Dis 12A EECS16A

1. Search and Rescue Dogs

Berkeley's Puppy Shelter needs your help! While Mr. Muffin was being walked, the volunteer let go of his leash and he is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the puppies at the shelter have a collar that sends a bluetooth signal to receiver towers, which are spread throughout the streets (pictured below). If the puppy/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of 5 city blocks. Can you help the shelter locate their lost puppy?

Note: A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map.) Mr. Muffin is constrained to running wild in the streets, meaning he won't be found in any buildings. If your TA asks 'Where is Mr. Muffin?' it is sufficient to answer with his intersection or 'between these two intersections.'

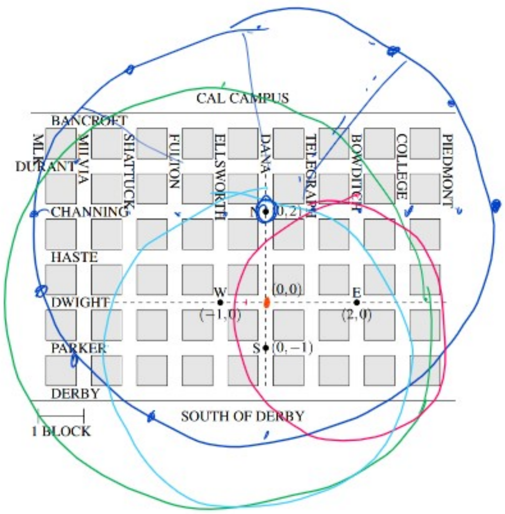

[1]

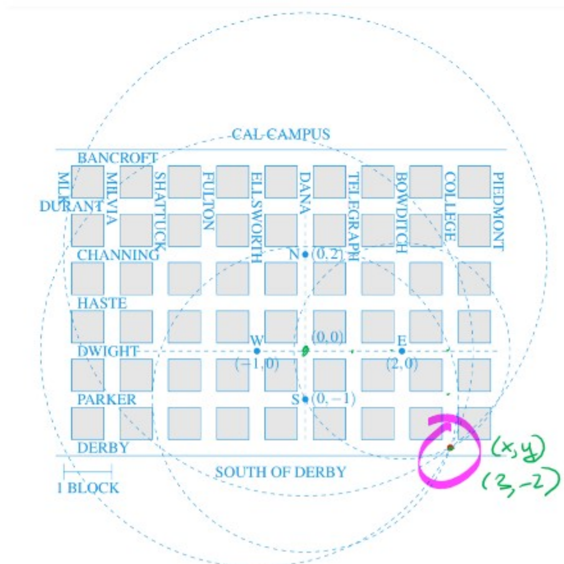
(a) You check the logs of the cell towers, and they have received the following messages:

Sensor	Distance	
N	5	
W	$\sqrt{20}$	≈ 4.472
E	$\sqrt{5}$	≈ 2.24
S	$\sqrt{10}$	≈ 3.16

On the map provided, identify where Mr. Muffin is!

[1] http://www.pupsmile.com/wp-content/uploads/2012/11/running_happy_dog-1024x684.jpeg





(b) Can you set this up as a system of equations? Are these equations linear? If not, can these equations be linearized? If you can linearize these equations, **write down a simplified form of your set of equations.**

Hint: Set (0,0) to be Dwight and Dana.

Hint 2: Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $\quad d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

Hint 3: You don't need all 4 equations. You have two unknowns, x and y. You know from lecture that you need three circles to uniquely find a point. How can you use the third circle/equation to get two equations and two unknowns?

