

Welcome to EECS 16A!

Designing Information Devices and Systems I

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Module 2
Lecture 3

Power and Voltage/Current Measurements
(Note 13)



Last Lecture

- Solve circuits for the currents and node potentials
- Set up a matrix problem of the form $A \vec{x} = \vec{b}$

where

\vec{x} consists of the unknown currents and potentials

\vec{b} contains the independent current and voltage sources

A describes the relationship between them.

$$A \vec{x} = \vec{b}$$

$\vec{x} \therefore$ unknowns

$\vec{b} \therefore$ knowns/constants

A \therefore knowns/constants

$$\vec{x} = \begin{bmatrix} i_1 \\ \vdots \\ i_k \\ v_1 \\ \vdots \\ v_n \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_{k+n} \end{bmatrix}$$

Rules:

• KVL

• KCL

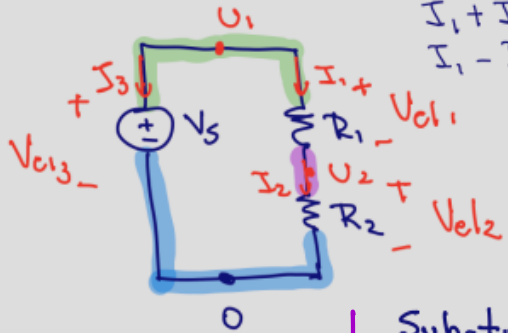
• Element definitions

• I & V relationship

* Started Voltage
Divider Analysis

Node Voltage Analysis – Voltage Divider

Step 7 – Use the IV relationships of each of the non-wire elements to fill in the remaining rows of A and



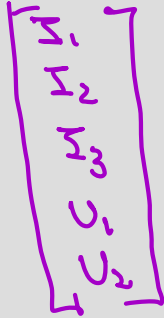
$$I_1 + I_3 = 0 \quad (1)$$

$$I_1 - I_2 = 0 \quad (2)$$

$$\underline{V_{el1}} = U_1 - U_2$$

$$\underline{V_{el2}} = U_2 - 0 = U_2$$

$$\underline{V_{el3}} = U_1 - 0 = U_1$$



$$\underline{V_{el1}} = R_1 \cdot I_1$$

$$\underline{V_{el2}} = R_2 \cdot I_2$$

$$\underline{V_{el3}} = V_S$$

Substitution:

$$\text{El}_1 \therefore U_1 - U_2 = R_1 \cdot I_1 \Rightarrow R_1 I_1 - U_1 + U_2 = 0 \quad (3)$$

$$\text{El}_2 \therefore U_2 = R_2 I_2 \Rightarrow R_2 I_2 - U_2 = 0 \quad (4)$$

$$\text{El}_3 \therefore U_1 = V_S \quad (5)$$

Node Voltage Analysis – Voltage Divider

Step 8 – Solve the system of equations to determine values of unknown variables.

$$I_1 + I_3 = 0 \quad (1)$$

$$-I_1 + I_2 = 0 \quad (2)$$

$$R_1 I_1 - U_1 + U_2 = 0 \quad (3)$$

$$R_2 I_2 - U_2 = 0 \quad (4)$$

$$U_1 = V_s \quad (5)$$

$$\overset{A}{\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ R_1 & 0 & 0 & -1 & 1 \\ 0 & R_2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}} \overset{x}{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix}} = \overset{b}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \end{bmatrix}}$$

$$I_1 = \frac{V_s}{R_1 + R_2}$$

$$I_2 = \frac{V_s}{R_1 + R_2}$$

$$I_3 = -\frac{V_s}{R_1 + R_2}$$

$$U_1 = V_s$$

$$U_2 = \frac{R_2}{R_1 + R_2} \cdot V_s$$

$$\hookrightarrow \alpha < 1$$

α is an operator

Electrical Circuit Analysis Algorithm (tool)

