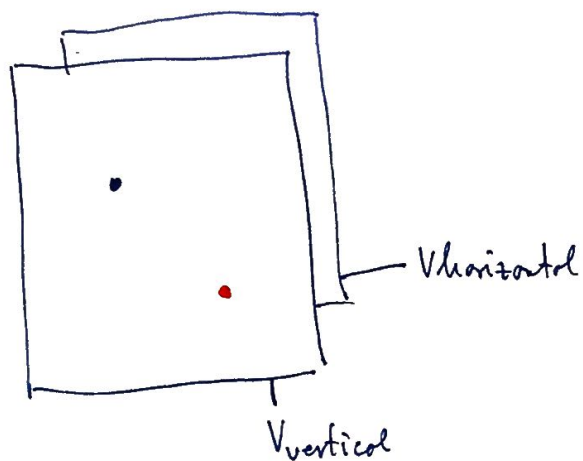
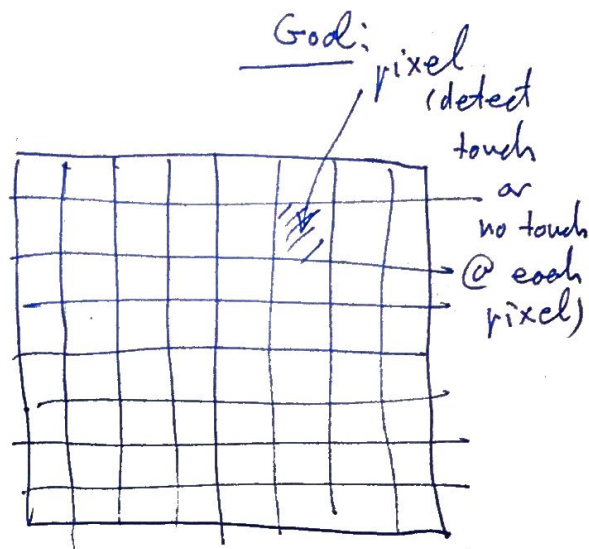


- Today: # Capacitive touchscreen
 Note 16 # Capacitor equivalence
 # Capacitor physics

An improved touchscreen:



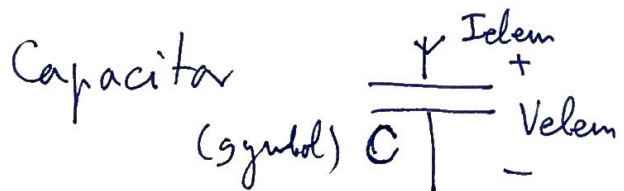
Arbitrary #
=> of touch points



Capacitive touch

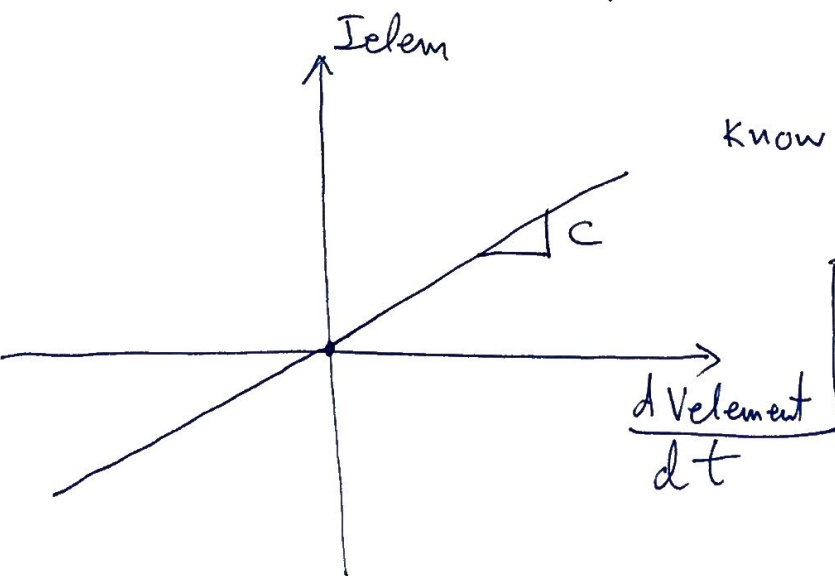
Circuit model:

