

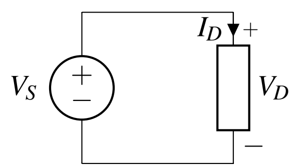
where α is some constant and V_D is the voltage across the device as in the above figure.
IMPORTANT: Note the passive sign convention.

(b) Your friend Gireeja sees you looking confused, and reads what you've done so far. She decides to help push you in the right direction. She reminds you that if you know how a device reacts when you put current through it or apply a voltage across it, then you are in a good spot to understand how it would behave in any circuit.

i. **In the following circuit, express the voltage across the device, V_D, as a function of the source current, I_S.**



ii. **In the following circuit, express the current through the device, I_D, as a function of the source voltage, V_S.**



Solution:

i. From the above IV characteristics we know that:

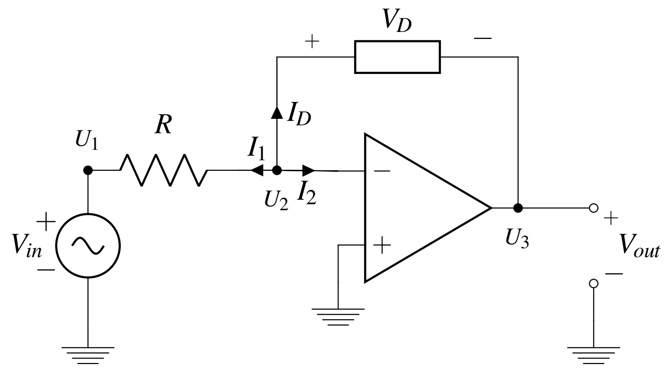
$$I_S = I_D = e^{\alpha V_D}$$
$$\rightarrow \ln(I_S) = \alpha V_D$$
$$\rightarrow V_D = \frac{1}{\alpha}\ln(I_S)$$

Putting a fixed current through the device causes a voltage to appear across the device that is related to the natural log of the current.

ii. This follows directly from the given IV relationship above $V_D = V_S$ therefore,
$I_D = e^{\alpha V_S}$

(c) By now we have all the tools necessary to make this a reality. Unfortunately, as you may remember from the touchscreen lab, ideal current sources don't exist. As such, we will use an op-amp to create the behavior we are interested in. Let's analyze the circuit below in steps.

Hint: *Your answers for some parts of this problem will not perfectly match with the ideal scenario in part (a). Don't be afraid of getting different answers! After all, a block diagram is just a starting point.*



i. **write V_D in terms of V_{out} only. You may assume the circuit is in negative feedback.** **Solution:** We know the following three things:

$$V_D = U_2 - U_3$$
$$V_2 = U^- - 0$$
$$V_{out} = U_3 - 0$$

Since we can assume negative feedback, then we know that $U^+ = U^-$. Since $U^+ = 0$, then $U^- = 0$. Substituting into the above node equations,

$$V_D = U^- - U_3$$
$$V_D = 0 - U_{out}$$
$$\rightarrow V_D = -V_{out}$$

ii. **Now write out the KCL expression at the U_2 node in terms of the three currents I_D, I_1 and I_2.** **Solution:** A common mistake was to attempt to use the inverting amplifier equation, or trying to find the resistance of the magic device. The inverting amplifier was derived under the assumption that both devices were resistors, which does not hold if that is violated. Also only resistors follow V=IR, so the magic device cannot fit into that.
Now for the actual solution: By KCL:

