1. Lateration with Linear Systems of Equations (Spring 2017 Final)

In this problem, we'll set up the system of equations you need in order to uniquely determine the microphone position while only knowing the relative time differences of arrival. We will use 5 beacons for the setup on this exam.

All beacons transmit their unique signal at time 0 that each arrive at the microphone at different time T_m depending on the microphone position. Let $\tau_m = T_m - T_0$ be the time difference of arrival between beacon m and beacon 0.

(a) Using the fact that $R_m = vT_m$, where R_m is the distance of the microphone from beacon m and v is the speed of the audio signals in air, show that we can write:

$$0 = v\tau_m + 2R_0 + \frac{R_0^2 - R_m^2}{v\tau_m}, \tag{1}$$

where τ_m is the time difference of arrival for beacon 0 and beacon m.

(b) We introduce coordinates based on beacon 0. Beacon 0 is located at $(0,0)$, beacon m is located at known positions (x_m, y_m). We try to find the microphone position in the sensor plane, given by (x,y). We see that R_m, the distance of the microphone from beacon m, is a function of the microphone position:

$$R_m = \sqrt{(x - x_m)^2 + (y - y_m)^2} \tag{2}$$

By plugging the relationship from Eqution **??** into Equation **??** above for R_0, R_1, and R_2, we can write a system of equations with two equations and two unknowns:

$$0 = v\tau_1 + 2\sqrt{x^2 + y^2} + \frac{2x_1x - x_1^2 + 2y_1y - y_1^2}{v\tau_1} \tag{3}$$

$$0 = v\tau_2 + 2\sqrt{x^2 + y^2} + \frac{2x_2x - x_2^2 + 2y_2y - y_2^2}{v\tau_2} \tag{4}$$

What is the minimum number of beacons you would need to use to create this system of equations? Justify why you cannot use linear algebra methods to solve the system of equations in this part.

(c) We can use extra information to make the system linear. Subtract Equation **??** from Equation **??**. This is a linear equation and should have the form of

$$ax + by + c = 0. \tag{5}$$

How many linear equations of this form can we construct with 3 beacons, and how many unknowns do we have? Can you use this system to find a unique solution for (x,y) using linear algebra methods? Why or why not?

(d) For our microphone lateration system, we want to use 5 beacons. Given 5 beacons, how many unique equations of the form $ax+by+c=0$ can you write? (You do not need to write out the expressions for a, b, and c.) Write a matrix equation, $\mathbf{A}\vec{x}=\vec{b}$ in terms of a_n, b_n, and c_n, where the subscript n corresponds to one of the unique equations. Also, use $\vec{x}=\begin{bmatrix} x \\ y \end{bmatrix}$.