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Lecture 5 - Module 2

EE16A

Today: * Superposition } Circuit jedi

Note 15 * Equivalence } techniques

Goal: Want to design interesting systems
- Need the tools that provide insight

Reminder: Circuit analysis objective:

Find $\vec{x} = \begin{bmatrix} I_1 \\ \vdots \\ I_m \\ V_1 \\ \vdots \\ V_k \end{bmatrix}$ for some ckt matrix A and some stimulus vector

$$\vec{b} = \underbrace{\begin{bmatrix} I_{s1} \\ \vdots \\ I_{sl} \\ V_{s1} \\ \vdots \\ V_{s_{m+k-l}} \end{bmatrix}}_{} \Leftarrow \text{independent sources}$$

$$\underbrace{A\vec{x} = \vec{b}}_{\text{formulation}} \Rightarrow \underbrace{\vec{x} = A^{-1}\vec{b}}_{\text{solution}}$$

$$I_i = \alpha_1 I_{s1} + \dots + \alpha_l I_{sl} + \dots + \alpha_{m+k} V_{s_{m+k-l}}$$

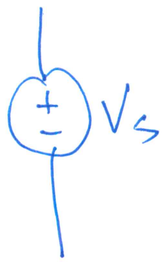
and

$$\underbrace{V_j}_{\text{lin. combination of sources}} = \beta_1 I_{s1} + \dots + \beta_{m+k} V_{s_{m+k-l}}$$

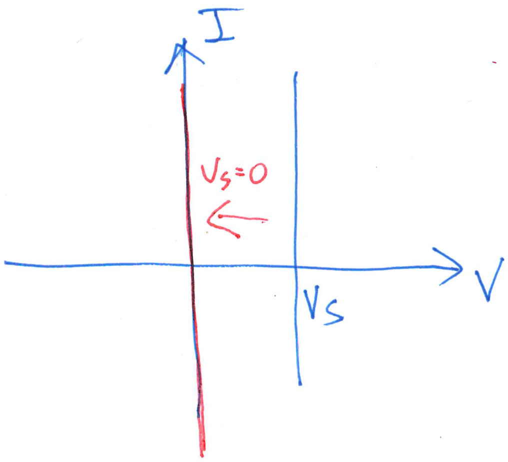
$$I_i = I_{i,1} + \dots + I_{i,l} + \dots + I_{i,m+k}$$

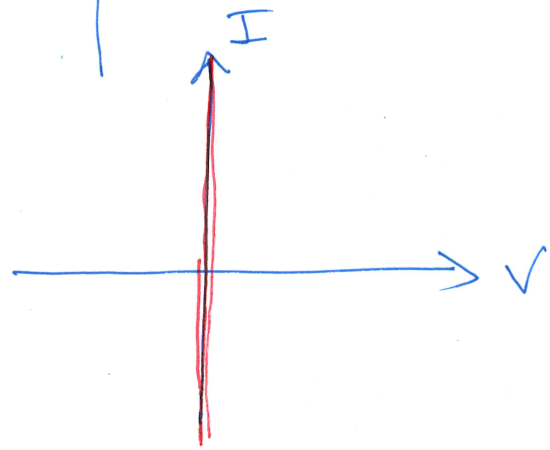
$\Rightarrow$ can calculate I_i by calculating $I_{i,1} \dots I_{i,m+k}$ separately (i.e. by nulling all the other sources)

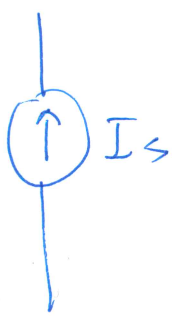
What does it really mean to "null a source"?