"I-V" element relationships



$$Q_{elem} = C \cdot V_{elem}$$

$[C]$ $[F]$ $[V]$
 Coulomb Farad Volt



Know:

$$I_{elem} = \frac{dQ_{elem}}{dt} = C \cdot \frac{dV_{elem}}{dt}$$

$$I_{elem} = C \cdot \frac{dV_{elem}}{dt}$$

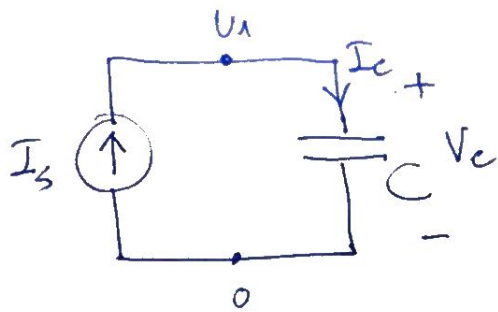
C = const in time

"Ohm's law"

for capacitors

can use the Z-step ckt analysis ...

(2) Simple circuit #1:



KCL: $I_s - I_c = 0$

elem. def. for C : $I_c = C \cdot \frac{dV_c}{dt}$

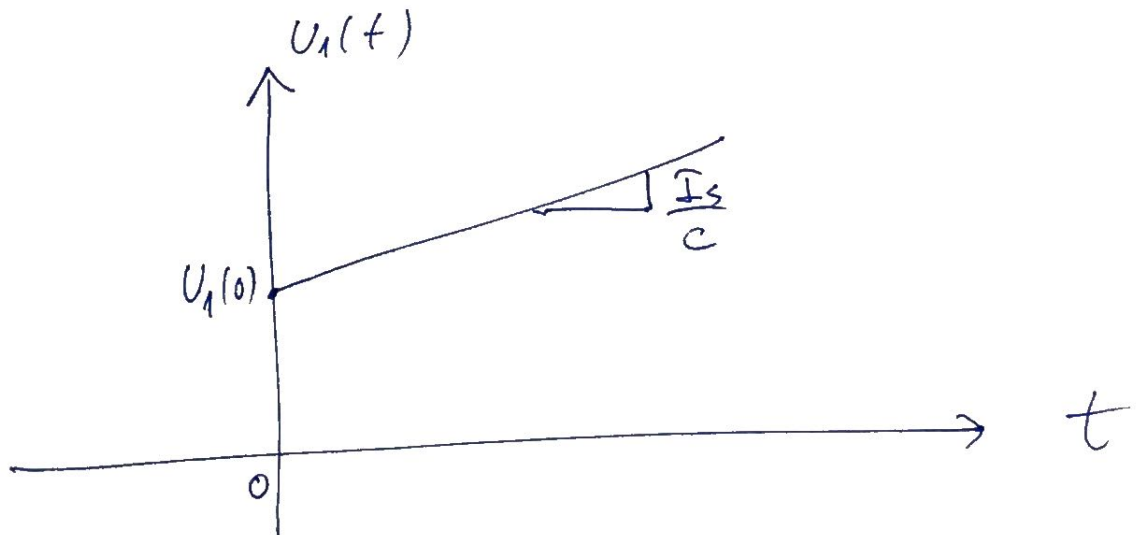
voltage def: $V_c = U_1 - 0 = U_1$

$$\int_0^t I_s dt = \int_{U_1(0)}^{U_1(t)} C dU_1$$

$$\underline{I_s} = C \frac{dV_c}{dt} = C \underline{\frac{dU_1}{dt}}$$

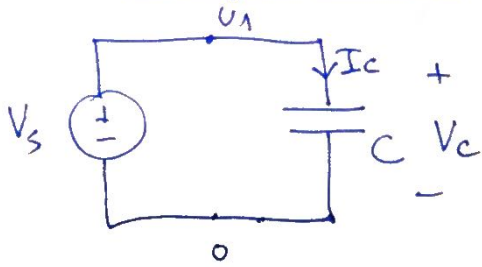
$$I_s \cdot \underbrace{(t-0)}_t = C \cdot (U_1(t) - U_1(0))$$