Note: Remember to check for consistency for all nonlinear equations after finding the coordinates.

sensor	dist	sensor location
* N	5	(0,2)
* W	$\sqrt{20}$	(-1,0)
* E	$\sqrt{5}$	(2,0)
* S	$\sqrt{10}$	(0,-1)

* unknown location of Mr. Muffin (x,y)

* linear? no

* $5^2 = (x-0)^2 + (y-2)^2$

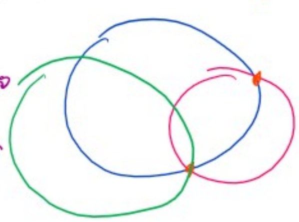
* $(\sqrt{20})^2 = (x+1)^2 + (y-0)^2$

* $(\sqrt{5})^2 = (x-2)^2 + (y-0)^2$

* $(\sqrt{10})^2 = (x-0)^2 + (y+1)^2$

$$\Rightarrow \begin{cases} x^2 + y^2 - 4y + 4 = 25 & (1) \\ x^2 + 2x + 1 + y^2 = 20 & (2) \\ x^2 - 4x + 4 + y^2 = 5 & (3) \\ x^2 + y^2 + 2y + 1 = 10 & (4) \end{cases}$$

* need at least 3 equations to find Mr. Muffin



(2) − (1) $\quad 2x + 4y + 1 - 4 = 20 - 25$

(3) − (1) $\quad -4x + 4y + 4 - 4 = 5 - 25$

2 linear eqns 2 unknowns

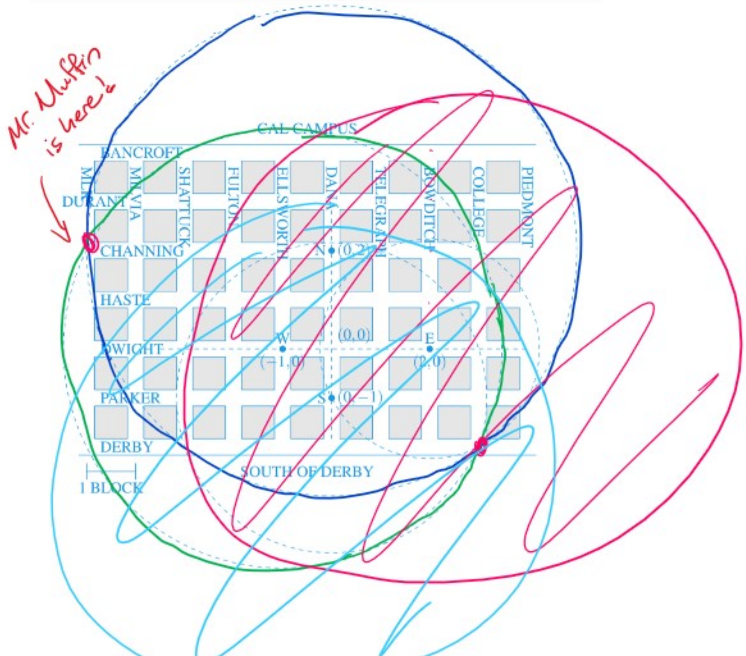
$$\Rightarrow \begin{cases} 2x + 4y = -2 \\ -4x + 4y = -20 \end{cases}$$

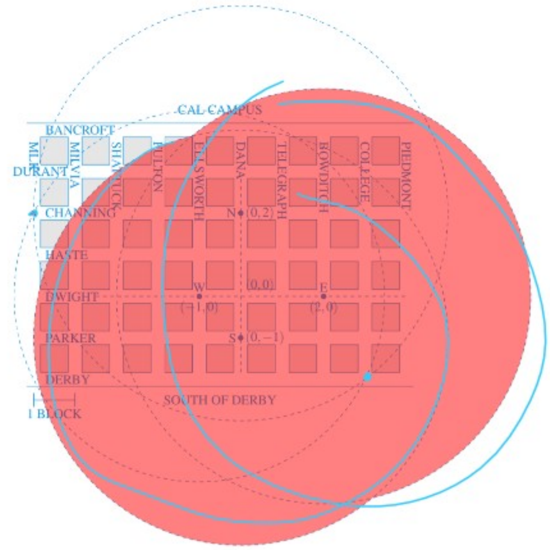
$$\Rightarrow \boxed{x = 3 \quad y = -2}$$

