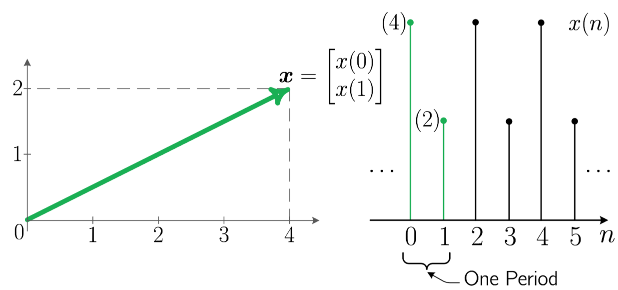
1. Periodic Signals

Periodic signals are ones that repeat themselves entirely after some time period. That is, after some time p, the signal $x(n)$ repeats itself so that $x(n+p) = x(n)$. Discrete periodic signals, during the period, do not update continuously through time. They instead update in specific discrete time steps, as if sampling a continuous signal.

Since there are a finite number of "unique" sequences in a discrete periodic signal, it is natural for us to represent the signal as a vector. We observe one period and treat the value at each time step as a different value in our vector.



Let us study the signal $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ that is periodic over $p = 2$.

(a) Write the signal as a linear combination of the standard/canonical basis. What signals do these vectors correspond to? How can we interpret the linear combination?

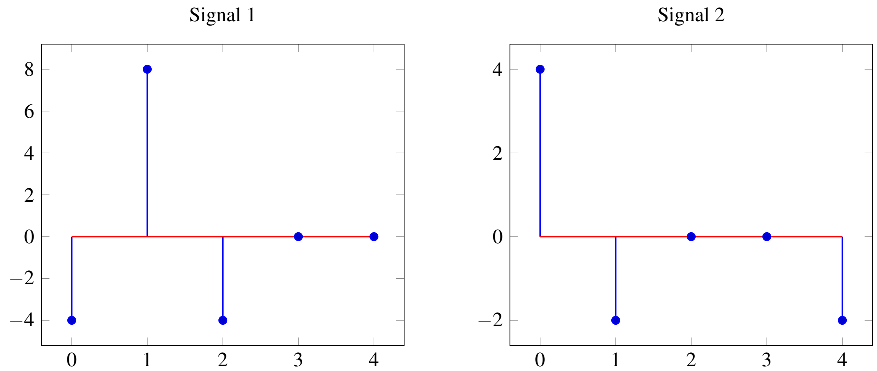
(b) Write the signal as a linear combination of the basis $\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\}$. What signals does these vectors correspond do? How can we interpret the linear combination?

(c) Project the signal $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ onto each of the vectors in the previous part. How does these vectors relate to the linear combination from the previous part?

(d) Given the above, what is an easy way to find the coefficients for describing the signal as a linear combination of our basis? What property must hold about our basis?

2. Correlation

You are given the following two signals:



Assume that both signals are periodic with period 5, that is, each plot shows one full period of a periodic signal.

(a) Sketch the autocorrelation (correlation with itself) of signal 1.

(b) Sketch the autocorrelation of signal 2.

(c) Sketch the cross-correlation of signal 1 with signal 2. Suppose we know signal 2 is a delayed (and attenuated) version of signal 1. What does the cross-correlation tell us about the delay?

3. Autocorrelation Peak

Let $\rho_{xx}[m]$ be the autocorrelation of an N-periodic signal $x[n]$. Prove that $\rho_{xx}[0] \geq |\rho_{xx}[m]|\ \forall m$. In other words, the autocorrelation peak (maximum value of autocorrelation) of any periodic signal always occurs at lag $m = 0$.

4. Search and Rescue Dogs

Berkeley's Puppy Pound needs your help! While Mr. Muffin was being walked, the volunteer let go of his leash and he is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the puppies at the pound have a collar that sends a bluetooth signal to receiver towers, which are spread throughout the streets (pictured below). If the puppy/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of 3 city blocks. Can you help the pound locate their lost puppy?

Note: A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map.) Mr. Muffin is constrained to running wild in the streets, meaning he won't be found in any buildings. If your TA asks 'Where is Mr. Muffin?' it is sufficient to answer with his intersection or 'between these two intersections'.

[1]

(a) You check the logs of the cell towers, and they have received the following messages:

Sensor	Distance
N	1.3
W	3
E	1.5
S	3

On the map provided, identify where Mr. Muffin is!

