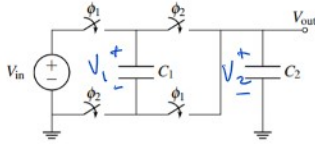


1. Charge Sharing

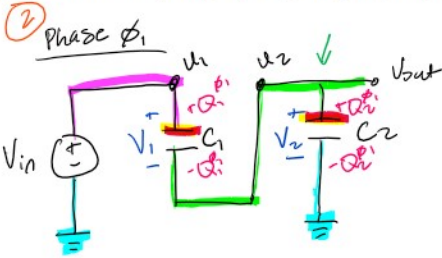
Consider the circuit shown below. In phase  $\phi_1$ , the switches labeled  $\phi_1$  are on while the switches labeled  $\phi_2$  are off. In phase  $\phi_2$ , the switches labeled  $\phi_2$  are on while the switches labeled  $\phi_1$  are off.



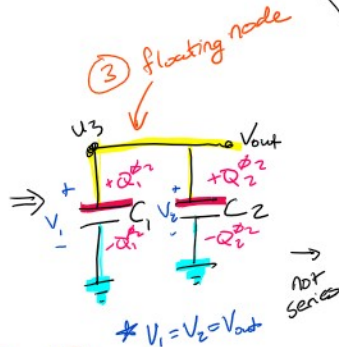
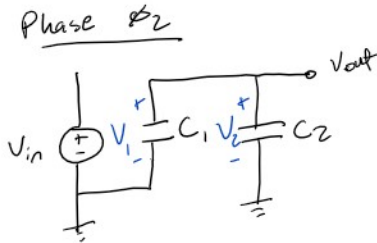
(a) Draw the polarity of the voltage (using + and - signs) across the two capacitors  $C_1$  and  $C_2$ . (It doesn't matter which terminal you label + or -; just remember to keep these consistent through phase 1 and 2!)

①

(b) Redraw the circuit in phase  $\phi_1$  and phase  $\phi_2$ . Keep your polarity from part (a) in mind.



$V_2 \neq V_3$



Charge Sharing Algorithm

- ① label voltages across caps, choose polarity and stick with it

- ② draw the circuit in both phases (label nodes differently)

- ③ identify floating nodes in phase 2

↳ node where charge is trapped

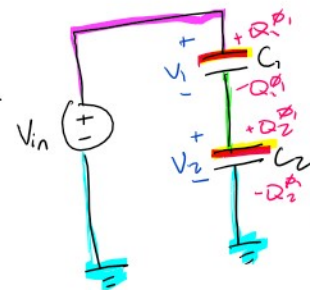
For each floating node:

- ④ Calculate total charge on floating node plates in phase 1  $Q_{float}^{\phi_1}$

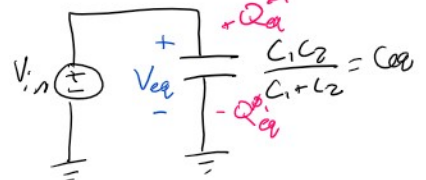
- ⑤ Find total charge on floating node plates in phase 2  $Q_{float}^{\phi_2}$

- ⑥ charge conservation:  $Q_{float}^{\phi_1} = Q_{float}^{\phi_2}$  (solve for unknowns)

- ⑦ repeat



\* series caps!  
 → assuming initially uncharged  
 Same charge  
 $Q_1^{\phi_1} = Q_2^{\phi_1}$   
 on green nodes  
 $0 = -Q_1^{\phi_1} + Q_2^{\phi_1}$



$$Q_{eq}^{\phi_1} = Q_1^{\phi_1} = Q_2^{\phi_1}$$

$$Q_{eq}^{\phi_1} = \frac{C_1 C_2}{C_1 + C_2} V_{in}$$

(c) Find  $V_{out}$  in phase  $\phi_2$  as a function of  $V_{in}$ ,  $C_1$ , and  $C_2$ . ④-⑦

④  $Q_{u_3}^{\phi_1} = +Q_1^{\phi_1} + Q_2^{\phi_1} = C_1 V_1 + C_2 V_2$

$$= Q_{eq}^{\phi_1} + Q_{eq}^{\phi_1} = 2 Q_{eq}^{\phi_1}$$

$$= 2 \frac{C_1 C_2}{C_1 + C_2} V_{in}$$

⑤  $Q_{u_3}^{\phi_2} = Q_1^{\phi_2} + Q_2^{\phi_2}$

$$= C_1 V_1 + C_2 V_2 = C_1 V_{out} + C_2 V_{out}$$

$$= (C_1 + C_2) V_{out}$$

⑥  $Q_{u_3}^{\phi_1} = Q_{u_3}^{\phi_2}$

$$2 \frac{C_1 C_2}{C_1 + C_2} V_{in} = (C_1 + C_2) V_{out}$$

$$V_{out} = \frac{2 C_1 C_2}{(C_1 + C_2)^2} V_{in}$$

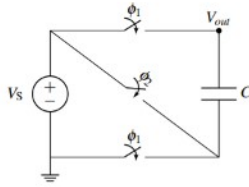
(d) How will the charges be distributed in phase  $\phi_2$  if we assume  $C_1 \gg C_2$ ?

$$Q_1^{\phi_2} = C_1 V_{out} \quad \& \quad C_1 \gg C_2 \quad \rightarrow \quad Q_1^{\phi_2} \gg Q_2^{\phi_2}$$

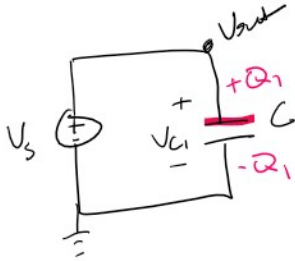
$$Q_2^{\phi_2} = C_2 V_{out}$$

## 2. Voltage Booster

We have made extensive use of resistive voltage dividers to reduce voltage. What about a circuit that boosts voltage to a value greater than the supply  $V_S = 5V$ ? We can do this with capacitors!