$$U_1(t) = \frac{I_s}{C} \cdot t + U_1(0)$$



(23)

Simple circuit # 2:



$$U_1 - 0 = V_s$$

(elem. def. source)

$$U_1 - 0 = V_c$$

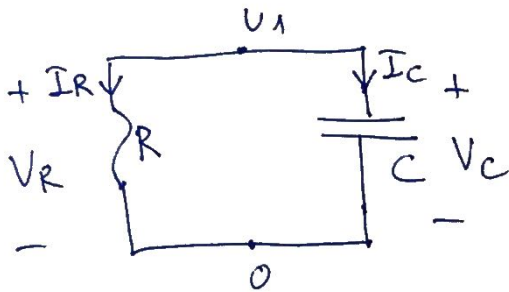
(C. voltage def)

$$V_c = V_s$$

$$I_c = C \frac{dV_c}{dt} \quad (\text{cap. elem. def})$$

$$I_c = C \cdot \frac{dV_c}{dt} = C \cdot \frac{dV_s}{dt} = 0$$

Simple circuit # 3:



$$\text{KCL: } I_c = -I_R \quad \text{or} \quad I_c + I_R = 0$$

since $I_c = 0$ (steady-state)

$$\Rightarrow I_c = -I_R \Rightarrow \boxed{I_R = 0}$$

steady-state means
when voltages settle.

$$\frac{dV_i}{dt} = 0$$

⇓

every $I_c = 0$

$$V_R = R \cdot I_R = 0$$

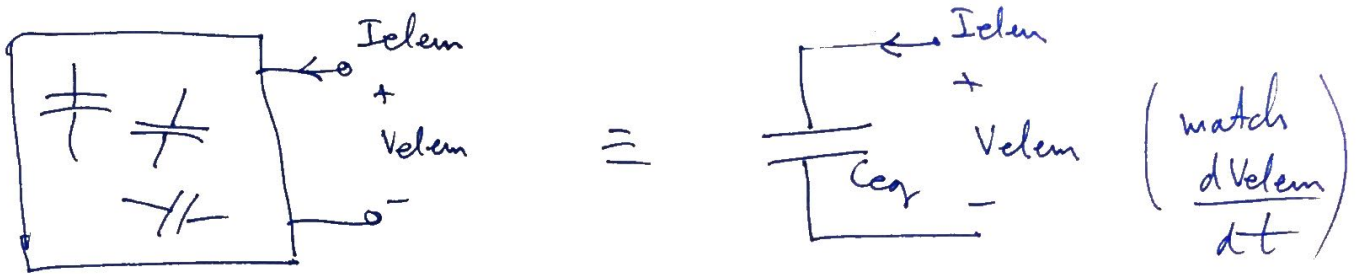
$$V_R = U_1 - 0 = \boxed{U_1 = 0}$$

C4

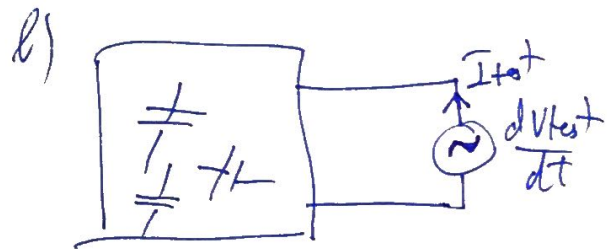
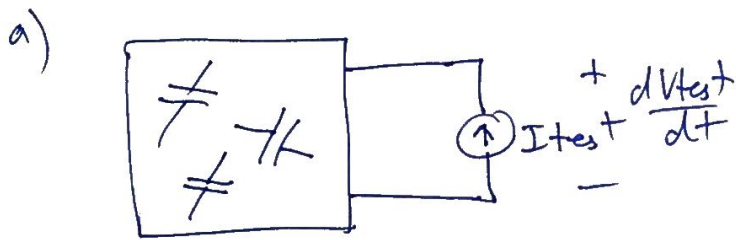
Equivalent circuits with capacitors:

* Capacitor - only analysis

* step 2: $C_{eq} = \frac{I_{elem}}{\frac{dV_{elem}}{dt}} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$



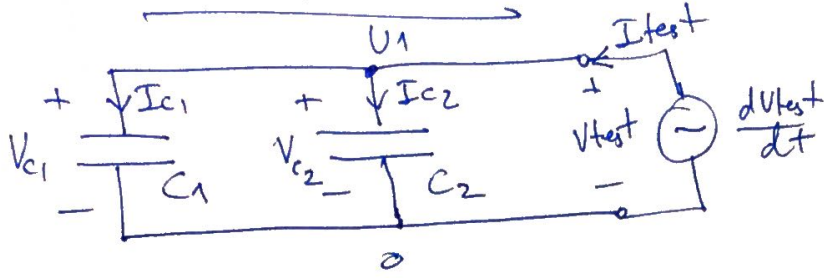
- a) Apply I_{test} and measure $\frac{dV_{test}}{dt}$
- b) Apply $\frac{dV_{test}}{dt}$ and measure I_{test}
- $\Rightarrow C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$



15

Example 1

"capacitors in parallel"



$$V_{c1} = U_1 \quad \underbrace{V_{c1} = V_{c2} = V_{test}}$$

$$V_{c2} = U_1$$

$$V_{test} = U_1$$

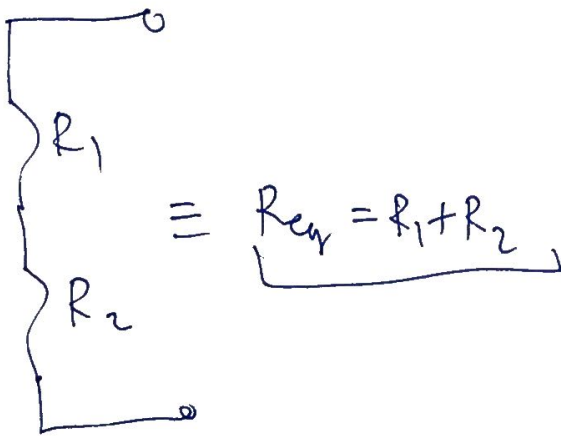
(elem. def) $I_{c1} = C_1 \cdot \frac{dV_{c1}}{dt} = C_1 \frac{dV_{test}}{dt}$

(elem) $I_{c2} = C_2 \frac{dV_{c2}}{dt} = C_2 \cdot \frac{dV_{test}}{dt}$

(KCL) $I_{test} = I_{c1} + I_{c2} = C_1 \frac{dV_{test}}{dt} + C_2 \frac{dV_{test}}{dt}$

$$I_{test} = (C_1 + C_2) \frac{dV_{test}}{dt}$$

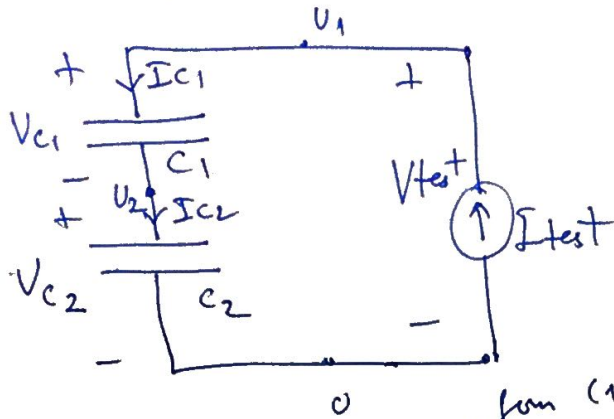
$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \underbrace{C_1 + C_2}$$



