Dis 6B EECS16A

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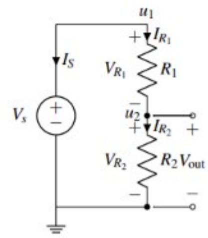
1. Voltage Divider

For the circuit below, your goal will be to find the voltage V_{out} in terms of the resistances R_1, R_2, and V_s, using NVA (Node Voltage Analysis). The labeling steps (steps 1-4) have already been done for you.

$V_s = u_1 - 0$

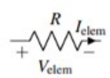


Here is a reminder of the labeling steps followed to get the circuit diagram above:

- **Step 1:** Select a reference (ground) node. Any node can be chosen for this purpose. We will measure all of the voltages in the rest of the circuit relative to this point.
- **Step 2:** Label all nodes with voltage set by voltage sources.
- **Step 3:** Label remaining nodes.
- **Step 4:** Label element voltages and currents, following **Passive Sign Convention** (discussed below).

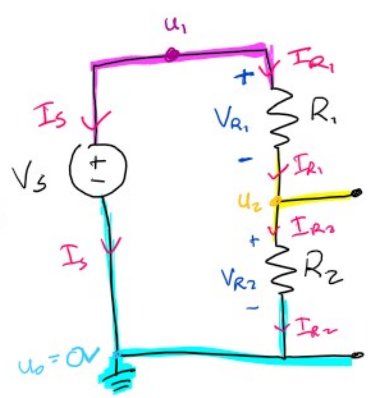
Passive sign convention

The **passive sign convention** dictates that positive current should *enter* the positive terminal and *exit* the negative terminal of an element. Below is an example for a resistor:



As long as this convention is followed consistently, it does not matter which direction you arbitrarily assigned each element current to; the voltage referencing will work out to determine the correct final sign. When we discuss *power* later in the module, you will see why we call this convention "passive."

$*$ $V_{element}$ = (node voltage @ + terminal) − (node voltage @ − terminal)



$V_s = u_1 - 0$

$\Rightarrow u_1 = V_s$ known node voltage

$V_{out} = u_2 - 0 = u_2$

To achieve your goal of ***finding*** V_{out}, perform the rest of the NVA steps in the boxes below:

Step 5: Write KCL equations for all nodes with unknown voltages.

u_2

KCL @ a node:
$I_{in} = I_{out}$
$0 = I_{out} - I_{in}$

KCL @ u_2:
$I_{in} = I_{out}$
$I_{R_1} = I_{R_2}$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics.

Ohm's Law:
$V = IR \qquad I = \frac{V}{R}$

$V_{R_1} = I_{R_1} R_1$
$V_{R_2} = I_{R_2} R_2$
$I_{R_1} = \frac{V_{R_1}}{R_1} \qquad I_{R_2} = \frac{V_{R_2}}{R_2}$

Step 7: Substitute all element voltages with node voltages found in your step 6 equations.

$V_{element}$ plug in step 6 eqns
$* u_1 = V_s$

$V_{R_1} = u_1 - u_2$
$V_{R_2} = u_2 - 0 = u_2$
$* \; I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - u_2}{R_1} = \frac{V_s - u_2}{R_1}$
$* \; I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_2}{R_2}$

Step 8: Substitute expressions found in step 7 into the KCL equations from step 5.

$I_{R_1} = I_{R_2}$
$\frac{V_s - u_2}{R_1} = \frac{u_2}{R_2}$
$*$ 1 eqn, 1 unknown u_2

Step 9: Solve for the node voltage values. At this point the analysis procedure is effectively complete - all that's left to do is solve the system of linear equations (by applying Gaussian Elimination, inverting **A**, etc.) to find the values for the u's. Then we can go back to our Step 7 equations and calculate the I's. Note that in our circuit, $V_{out} = u_2 - 0 = u_2$.

$\frac{V_s - u_2}{R_1} = \frac{u_2}{R_2}$
$\frac{V_s}{R_1} = \frac{u_2}{R_2} + \frac{u_2}{R_1} = u_2\left(\frac{1}{R_2} + \frac{1}{R_1}\right)$
$\frac{V_s}{R_1} = u_2\left(\frac{R_1 + R_2}{R_1 R_2}\right)$
$u_2 = \left(\frac{R_2}{R_1 + R_2}\right) V_s$

$V_{out} = u_2 = \frac{R_2}{R_1 + R_2} V_s$

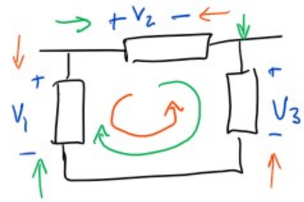
$*$ $V_{out} = V_{R_2}$

KCL @ a node:

$I_{in} = I_{out}$

KVL for a loop:

$\sum V = 0$



If arrow goes through negative terminal first → has + sign

$V_1 - V_2 - V_3 = 0$

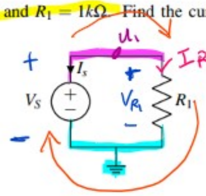
$-V_1 + V_2 + V_3 = 0$

Note: don't see KVL in NVA because element/node voltage eqns ($V_{element}$) provides the same info

2. A Simple Circuit

Use KVL and/or KCL to solve the following circuits.

(a) For this problem assume $V_S = 1V$ and $R_1 = 1k\Omega$. Find the current, I_s flowing through the voltage source.



KVL: $+V_S - V_{R_1} = 0$

$V_S = V_{R_1}$

$V_{element}$

★ if we do element/node voltage eqns

$V_S = u_1 - 0$

$V_{R_1} = u_1 - 0$

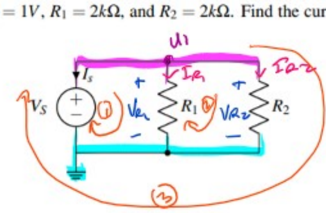
$\Rightarrow V_S = V_{R_1}$

Ohm's Law: $V_{R_1} = I_{R_1} R_1$ $\quad I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1}$

KCL @ u_1: $I_{in} = I_{out}$

$0 = I_S + I_{R_1} \rightarrow I_S = -I_{R_1} = -\frac{V_S}{R_1} = -1mA$

(b) For this problem assume $V_S = 1V$, $R_1 = 2k\Omega$, and $R_2 = 2k\Omega$. Find the current, I_s flowing through the voltage source.



KVL:

① $V_S - V_{R_1} = 0$

② $V_{R_1} - V_{R_2} = 0$

③ $V_S - V_{R_2} = 0$

$\rightarrow V_S = V_{R_1} = V_{R_2}$

note ① + ② = ③

only 2 lin ind eqns

★ $V_{element}$ would give us the same info

Ohm's Law

$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1}$ $\quad I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_S}{R_2}$

KCL @ u_1: $I_{in} = I_{out}$

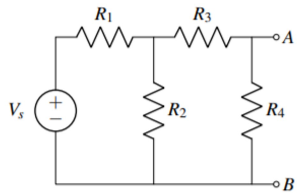
$0 = I_S + I_{R_1} + I_{R_2}$

$I_S = -(I_{R_1} + I_{R_2})$

$= -\left(\frac{V_S}{R_1} + \frac{V_S}{R_2}\right) = -1mA$

3. (PRACTICE) KVL and KCL

For the circuit shown below, $V_s = 5\,\text{V}$, $R_1 = R_2 = 4\,\text{k}\Omega$, and $R_3 = R_4 = 2\,\text{k}\Omega$.



(a) For the circuit above, write KVL equations for each loop and KCL equations for each node.

(b) Solve for the voltage between A and B using the equations from part (a).