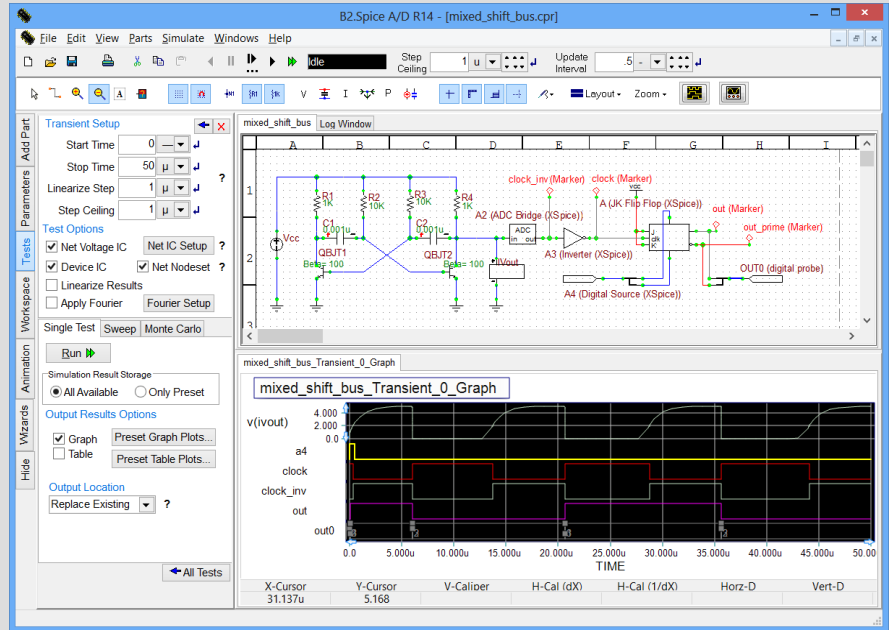
SPICE (Simulation Program with Integrated Circuit Emphasis): started as a student project at Berkeley!

Now the basis for open-source electronic circuit simulation, to design and model device characteristics and check circuit boards

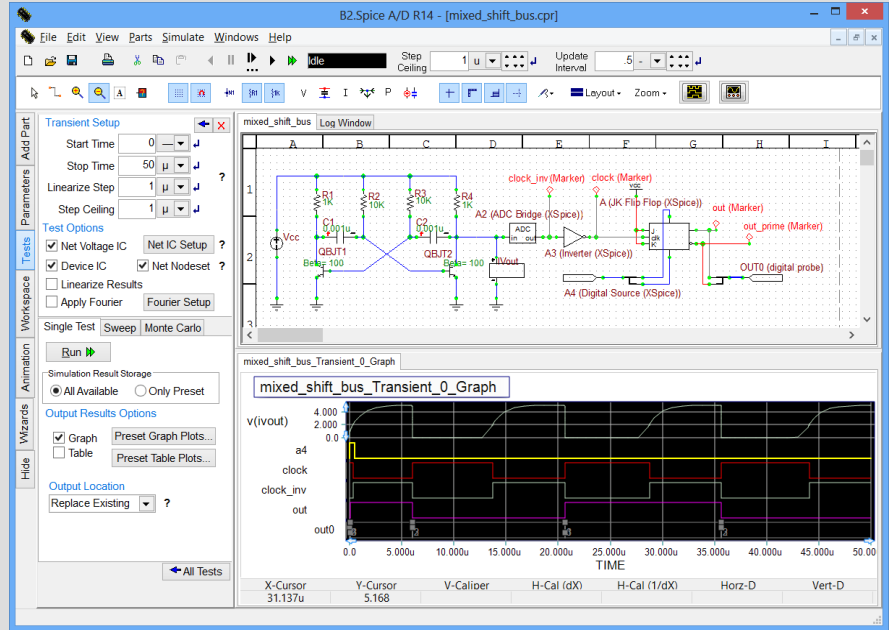
cādence[®]
SYNOPSYS[®]



Prof. Alberto L. Sangiovanni-Vincentelli



Electrical Circuit Analysis Algorithm (tool)



How to think about Energy and Power in circuits?

Current: flow of charges (electrons moving from point A to B inside a material) $I = \frac{dQ}{dt}$

It takes **energy** to move charge from A \rightarrow B \Rightarrow Voltage $V_{AB} = \frac{dE}{dq}$

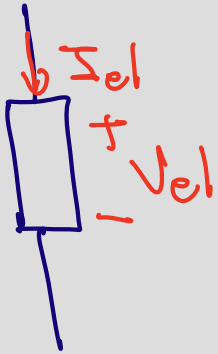
Power: is the rate of change of energy

$$P = \frac{dE}{dt} \cdot \frac{dq}{dt} = V \cdot I$$

(V) \cdot (A) = (W)