$$I_1 + I_D + I_2 = 0$$

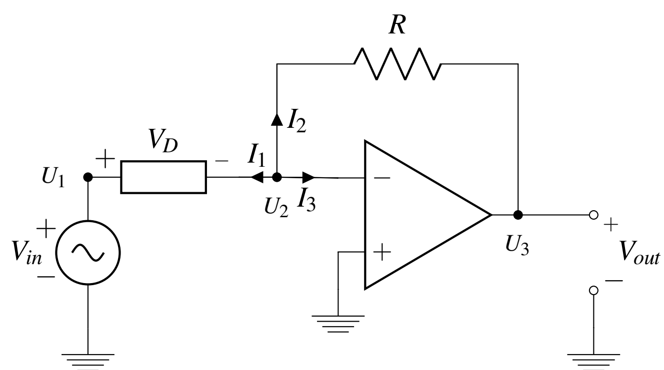
iii. **Use the IV relations for each device and the KCL expression above to find V_{out} as a function of V_{in}.**
Solution: We immediately know that $I_2 = 0$ from the golden rules. Substituting $V = IR$ for I_1 and the mystery device IV relationship for I_D gives:

$$\frac{U_2 - U_1}{R} + e^{\alpha(U_2 - U_3)} + 0 = 0$$

We know $U_2 = U^- = U^+ = 0$ from the golden rules. We also know $U_1 = V_{in}$. From part (i) we also know that $U_2 - U_3 = -V_{out}$, thus

$$\frac{0 - V_{in}}{R} + e^{-\alpha V_{out}} = 0$$
$$e^{-\alpha V_{out}} = \frac{V_{in}}{R}$$
$$-\alpha V_{out} = \ln(\frac{V_{in}}{R})$$
$$V_{out} = -\frac{1}{\alpha}\ln(\frac{V_{in}}{R})$$

(d) Just one more piece left to analyze. You may assume the circuit is in negative feedback. **In the circuit below, write V_{out} as a function of V_{in}.**
Hint: *Note the direction of the voltage drop, V_D. Be careful when writing your KCL expressions to account for this!*



Solution: The strategy here is very similar to part (c):
KCL tells us:

$$I_1 + I_2 + I_3 = 0$$

but we know that $I_3 = 0$ by the golden rules. The current through the mystery device must enter the positive side, so I_1 is in the opposite direction of the defined I_D, so $I_1 = -I_D$. Therefore:

$$-I_D = -I_2$$
$$I_D = I_2$$

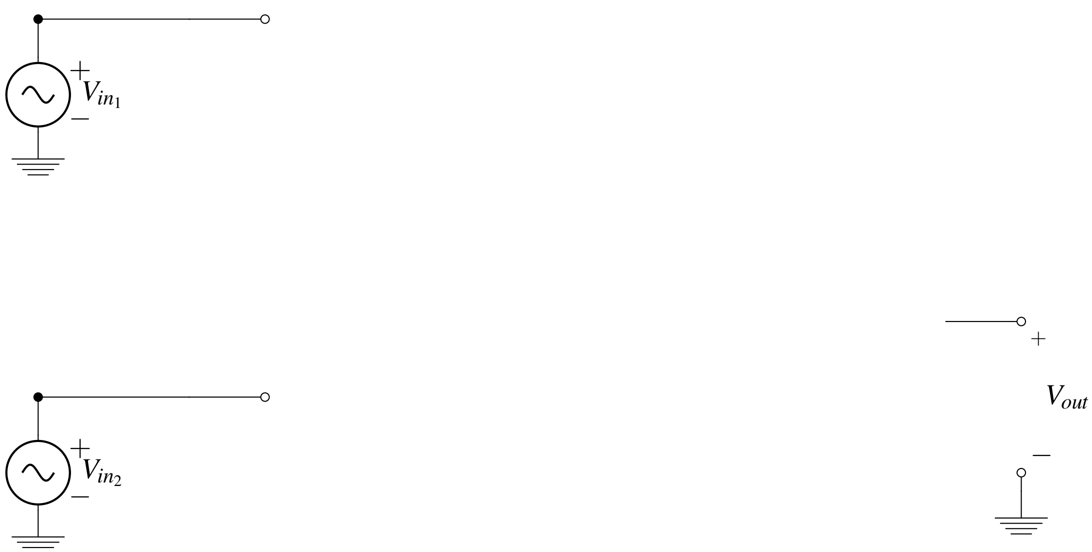
Substitute in IV relationships and simplify:

$$e^{\alpha(U_1 - U_2)} = \frac{U_2 - U_3}{R}$$
$$e^{\alpha V_{in}} = \frac{0 - V_{out}}{R}$$

