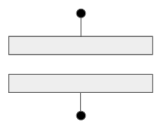
EECS 16A Designing Information Devices and Systems I
Spring 2021 Discussion 8B

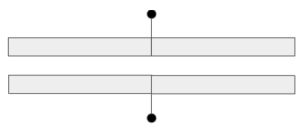
1. Capacitance Equivalence

For the structures shown below, assume that the plates have a depth L into the page and a width W and are always a distance d apart. The dielectric between the plates has absolute permitivity ε. For the following calculations, assume the capacitance is purely parallel plate, i.e. ignore fringing field effects.

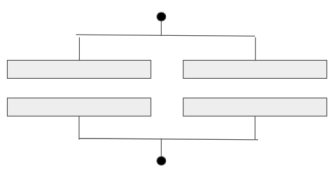
(a) What is the capacitance of the structure shown below?



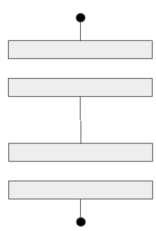
(b) Suppose that we take two such structures and put them next to each other as shown below. What is the capacitance of this new structure?



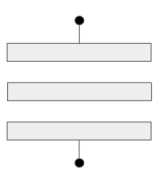
(c) Now suppose that rather than connecting the together as shown above, we connect them with an ideal wire as shown below. What is the capacitance of this structure?



(d) Suppose that we now take two capacitors and connect them as shown below. What is the capacitance of the structure?



(e) What is the capacitance of the structure shown below?



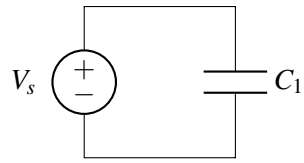
2. Voltages Across Capacitors

For the circuits given below, determine the voltage across each capacitor and calculate the charge and energy stored on each capacitor (assume all capacitors start *uncharged*, and then we've let the system reach steady state). We are also given $C_1 = 1\,\mu\text{F}$, $C_2 = 3\,\mu\text{F}$, and $V_s = 1\,\text{V}$.

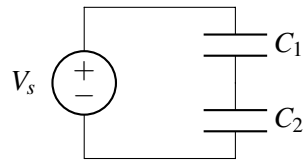
Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) $= \frac{\text{Coulomb}}{\text{Volt}}$.

It may also help to note metric prefix examples: $3\,\mu\text{F} = 3 \times 10^{-6}\text{F}$.

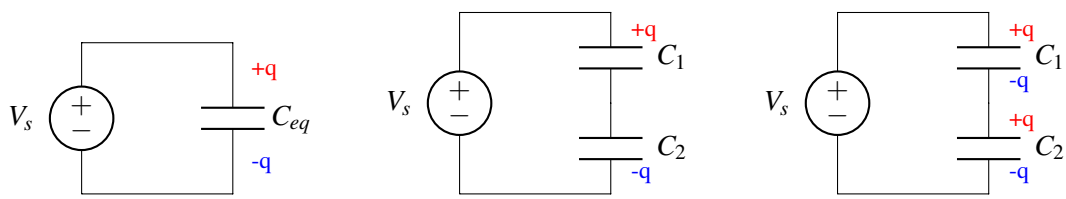
(a)



(b)



Helpful diagrams for considering the charges capacitors linked in series:
(without any initial charges)



Left: Our series capacitors may be modeled as one equivalent capacitor C_{eq}, which after some time is charged up by V_s to have $+q$ on the top plate and $-q$ on the bottom plate.

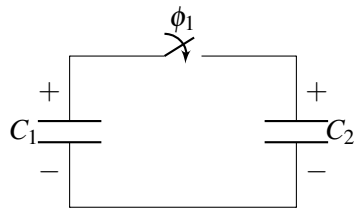
Middle: We return to the 2-capacitor picture, but carry this insight of equivalent charge with us. Now the charge $+q$ is on the top plate *of capacitor* C_1, and $-q$ is on the bottom plate *of capacitor* C_2.

Right: Since capacitor plates have opposite & equal charges, we attain this final right digram.

As another conceptual check, we notice that the node between C_1 and C_2 is isolated from any other connections and should always remain *charge neutral*. From the diagram right we see this is maintained since $(+q) + (-q) = 0$.

3. Capacitors and Charge Conservation

(a) Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and an open switch. Suppose that C_1 is initially charged to $+1\,\text{V}$ and that C_2 is charged to $+2\,\text{V}$. How much charge is on C_1 and C_2?



(b) Now the switch is closed (i.e. the capacitors are connected together.) What are the voltages across and the charges on C_1 and C_2?