Energy and Power

$$P_{el} = V_{el} \cdot I_{el}$$



if element is a resistor

$$P = V \cdot I = R \cdot I \cdot I = R \cdot I^2 \geq 0$$

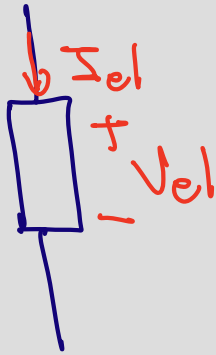
Power dissipated is positive

$$V_{el} = R \cdot I_{el}$$

$$I_{el} = \frac{V_{el}}{R}$$

$$P = V \cdot I = V_{el} \cdot V_{el}/I = V_{el}^2/I \geq 0$$

Energy and Power



$$P_{el} = V_{el} \cdot I_{el}$$

Open circuit



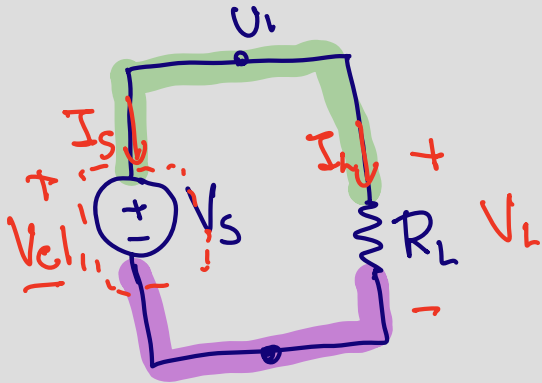
$$P_{el} = V_{el} \cdot \cancel{I_{el}}^0 = 0$$

wire



$$P_{el} = \cancel{V_{el}}^0 \cdot I_{el} = 0$$

Example



Elem 1 \circ

$$P_s = I_s \cdot V_{el.} \text{ (def.)}$$

$$P_s = I_s \cdot V_s$$

* Conservation of Energy

$$\text{KCL: } I_L + I_s = 0$$

$$\text{KVL: } U_1 - 0 = V_L$$

$$U_1 - 0 = V_{el.} = V_s$$

Power: Elem h

$$P_h = I_h \cdot V_h \text{ (def.)}$$

$$P_L = (-I_s) \cdot V_L$$

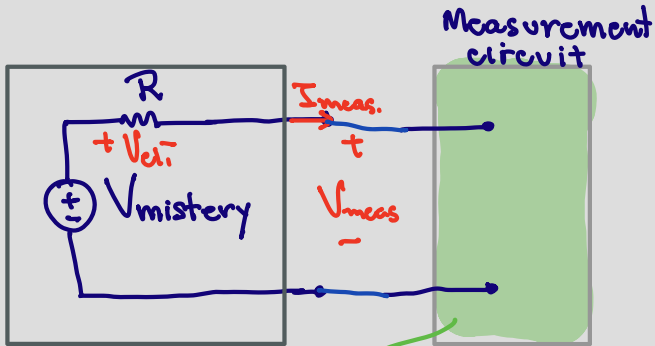
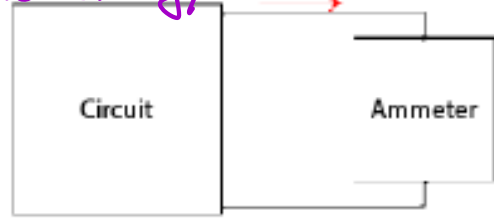
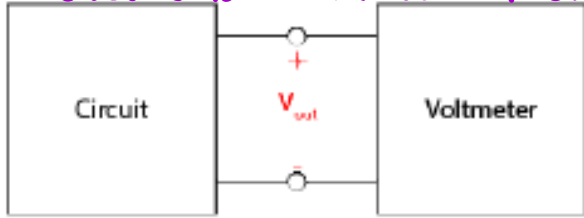
$$P_h = (-I_s) \cdot V_s$$

$$* P_h = -P_s$$

$$P_h + P_s = 0$$

How to measure Voltage and Current?