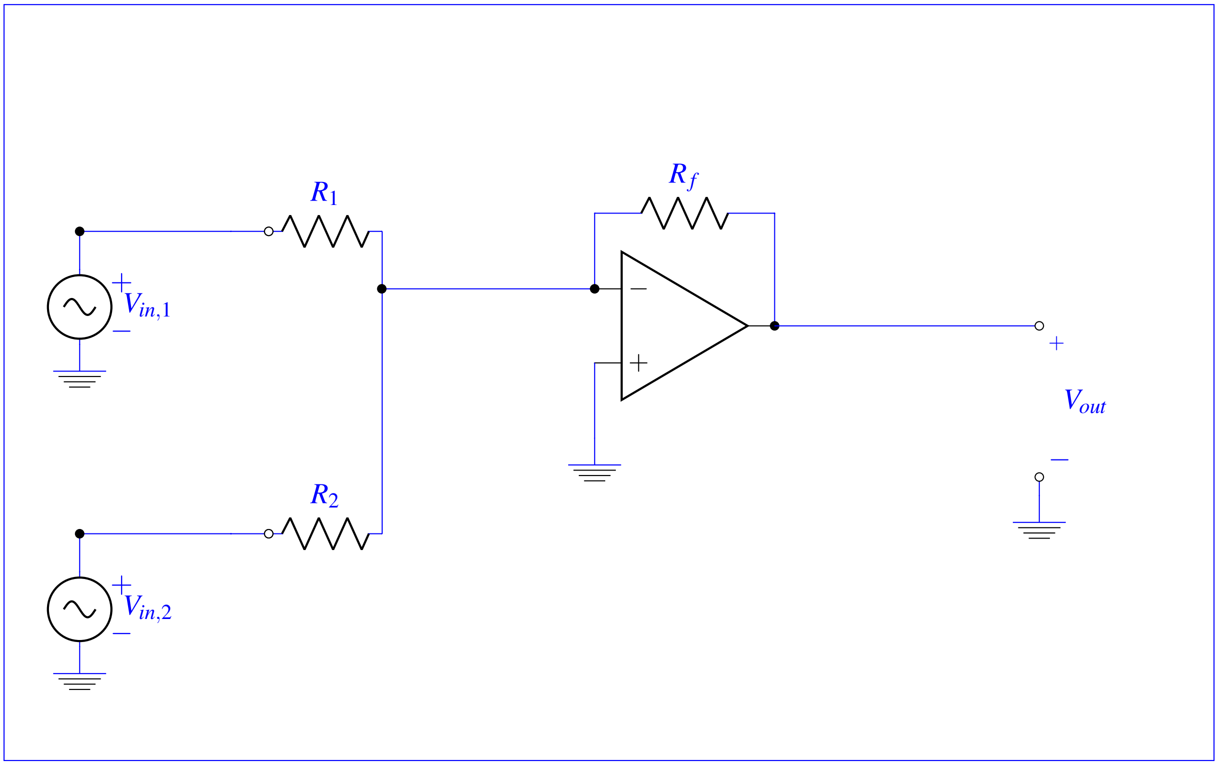
The first line uses the fact that $V_D = U_1 - U_2$. The second line follows from the golden rules. We know $U_2 = U^- = U^+ = 0$ and we also know $U_1 = V_{in}$ and $V_{out} = U_3 - 0$. Continuing:

$$V_{out} = -Re^{\alpha V_{in}}$$

(e) Given the polarity of circuits in parts (c) and (d), to put everything together, you need to implement **an inverting summer circuit, where $V_{out} = -(V_{in_1} + V_{in_2})$.**
You may use up to 1 op-amp (you do not need to label the power supplies) and as many resistors as you wish. **Any resistors used must be assigned a value. You must mathematically justify your circuit's behavior for full credit.**