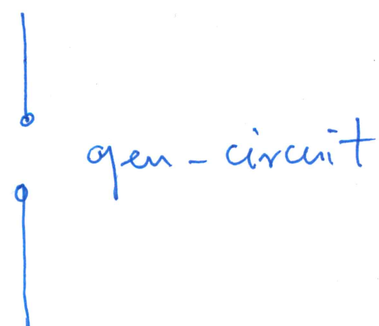
$\Rightarrow$
$V_s = 0$

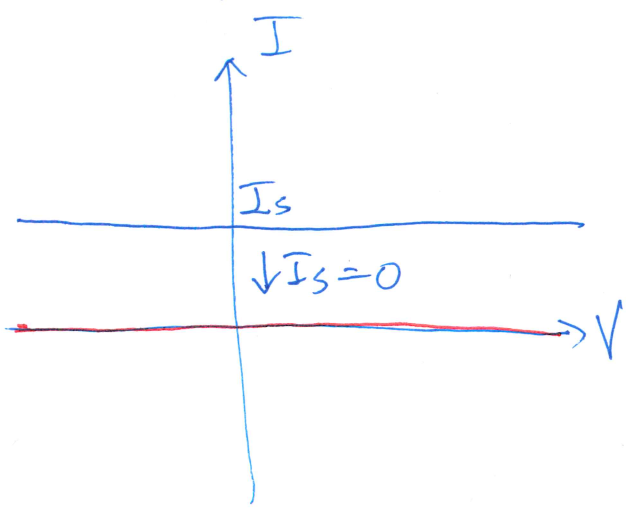


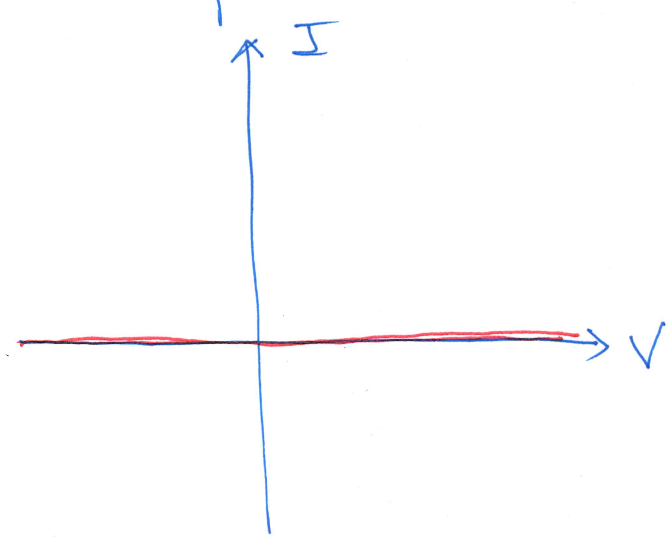




$\Rightarrow$
$I_s = 0$

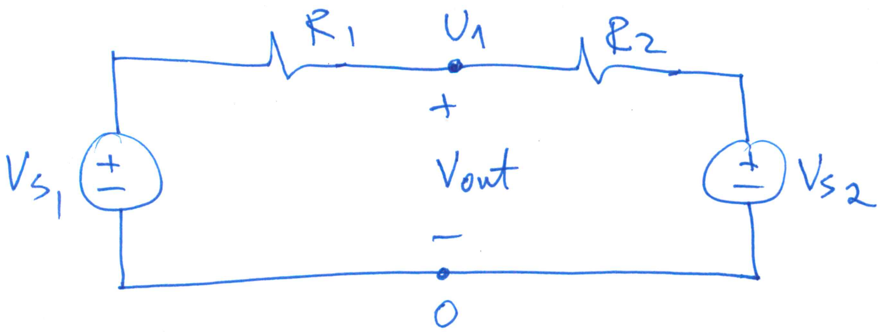






Superposition: Find I's and U's by turning independent sources "on" one at a time and solving the circuit.

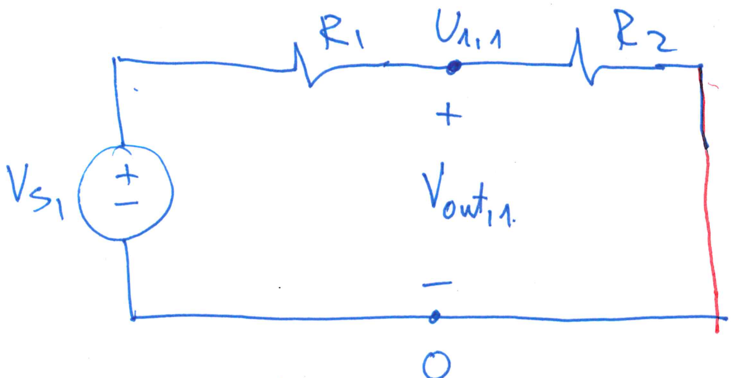
Example: "Mystery circuit" function ? $\underbrace{\text{voltage summer}}$