2.6

Example 2:

"capacitors in series"



KCL: $I_{test} = I_{c1} = I_{c2}$ (1)

elements: $I_{c1} = C_1 \frac{dV_{c1}}{dt}$ (2)

$\rightarrow I_{c2} = C_2 \frac{dV_{c2}}{dt}$ (3)

from (1), (3) & (5) voltage defs:

$I_{test} = C_2 \left(\frac{dV_{c2}}{dt} \right)$

$V_{c1} = U_1 - U_2$ (4)

$V_{c2} = U_2 - 0$ (5)

$V_{test} = U_1 - 0$ (6)

$\frac{dV_{c1}}{dt} \stackrel{(4)}{=} \frac{dU_1}{dt} - \frac{dU_2}{dt} \stackrel{(2)}{=} \frac{I_{c1}}{C_1}$

$\frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1}$

\downarrow
 $\frac{dV_{test}}{dt} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right) I_{test}$

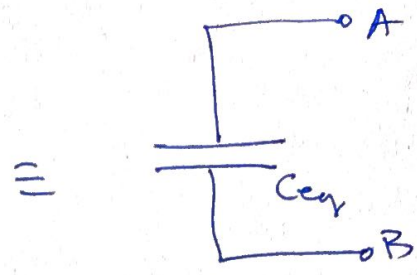
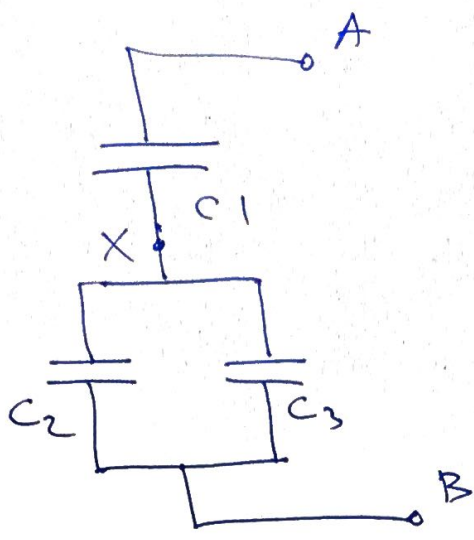
$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 || C_2$

$C_{eq} = C_1 || C_2$

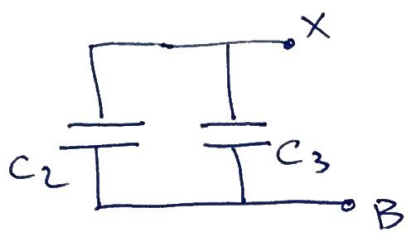
\uparrow parallel operator

(7)

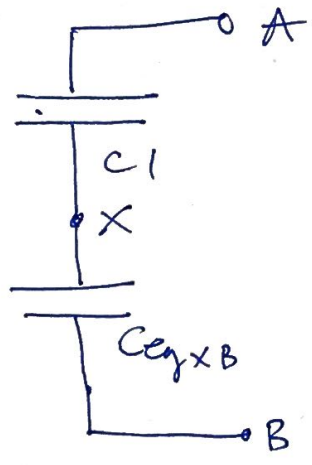
Example #3:



$$C_{eq} = ? = C_1 \parallel (C_2 + C_3)$$



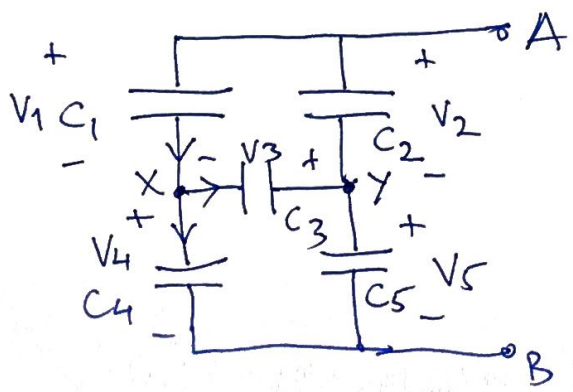
$$\Rightarrow C_{eqXB} = C_2 + C_3$$



$$\begin{aligned} \Rightarrow C_{eqAB} &= C_1 \parallel C_{eqXB} \\ &= C_1 \parallel (C_2 + C_3) \end{aligned}$$

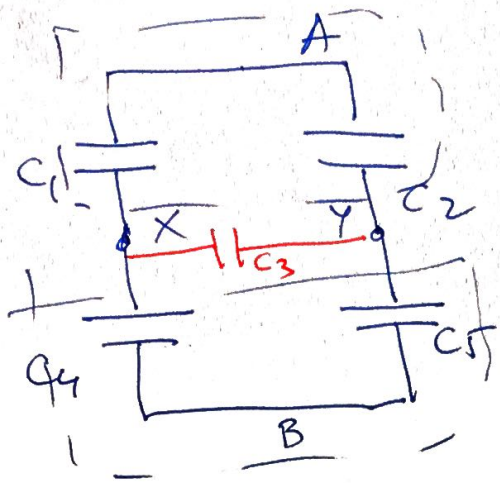
$$= \frac{C_1 (C_2 + C_3)}{C_1 + (C_2 + C_3)}$$

challenge ckt:



$$C_{eqXY} =$$

Q8

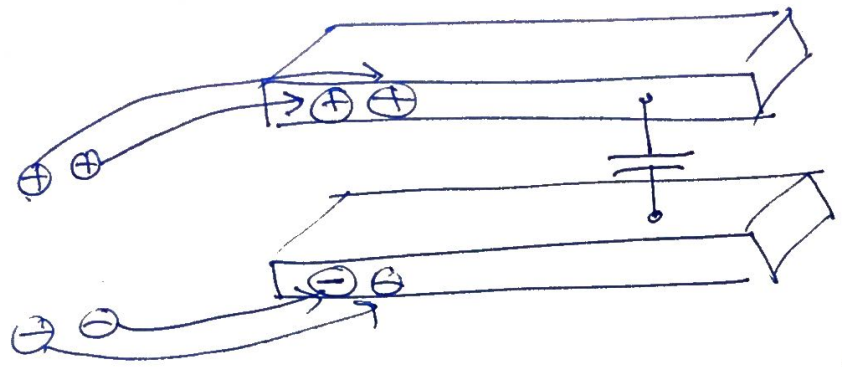


$$C_{xy} = (C_1 || C_2) + (C_4 || C_5) + C_3$$

Q3

16A Capacitor physics

Capacitor : Any two conductors separated by an insulator (cannot carry current)



$$\equiv \frac{Q}{C} \quad \underline{Q = C \cdot V}$$

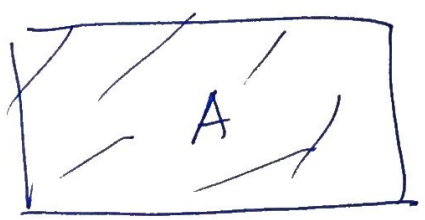
Capacitor is a "bucket" $\frac{dE}{dq}$ for charge

Capacitance depends on : 1) geometry of the conductor
2) material properties of the insulator

side view :



top view :



$$C = \epsilon \frac{A}{d} \quad \left[\frac{\text{m}^2}{\text{m}} = \text{m} \right]$$

↑
permittivity $\left[\frac{\text{F}}{\text{m}} \right]$

$$\epsilon_0 = 8.85 \frac{\mu\text{F}}{\text{mm}}$$

$\mu = \text{pico} = 10^{-12}$