Measurement should not change the energy of the circuit



Must behave as an open-circuit

Goal : $V_{\text{meas}} = V_{\text{mistery}}$

$$V_{el1} = I_{\text{meas}} \cdot R$$

$$\text{KVL} : V_{\text{mistery}} - V_{el1} - V_{\text{mea}} = 0$$

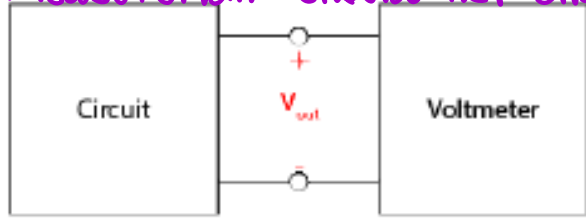
$$V_{\text{mistery}} = V_{el1} + V_{\text{meas}}$$

$$V_{\text{mistery}} = I_{\text{mea}} \cdot R + V_{\text{meas}}$$

$$V_{\text{mistery}} = V_{\text{meas}} \quad |_{\text{if}} \quad I_{\text{meas}} = 0$$

How to measure Voltage and Current?

Measurement should not change the energy of the circuit



Task/Goal:

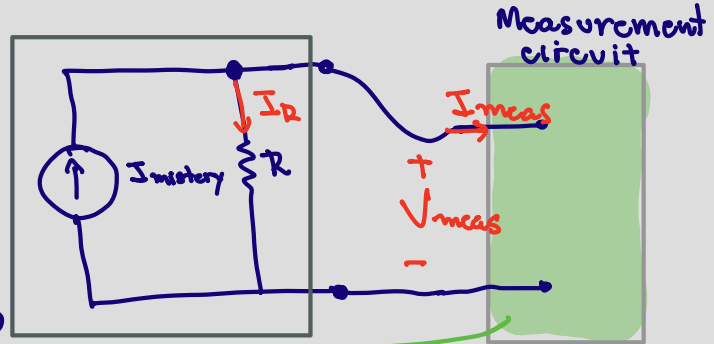
Measure $I_{mystery}$

KCh: $I_{mystery} = I_R + I_{meas}$

$I_{mystery} = I_{meas}$ | if $I_R = 0$

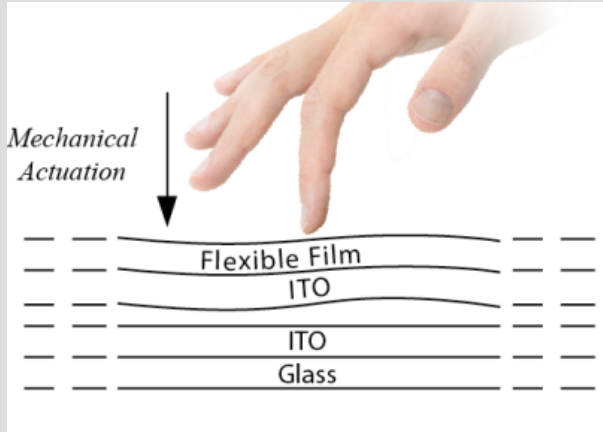
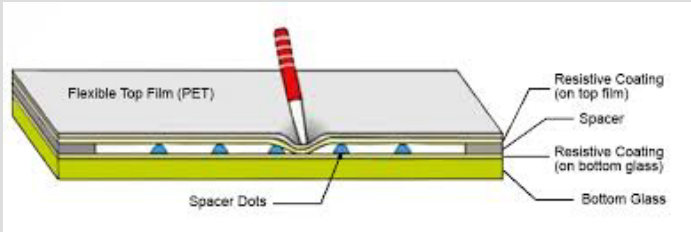
$I_R = \frac{V_{meas}}{R}$