$$U_1 - 0 = V_{out}$$

$$U_1 = V_{out}$$

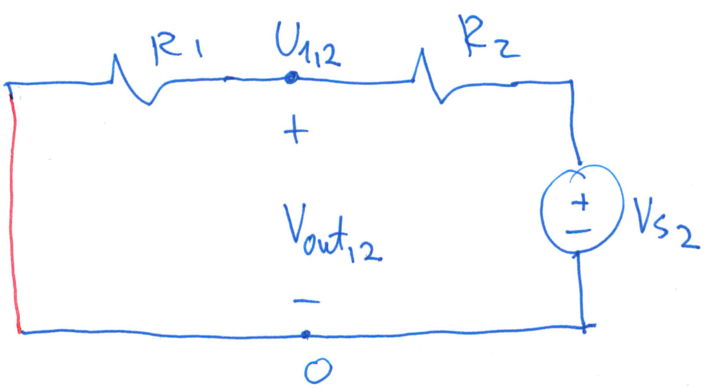
step 1: Compute a response to V_{s1} (set $V_{s2} = 0$)



voltage divider ☺

$$V_{out,1} = \frac{R_2}{R_1 + R_2} \cdot V_{s1}$$

step 2: Compute a response to V_{s2} (set $V_{s1} = 0$)



Voltage divider ☺

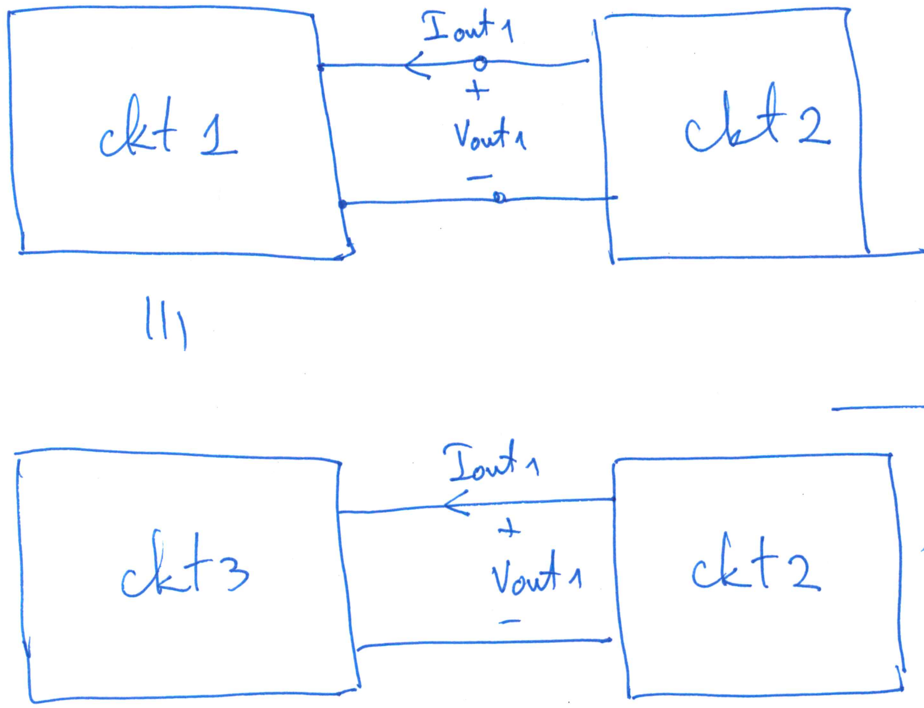
$$V_{out,2} = \frac{R_1}{R_1 + R_2} \cdot V_{s2}$$