(c) Suppose Mr. Muffin is moving fast, and by the time you get to destination in part (a) he's already run off! You check the logs of the cell towers again, and see the following updated messages:

Sensor	Distance
N	5
W	$\sqrt{20}$
E	Out of Range
S	Out of Range

Can you find Mr. Muffin? If so, on the map provided, identify where Mr. Muffin is! (*Note:* Each cell tower has a range of 5 city blocks)





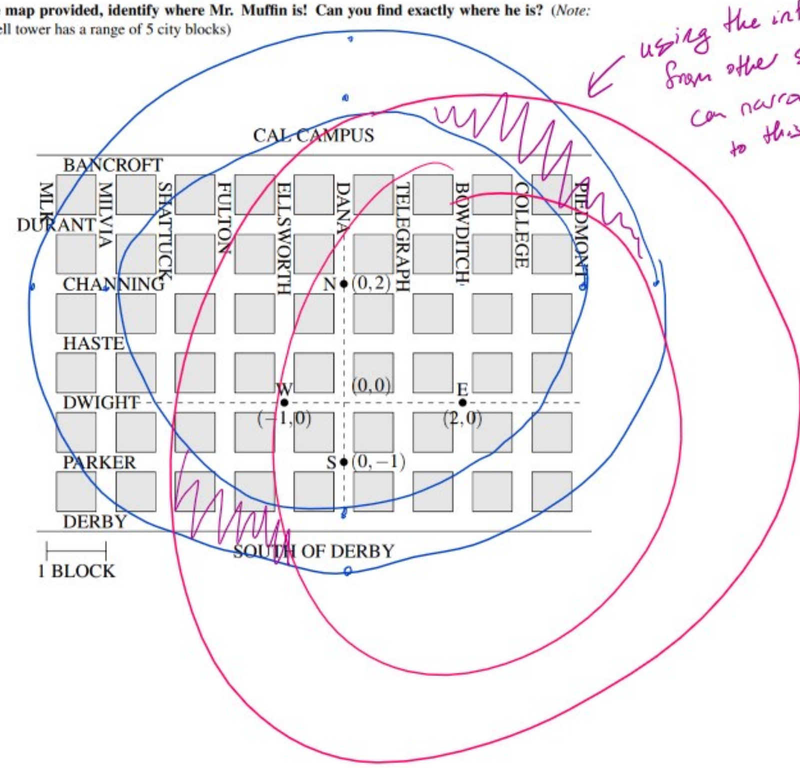
(d) Mr. Muffin is a very mischievous puppy, and while playing and running around he damaged his collar. The transmitter on his collar will still send a signal to the receiver towers, but the distance sensor has noise. You check the logs of the cell towers, and they have received the following messages:

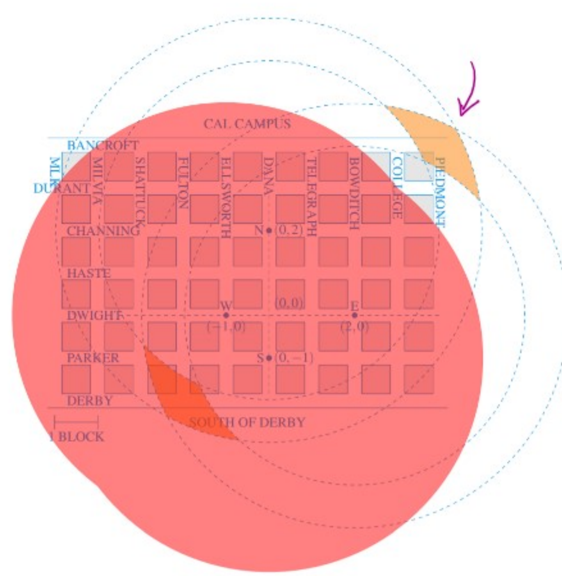
Sensor	Distance
N	4.5 ± 0.5
W	Out of Range
E	4.5 ± 0.5
S	Out of Range