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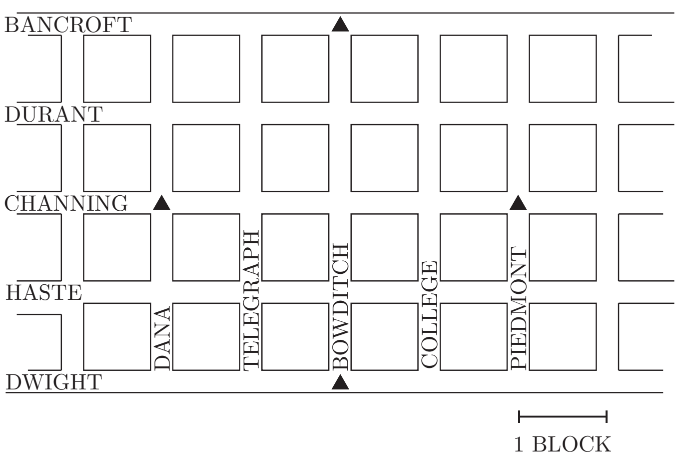


SOUTH OF DWIGHT

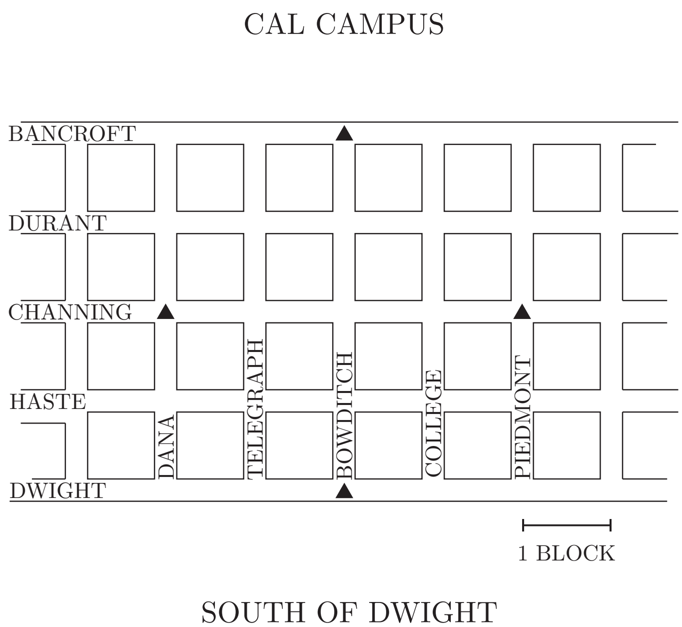
(b) Can you set this up as a system of equations? Is it linear? If it's not linear, can you think of a way to make it linear? Now, how do you set this up in matrix form?
Hint: Set (0,0) to be Channing and Bowditch.
Hint 2: distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Hint 3: You don't need all 4 equations. You have two unknowns, x and y. You know from lecture that you need three circles to uniquely find a point. How can you use the third circle/equation to get two equations and two unknowns?

[1] http://www.pupsmile.com/wp-content/uploads/2012/11/running_happy_dog-1024x684.jpeg

(c) Suppose Mr. Muffin is moving fast, and by the time you get to destination in part (a) he's already run off! You check the logs of the cell towers again, and see the following updated messages:

Sensor	Distance
N	2.2
W	Out of Range
E	1.1
S	Out of Range

Can you find Mr. Muffin?



(d) Mr. Muffin is a very mischievous puppy, and while playing and running around he damaged his collar. The transmitter on his collar will still send a signal to the receiver towers, but the distance sensor has noise. You check the logs of the cell towers, and they have received the following messages:

Sensor	Distance
N	1.7 ± 0.5
W	2.1 ± 0.2
E	Out of Range
S	Out of Range

On the map provided, identify where Mr. Muffin is! Can you find exactly where he is?