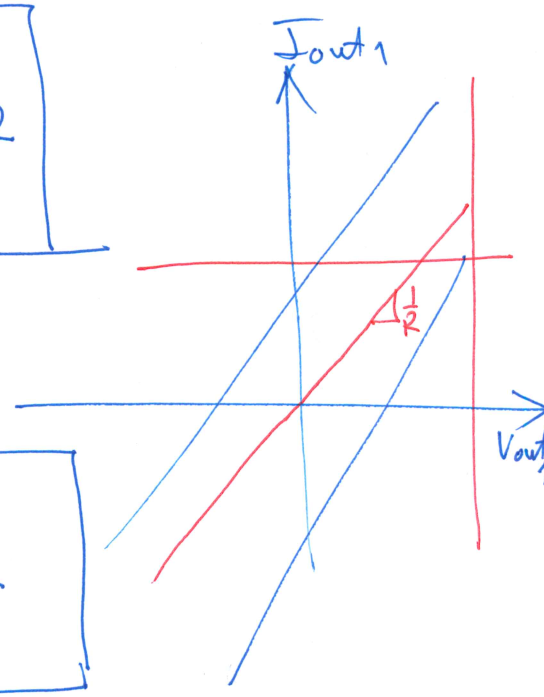
$$V_{out} = V_{out,1} + V_{out,2} = \underbrace{\frac{R_2}{R_1 + R_2}}_{\alpha < 1} V_{s1} + \underbrace{\frac{R_1}{R_1 + R_2}}_{\beta < 1} V_{s2}$$

$$U_1 = U_{1,1} + U_{1,2}$$

L4

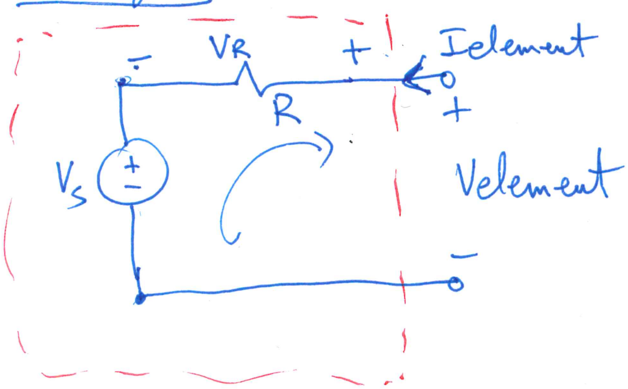
Equivalence:

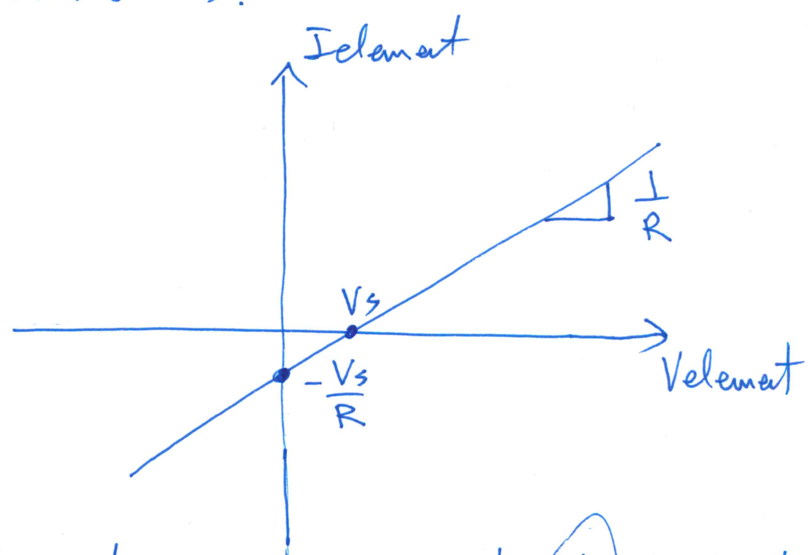




In circuits, two elements are equivalent if they have the same I-V characteristics.

example:



