

1. Solving Systems of Equations

- (a) Systems of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following system of equations, state whether or not a solution exists. If a solution exists, list all of them.

$$\text{i. } \begin{cases} 49x + 7y = 49 \\ 42x + 6y = 42 \end{cases} \quad \text{ii. } \begin{cases} 5x + 3y = -21 \\ 2x + y = -9 \end{cases} \quad \text{iii. } \begin{cases} 49x + 7y = 60 \\ 42x + 6y = 30 \end{cases}$$

$$\text{iv. } \begin{cases} 2x + 2y + 4z = -1 \\ \quad y + z = -2 \\ x + 2y + 3z = 2 \end{cases} \quad \text{v. } \begin{cases} 2x + 2y + 4z = 6 \\ \quad y + z = 1 \\ x + 2y + 3z = 4 \end{cases} \quad \text{vi. } \begin{cases} x + y + z = 4 \\ \quad \quad 3z = 6 \\ \quad y + z = 3 \end{cases}$$

- (b) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow along as your TA walks through `dis1B.ipynb`.

2. Gaussian Elimination

Use Gaussian elimination to solve the following systems. Does a solution exist? Is it unique?

- (a)

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 8 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$