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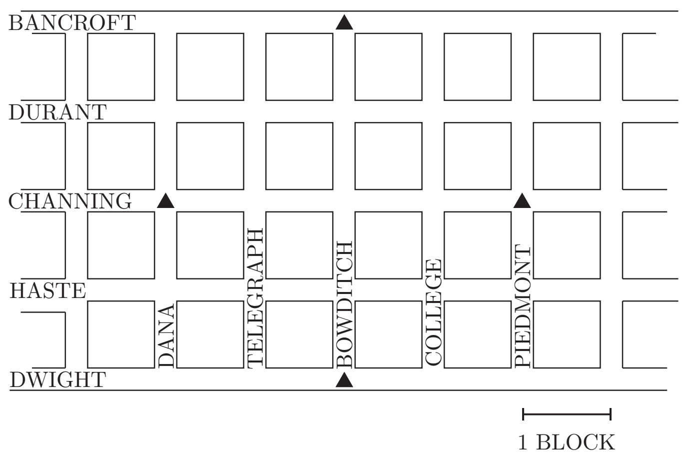

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5. Retail Store Marketing

Intro The retail store EehEeh Sixteen would like to create a smart system where it decides which promotion to give to its customers when they checkout, depending on things they may be interested in. The promotion is supposed to be printed alongside the receipt and be used during their next purchase. The problem is, the customers don't disclose what their interests are when they checkout, and the only data the retail store can use are their current purchase data.

The Setting The store uses the following set of attributes in their decision making process: interest in party products, interest in family products, interest in student products and interest in office products. These attributes are used to to describe each of the promotions the store offers. More concretely, the store attaches to each promotion A, a "score" vector $\vec{x_A} \in \mathbb{R}^4$ such that $\vec{s_A} = \begin{bmatrix} \text{party-related score} \\ \text{family-related score} \\ \text{student-related score} \\ \text{office-related score} \end{bmatrix}$ which describes the ideal target customer. Therefore, the store would like to infer these same attributes about each customer at time of checkout so that they can print a promotion tailored to that customer on the receipt.

The data that the algorithm is allowed to use are the subtotals (in the current purchase) in the following four categories: Food, movies, art, and books & supplies.

The Goal EehEeh Sixteen hired the same intern from the Framingham heart study to devise an algorithm that takes a customer's purchase subtotals in the four categories listed above (food, movies, art and books & supplies), and decides which promotion to print on the receipt. The intern is lost and given the awesomeness of your help last time, he needs your help again. In this problem, you will walk him through a possible design of such an algorithm.

(a) Assuming we somehow have the interests of a customer c in a vector $\vec{x_c} = \begin{bmatrix} c_{\text{party}} \\ c_{\text{family}} \\ c_{\text{student}} \\ c_{\text{office}} \end{bmatrix}$ and a set of promotions $A_1, A_2, \ldots, A_N$, with their attached vectors of scores $\vec{s_{A_1}}, \vec{s_{A_2}}, \ldots, \vec{s_{A_N}}$. We would like to select which promotion is best aligned with the preferences of the customer. Assuming we have a function $sim(\vec{x_c}, \vec{s_A})$ which outputs a similarity score (higher score means more similar) between the customer c and the promotion A, how can we select which promotion to print to the customer on her receipt?

(b) Would $sim_1(\vec{x_c}, \vec{s_A}) = \|\vec{x_c} - \vec{s_A}\|$ be a good similarity measure? Why? What about $sim_2(\vec{x_c}, \vec{s_A}) = \frac{1}{\|\vec{x_c} - \vec{s_A}\|}$? Why? What about $sim_3(\vec{x_c}, \vec{s_A}) = \langle \vec{x_c}, \vec{s_A} \rangle$? Why? What about $sim_4(\vec{x_c}, \vec{s_A}) = \left\langle \vec{x_c}, \frac{\vec{s_A}}{\|\vec{s_A}\|} \right\rangle$? Why?

(c) The intern hands you research that the EehEeh Sixteen research division conducted, which calculated the distribution of spending in the store for people who are purely interested in only one category. The results are depicted in Table **??**. Use this information to devise a system of linear equations, such that solving this system will result in the customer's preferences given her spending.

	Spending Category			
Interest Category	**Food**	**Movies**	**Art**	**Books & Supplies**
Party	40%	33%	22%	5%
Family	70%	10%	10%	10%
Student	20%	10%	15%	55%
Office	5%	2%	20%	73%

Table 1: The distribution of spending of people in each category.

(d) Combine these results into a complete algorithm.

(e) Run the algorithm on a customer, Jane Doe, that spent \$6 on food, \$4 on movies, \$1 on art and \$5 on books. With promotions A_1, A_2, A_3 and A_4 targeted at customers with preferences $\vec{s_{A_1}} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$, $\vec{s_{A_2}} = \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$, $\vec{s_{A_3}} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{5}{2} \\ -\frac{1}{2} \end{bmatrix}$ and $\vec{s_{A_4}} = \begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$

(f) Will there ever be a customer for which the system devised in part **??** will yield no solutions or infinite solutions?