4.0 – 5.0

On the map provided, identify where Mr. Muffin is! Can you find exactly where he is? (*Note:* Each cell tower has a range of 5 city blocks)

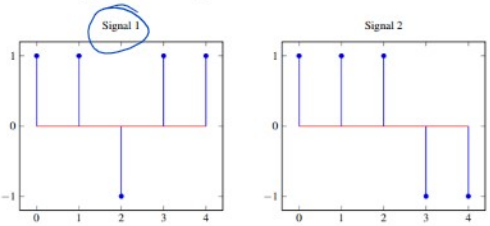
using the info from other sensors (W, S) can narrow it down to this area

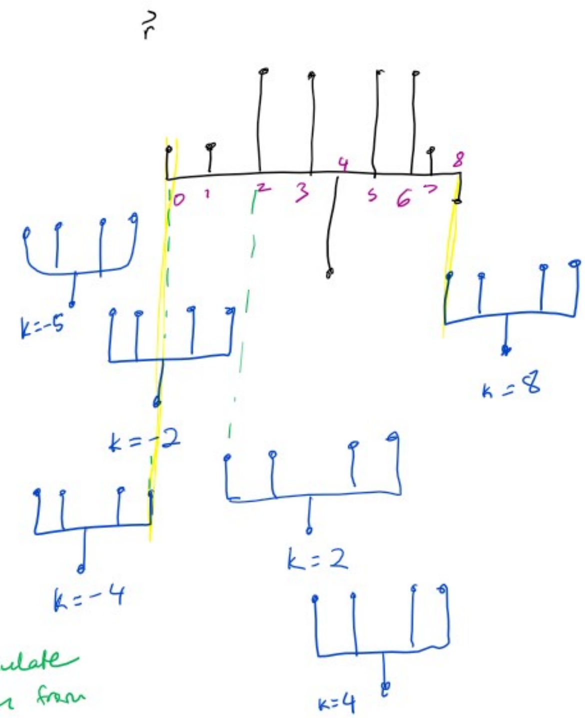




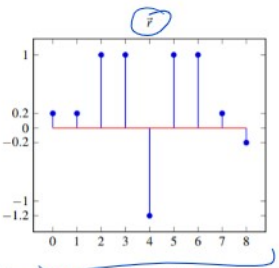
2. Identifying satellites and their delays

We are given the following two signals, $\vec{s}_1$ and $\vec{s}_2$ respectively, that are signatures for two satellites. Your cell phone recieves signals from these two satellites and given a recieved signal $r[n]$ you can identify which, if any, satellite sent the message based on their personal codes.





(a) Your cellphone antenna receives the following signal $r[n]$. You know that there may be some noise present in $r[n]$ in addition to the transmission from the satellite.



* by observation satellite 1 sent a signal at shift $k=+2$

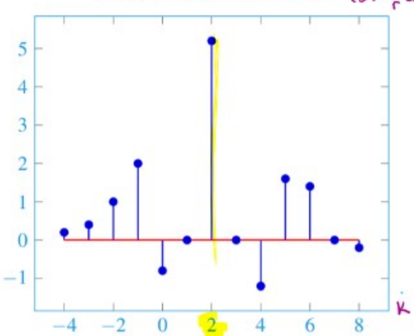
By computing the cross-correlations, can you identify which satellite(s) most likely sent the signal, and by what shift the code is identified relative to our received signal? You can use iPython to compute the cross-correlation. When using iPython to plot, think about the range of shifts k that we are interested in plotting based on the lengths of the signals.