Solution: There are many ways to implement summation with a negation. A non-inverting summer into an inverting amplifier, passive summer circuit with inverting amplifier, etc. Many of these topologies however, require multiple op-amps or do not output exactly $-(V_{in,1}+V_{in,2})$. Since we can only use a single op-amp, we must use the inverting summer configuration.



The output of this amplifier is

$$V_{out} = -R_f\left(\frac{V_{in,1}}{R_1}+\frac{V_{in,2}}{R_2}\right)$$

We want no fractional terms, which tells us we need to figure out how to cancel the resistor terms. If we make $R_1 = R_2 = R$ then the overall output becomes:

$$V_{out} = -\frac{R_f}{R}(V_{in,1}+V_{in,2})$$

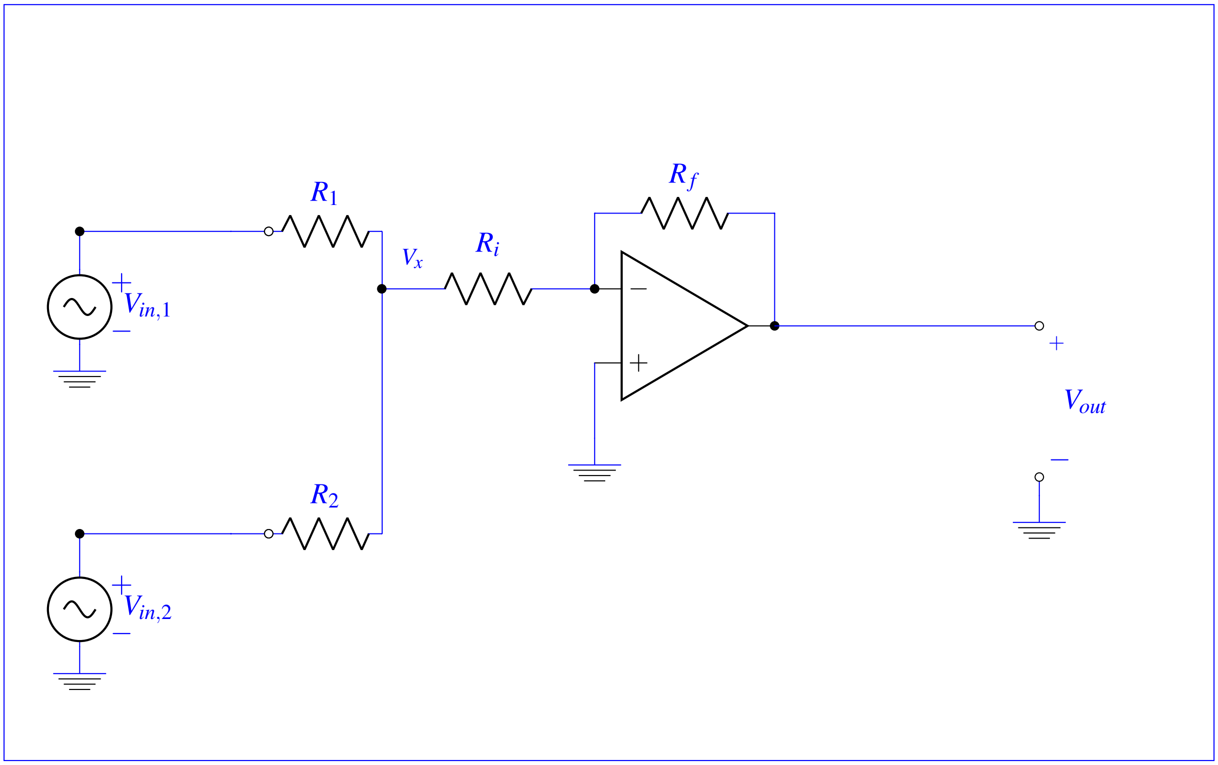
We need this fraction to become 1, so we pick $R_f = R$. The actual value is arbitrary, so we'll pick $1k\Omega$ for all resistors. Then we get

$$V_{out} = -(V_{in,1}+V_{in,2})$$

as desired.