$I_R = 0 ; \text{ if } V_{meas} = 0$

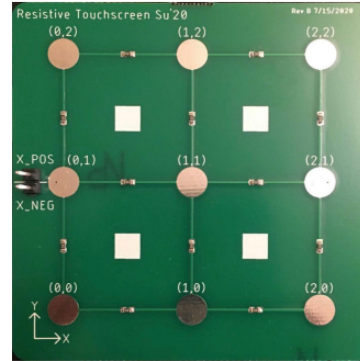


Must behave as a wire

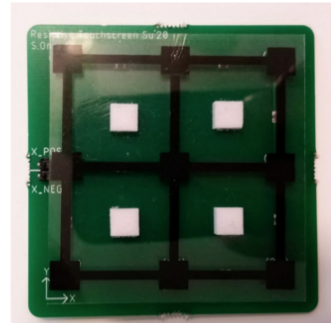
Resistive Touch Screen



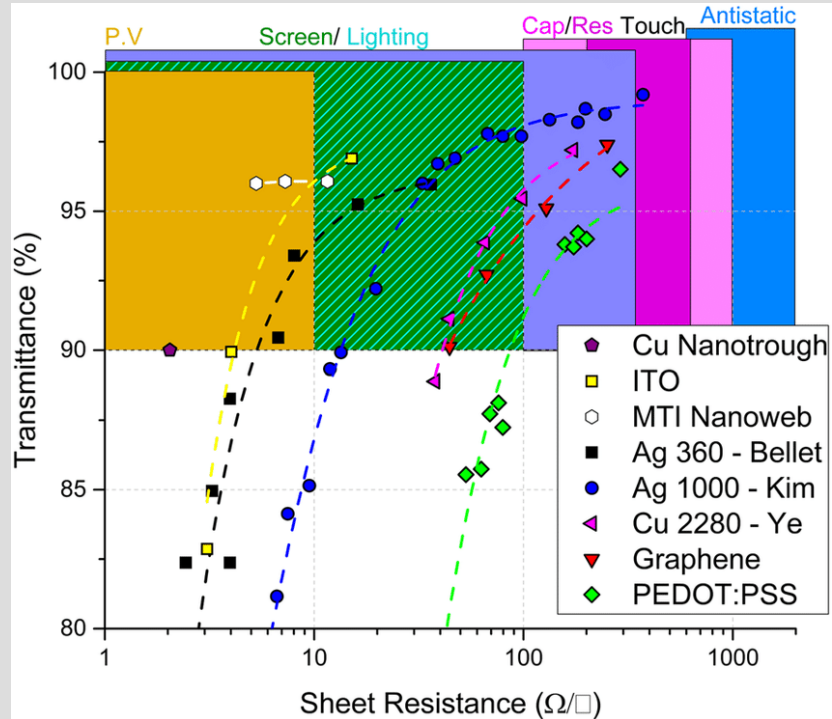
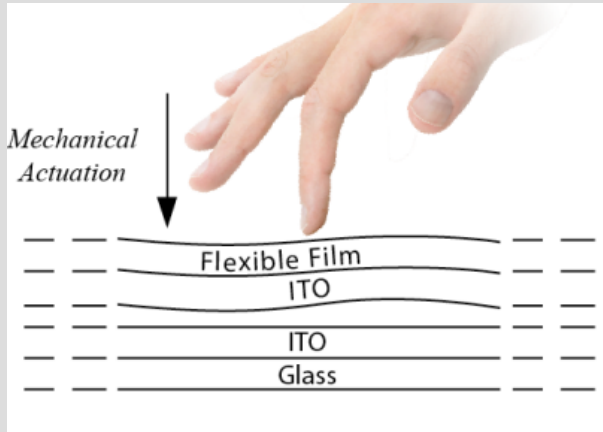
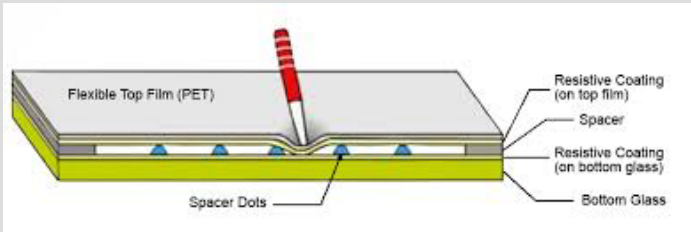
Bottom Layer: Resistive Layer