* want to calculate the correlation from $k=-4$ to $k=8$

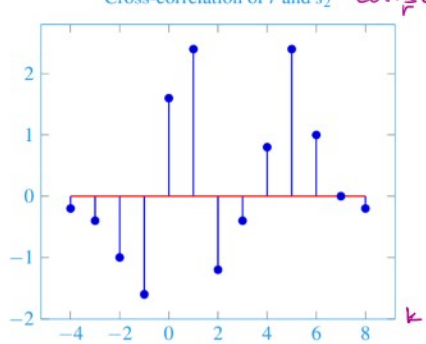
k_{start} → depends on length of the signal that ($\vec{s}_1$) is shifted

k_{end} → depends on length of the signal that ($\vec{r}$) is not shifted

Cross-correlation of $\vec{r}$ and $\vec{s}_1$ $corr_{\vec{r}}(\vec{s}_1)$



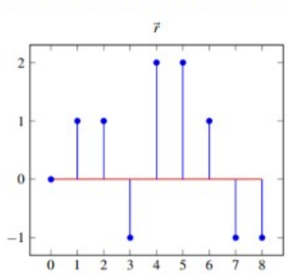
Cross-correlation of $\vec{r}$ and $\vec{s}_2$ $corr_{\vec{r}}(\vec{s}_2)$

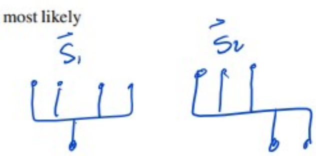


* peak tell us that $\vec{s}_1$ is in $\vec{r}$ at shift $k=2$

* no peaks → $\vec{s}_2$ is not in $\vec{r}$

(b) Now your cellphone receives a new signal $r[n]$ as below. Can you identify which satellite(s) most likely sent the signal, and by what shift the code is identified relative to our received signal?

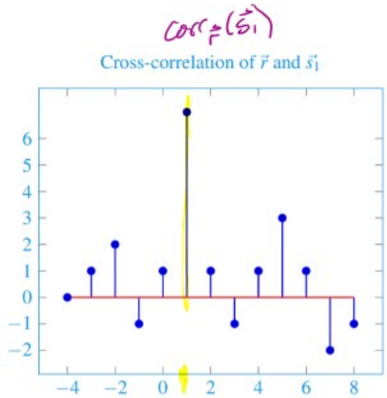


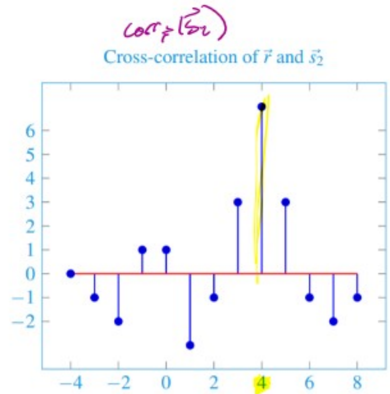


by observation

$\vec{s}_1$ is in $\vec{r}$ at shift $k=+1$

$\vec{s}_2$ is in $\vec{r}$ at shift $k=+4$





3. (Practice) Projections Derivation

Let us explore projections in 2D space. Consider two vectors $\vec{a}$ and $\vec{b}$.

Theorem: The point along $\vec{a}$ (in the span of $\vec{a}$) that is closest to $\vec{b}$ is the vector $\vec{z}$ such that $\vec{b}-\vec{z}$ is orthogonal to $\vec{a}$. The vector $\vec{z}$ is given by $\vec{z} = \frac{\langle \vec{a}, \vec{b} \rangle}{\|\vec{a}\|^2} \vec{a}$.

Hint: Let $\vec{z}$ be the solution to the above theorem. What does it mean for $\vec{z}$ to be in the span of $\vec{a}$?

(a) Derive the theorem using a geometrical interpretation. You may use Figure 1 as the starting point for your proof.

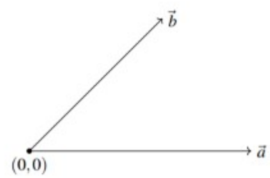


Figure 1: Example vectors $\vec{a}$ and $\vec{b}$ as starting point for geometric proof.

(b) Derive the theorem algebraically by solving a problem minimizing the distance between $\vec{b}$ and $\vec{z}$.