ALTERNATE SOLUTION:

A common pattern that students used is a voltage summer feeding directly into an inverting amplifier, as shown below



This can also work, but is tricky since the inverting op amp loads the summer circuit. The common thought process was to notice that the passive summer circuit can give $\frac{1}{2}(V_{in,1} + V_{in,2})$ if $R_1 = R_2 = R$. Then students tried to cancel out this factor of 0.5 with an inverting amplifier with a gain of 2. But the amplifier loads the output of the summer circuit, which needs to be accounted for.

Let's first assume that every resistor except R_f is equal to some resistance R. Doing superposition, first ground $V_{in,2}$. At the node connecting R_1 to R_i (call it V_x) you will have:

$$V_{x,1} = V_{in,1}\frac{R||R}{R+R||R}$$
$$\rightarrow V_{x,1} = \frac{1}{3}V_{in,1}$$

The resistors are "parallel" because the inverting terminal is a virtual ground. Therefore R_2 is parallel to R_i and the formula follows from voltage divider.

Now canceling $V_{in,1}$ gives:

$$V_{x,2} = V_{in,2}\frac{R||R}{R+R||R}$$
$$\rightarrow V_{x,2} = \frac{1}{3}V_{in,2}$$

For the same reason as before. Therefore by superposition, $V_x = V_{x,1} + V_{x,2} = \frac{1}{3}(V_{in,1} + V_{in,2})$. Now do KCL at the - terminal:

$$\frac{0-V_x}{R} + \frac{0-V_{out}}{R_f} = 0$$
$$V_{out} = -\frac{R_f}{3R}(V_{in,1} + V_{in,2})$$

So $R_f = 3R$ to get the desired answer. A common mistake was to use $R_f = 2R$ which you would get if you do not assume that the op amp loads the circuit. Due to the presence of R_i the typical answer changes.

And that's it! Now that you have all the individual parts, you can string them together and start selling calculators. You may need to make a few small changes to your original design, but you can do it!

3. APS Prelab - Cross Correlation

Complete the included iPython notebook to prepare for the upcoming APS lab, prob11.ipynb.

Solution: See ipynb solution file

4. Midterm Problem 3

Redo Midterm Problem 3.

(a) **Solution:** See midterm solutions.

(b) **Solution:** See midterm solutions.

5. Midterm Problem 4

Redo Midterm Problem 4.

(a) **Solution:** See midterm solutions.

(b) **Solution:** See midterm solutions.

(c) **Solution:** See midterm solutions.

6. Midterm Problem 5

Redo Midterm Problem 5.

(a) **Solution:** See midterm solutions.

(b) **Solution:** See midterm solutions.

7. Midterm Problem 6

Redo Midterm Problem 6.

(a) **Solution:** See midterm solutions.

(b) **Solution:** See midterm solutions.

8. Midterm Problem 7

Redo Midterm Problem 7.

(a) **Solution:** See midterm solutions.

(b) **Solution:** See midterm solutions.

9. Midterm Problem 8

Redo Midterm Problem 8.

(a) **Solution:** See midterm solutions.

(b) **Solution:** See midterm solutions.

(c) **Solution:** See midterm solutions.

(d) **Solution:** See midterm solutions.

(e) **Solution:** See midterm solutions.

10. Midterm Problem 9

Redo Midterm Problem 9.

(a) **Solution:** See midterm solutions.

(b) **Solution:** See midterm solutions.

11. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.