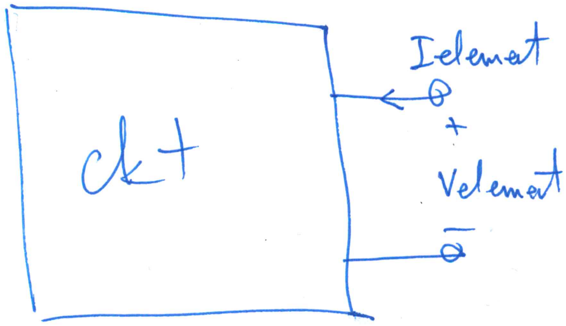


$$\frac{V_{element} - V_s}{R} = I_{element} \Rightarrow \underline{I_{element} = \frac{1}{R} V_{element} - \frac{V_s}{R}}$$

KVL: $V_s + V_R = V_{element}$

$V_R = V_{element} - V_s$

$I_{element} \cdot R = V_{elem} - V_s$



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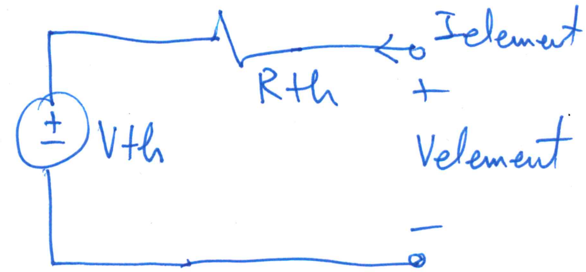
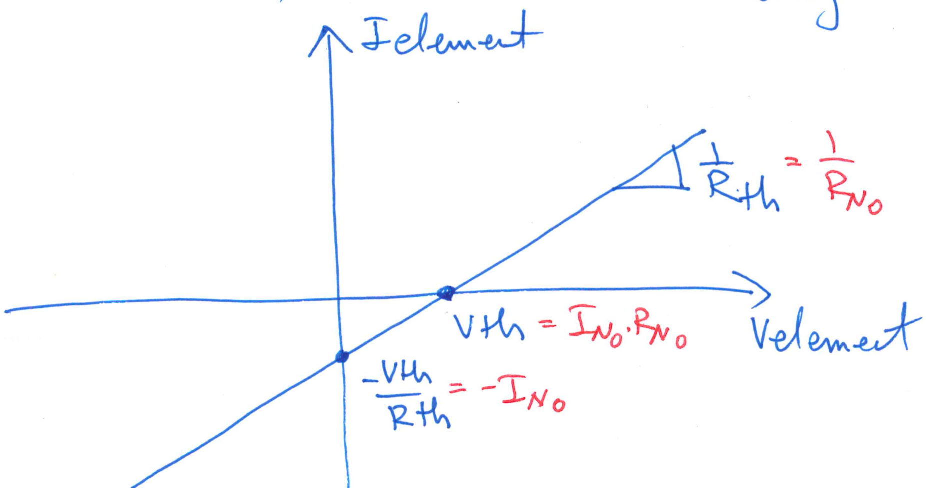
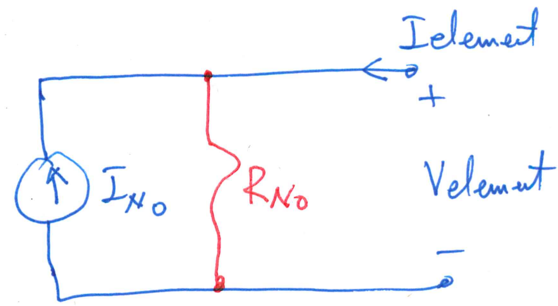
Thevenin equivalent:



Need a min. of two elements (a resistor and a source) to create any I-V line.