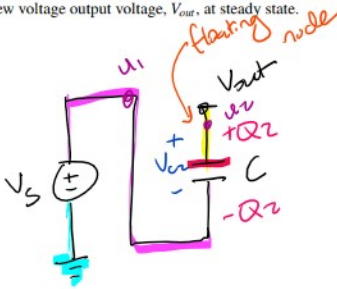
- (a) In the circuit above switches  $\phi_1$  are initially closed and switch  $\phi_2$  is initially open. Calculate the value of the output voltage,  $V_{out}$  with respect to ground, and the amount of charge stored on capacitor,  $C$ , at that state (phase 1).



$$Q_1 = C V_{C1} = C V_{out} = C V_S$$

$$V_{out} = V_S$$

- (b) Now, after the capacitors are charged, switches  $\phi_1$  are opened and switch  $\phi_2$  is closed. Calculate the new voltage output voltage,  $V_{out}$ , at steady state.



by charge conservation:  $Q_{u2}^{\phi_1} = Q_{u2}^{\phi_2}$

$$Q_{u2}^{\phi_1} = +Q_1 = C V_S$$

$$Q_{u2}^{\phi_2} = +Q_2 = C V_{C2} = C (u_2 - u_1) = C (V_{out} - V_S)$$

$$C V_S = C (V_{out} - V_S)$$

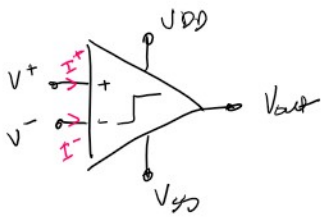
$$\rightarrow V_{out} = 2V_S$$

\* by observation:  
open circuit at  $V_{out}$   
in phase  $\phi_2$   
means  $Q_1 = Q_2$   
 $\rightarrow V_{C1} = V_{C2} = V_S$

$$\rightarrow V_{C2} = V_{out} - V_S = V_S$$

$$V_{out} = 2V$$

# Comparators



$I^+ = I^- = 0$

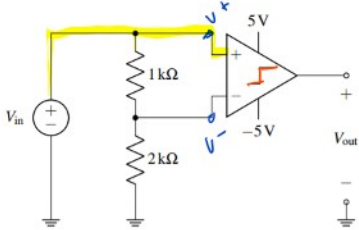
$V_{out} = \begin{cases} V_{DD} & \text{if } V^+ > V^- \\ V_{SS} & \text{otherwise } (V^+ \leq V^-) \end{cases}$

### 3. Comparators

For each of the circuits shown below, plot  $V_{out}$  for  $V_{in}$  ranging from  $-10V$  to  $10V$  for part (a) and from  $0V$  to  $10V$  for part (b).

\* flip point (from  $V_{SS}$  to  $V_{DD}$ ) or other way

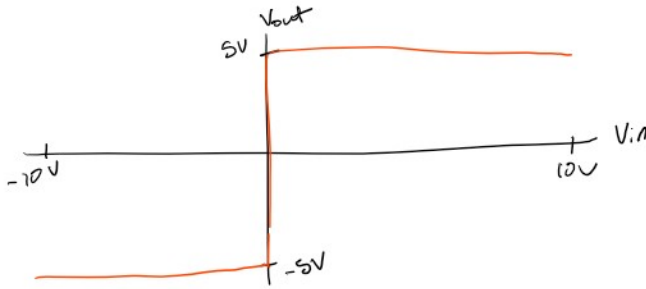
(a)



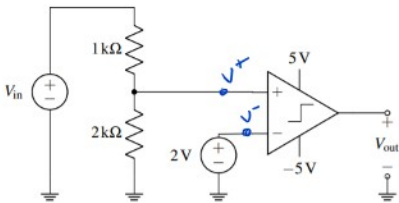
$V^+ = V_{in}$   
 $V^- = \frac{2k\Omega}{1k\Omega + 2k\Omega} V_{in} = \frac{2}{3} V_{in}$

$V^+ = V^-$   
 $V_{in} = 0$

if  $V_{in} > 0$   $V^+ > V^-$   $V_{out} = 5V$   
 if  $V_{in} < 0$   $V^+ < V^-$   $V_{out} = -5V$



(b)



$V^+ = \frac{2}{3} V_{in}$

$V^- = 2V$

\* flip point:  $V^+ = V^-$   
 $V_{in} = 3V$

if  $V_{in} > 3V$   $V^+ > V^-$   $V_{out} = 5V$

$V_{in} < 3V$   $V^+ < V^-$   $V_{out} = -5V$