Top Layer: Flexible Resistive Layer

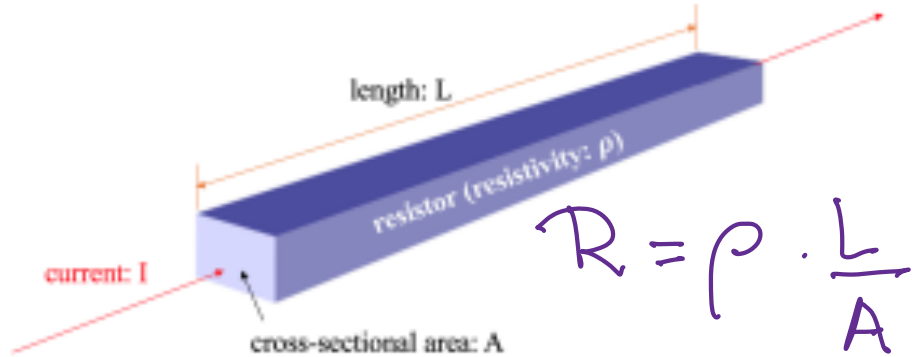


Resistive Touch Screen



Resistance, Resistivity, Conductivity – Properties of Materials

Material	Electrical characteristics	
	Electrical Resistivity ($\Omega \times \text{cm}$)	Electrical Conductivity ($\Omega^{-1} \times \text{cm}^{-1}$)
Cu	0.034×10^{-5}	29×10^5
Fe	32.54×10^{-5}	0.031×10^5
Ag	0.36×10^{-5}	2.8×10^5
Al	0.03×10^{-5}	33.3×10^5
Ni	0.046×10^{-5}	21.7×10^5
Cu-Fe	33.37×10^{-5}	0.030×10^5
Cu-Ag	2.71×10^{-5}	0.37×10^5
Al-Ni	0.564×10^{-5}	1.77×10^5



$$R = \rho \cdot \frac{L}{A}$$

Note 12

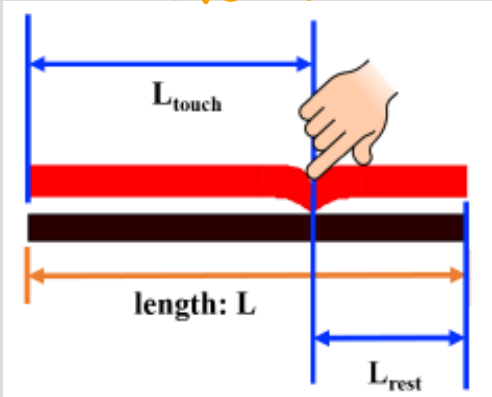
- longer the wire \rightarrow the more E is lost
- Wide wires \rightarrow lower resistance
- Wire properties depend on materials choice.

ρ = resistivity
(property of materials)

$\frac{L}{A}$ \therefore geometric parameters
(property of the wire)

Resistive Touch Screen

Problem: to find the location of touch.



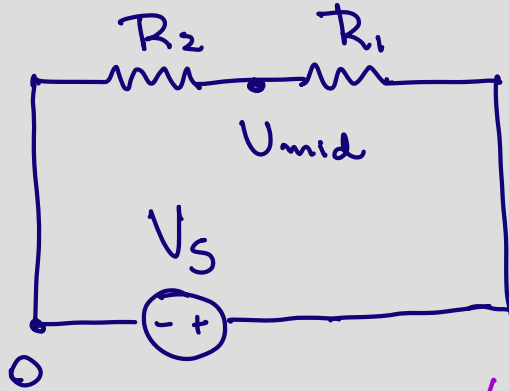
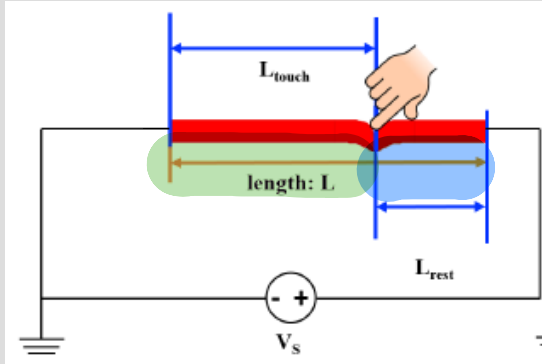
Go from mechanical to electrical quantity.

Want to measure $\frac{l_{\text{touch}}}{L}$

l_{touch} is unknown

Resistive Touch Screen – First model

$U_{mid} = ?$



$$U_{mid} = \frac{R_2}{R_2 + R_1} \cdot V_s \quad (\text{Voltage Divider}^*)$$

$$R_1 = \rho \cdot \frac{L_{rest}}{A}$$

$$U_{mid} = \frac{\cancel{\rho} \cdot L_{touch} / A}{\cancel{\rho} \cdot \frac{L_{touch}}{A} + \cancel{\rho} \cdot \frac{L_{rest}}{A}} \cdot V_s$$

$$R_2 = \rho \cdot \frac{L_{touch}}{A}$$

$$U_{mid} = \frac{L_{touch}}{L_{touch} + L_{rest}} \cdot V_s = \frac{L_{touch}}{L} \cdot V_s$$