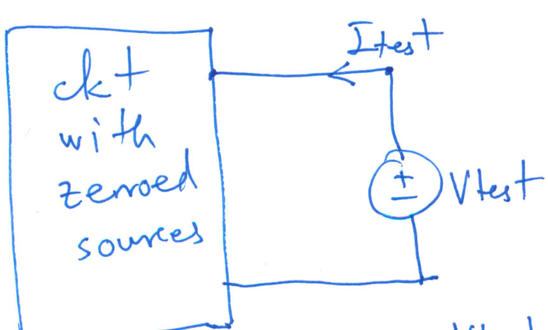




Norton equivalent:

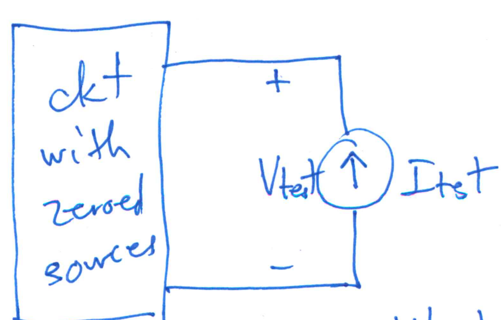
To find V_{th}: "Connect" an "open-circuit", across two terminals and measure $V_{open\text{-}circuit} = V_{th}$

To find R_{th}: Zero-out ("null") all independent sources (to find a slope)



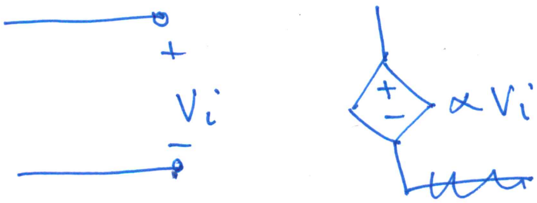
$$R_{th} = \frac{V_{test}}{I_{test}}$$

or

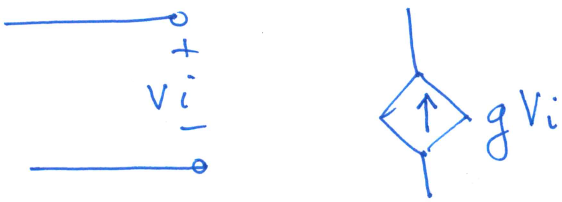


$$R_{th} = \frac{V_{test}}{I_{test}}$$

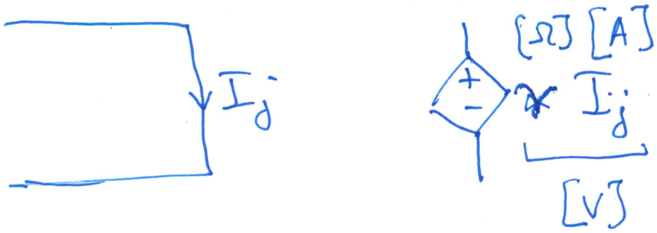
Dependent sources:



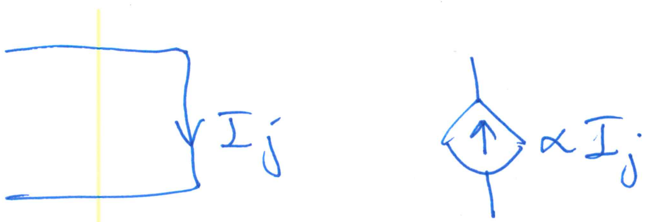
Voltage-controlled voltage source (VCVS)



Voltage-controlled current-source (VCCS)

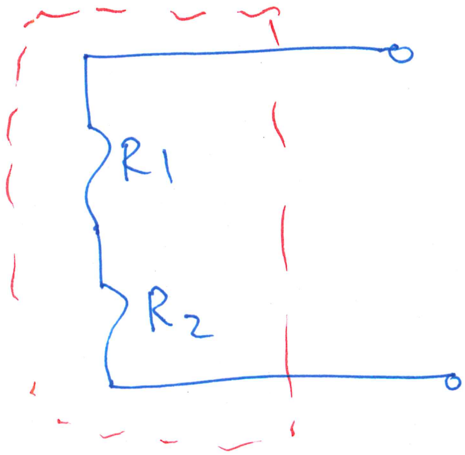


Current-controlled voltage-source (CCVS)

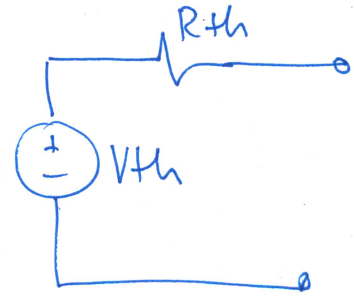


Current-controlled current-source (CCCS)

