# EECS 16A Designing Information Devices and Systems I
# Summer 2017 D. Aranki, F. Maksimovic, V. Swamy Discussion 2D

**1. Mechanical Problems – Inverse**

In each part, find the inverse of **A** or determine that the inverse doesn't exist.

(a) $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 0 & 4 \end{bmatrix}$

(d) $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

**2. Mechanical Problems – Eigenspaces**

In each part, find the eigenspace of **M** associated with the eigenvalue $\lambda$.

(a) $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$, $\lambda = 1$.

(b) $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$, $\lambda = 9$.

(c) $\mathbf{M} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $\lambda = 3$.

**3. Eigenvalues and Special Matrices – Visualization** The following parts don't require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

(a) Does a rotation matrix in $\mathbb{R}^2$ have any eigenvalue $\lambda \in \mathbb{R}$?

(b) Does a reflection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

(c) Does a projection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

(d) If a matrix **M** has an eigenvalue 0, what does this say about its null space? What does this say about the solutions of the system of linear equations $\mathbf{M}\vec{x} = \vec{b}$?

**4. Steady State Reservoir Levels**

We have 3 reservoirs: $A, B$ and $C$. The pumps system between the reservoirs is depicted in Figure **??**.

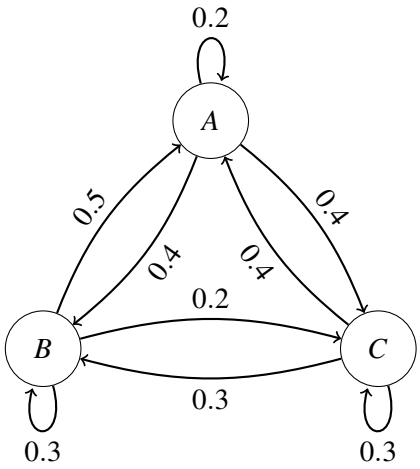


Figure 1: Reservoir pumps system.

(a) Write the transition matrix representing the pumps system in the problem.

(b) Assuming you start the pumps with the water levels of the reservoirs at $A_0 = 129, B_0 = 109, C_0 = 0$ (in kiloliters). What would be the steady state water levels (in kiloliters) according to the pumps system described in the problem?

*Hint:* If $\vec{x}_{ss} = \begin{bmatrix} A_{ss} \\ B_{ss} \\ C_{ss} \end{bmatrix}$ is a vector describing the steady state levels of water in the reservoirs (in kiloliters), what happens if you fill the reservoirs $A, B$ and $C$ with $A_{ss}, B_{ss}$ and $C_{ss}$ kiloliters of water, respectively, and apply the pumps once?

*Hint II:* Note that the pumps system preserves the total amount of water in the reservoirs. That is, no water is lost or gained by applying the pumps.