1. Gram-Schmidt and QR Factorization

Compute the QR factorization of the following matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2. QR Proofs

(a) Let $\mathbf{A} = \mathbf{QR}$, where $\mathbf{Q}$ is an orthonormal matrix and $\mathbf{R}$ is an upper triangular matrix. Show that if $\mathbf{A}$ is invertible, then $\mathbf{R}$ is invertible.

(b) Let $\mathbf{A} = \mathbf{QR}$, where $\mathbf{A}$ has linearly independent columns, $\mathbf{Q}$ is an orthonormal matrix, and $\mathbf{R}$ is an upper triangular matrix. Show that $\mathbf{A}$ and $\mathbf{Q}$ have the same column space.

3. QR Factorization (Fall 2016 Final)

Recall that the solution to a linear least squares problem is a minimization of $\left\|\vec{b} - \mathbf{A}\vec{x}\right\|^2$. Show that the approximation of $\vec{x}$, $\vec{\hat{x}}$, in this linear least squares formula has an equivalent representation using the QR factorization of $\mathbf{A}$, ($\mathbf{A} = \mathbf{QR}$). In other words, express $\vec{\hat{x}}$ in terms of $\mathbf{R}$, $\mathbf{Q}$, and $\vec{b}$. Assume the matrix $\mathbf{A}$ is full rank.