Example 1:

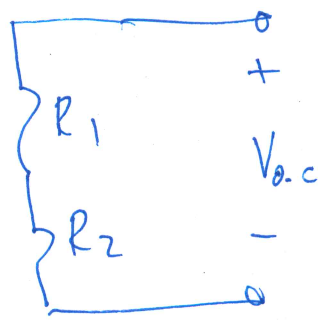


$\Rightarrow$

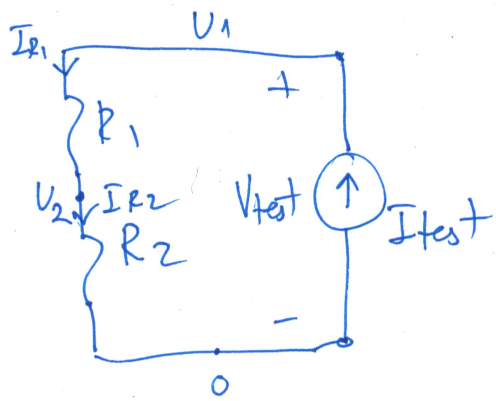


step 1: attach o.c. & measure $V_{th} = V_{o.c.}$

$$V_{th} = V_{o.c} = 0$$



step 2: Null indep. sources & apply test current



$$V_{test} = I_{test} \cdot R_2 + I_{test} \cdot R_1$$

$$= (R_1 + R_2)\, I_{test}$$

$$R_{th} = \frac{V_{test}}{I_{test}} = R_1 + R_2$$

KCL:

$$\boxed{\begin{aligned} I_{R1} &= I_{test} \\ I_{R2} &= I_{R1} \end{aligned}}$$

"In series" means that <u>same</u> current flows through the elements in series.

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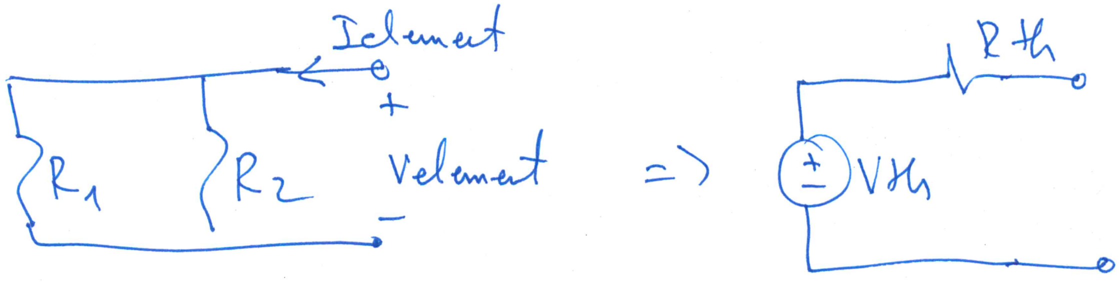
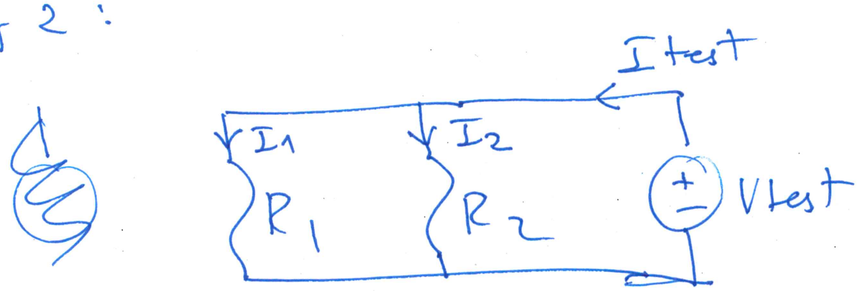


step 1: $V_{th} = 0$

step 2:



$$I_1 = \frac{V_{test}}{R_1}$$

$$I_2 = \frac{V_{test}}{R_2}$$

KCL: $I_1 + I_2 = I_{test}$

$$V_{test}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = I_{test}$$

$$R_{th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} = R_1 \| R_2$$

parallel operator

"In parallel" means voltage across them is the same.