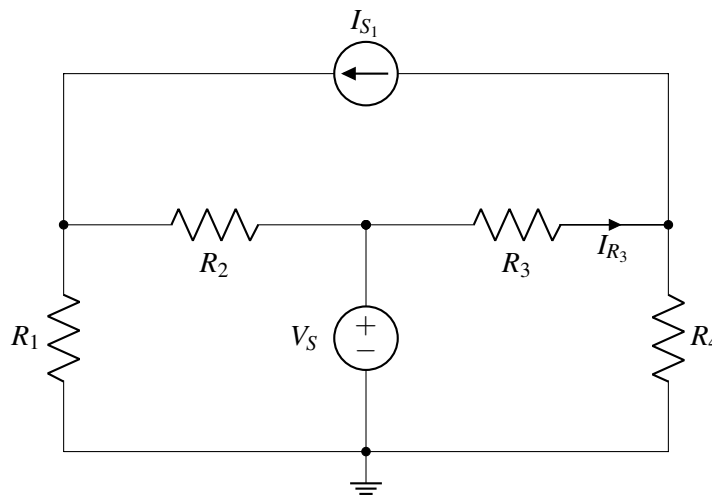


# EECS 16A Designing Information Devices and Systems I

## Summer 2020 Discussion 3B

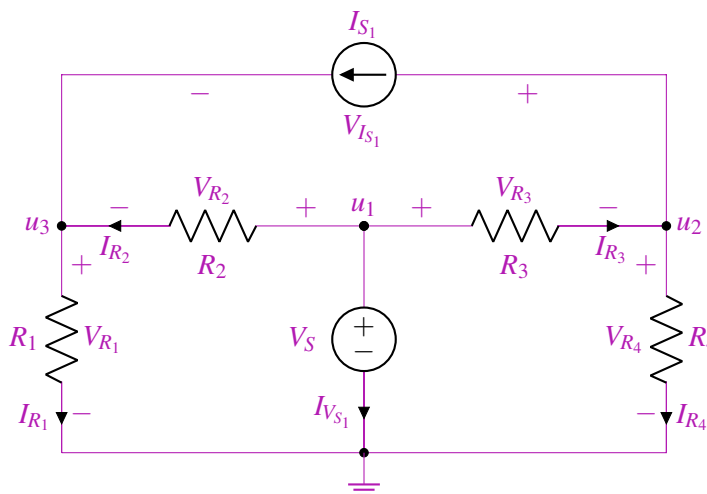
### 1. Circuit Analysis

- Use nodal analysis to solve for all node voltages.
- Find current  $I_{R_3}$  flowing through resistor  $R_3$ .



#### Answer:

- Let us follow the procedure illustrated in lecture. Step 1 (selecting a ground node) has already been done for us, we will go through steps 2-4 in the following circuit diagram. As a reminder the steps are the following:
  - Label all known voltages determined by voltage sources (denoted below as  $u_1$ ),
  - Label all remaining unknown nodes (denoted below as  $u_2, u_3$ ),
  - Label element voltages and currents ( $V_{R_1}, I_{R_1}$ , etc.)



**Note:** You can pick any direction you want for the currents but once you have done so you the voltages you choose have to abide by the passive sign convention!! Vice versa, you can pick the element voltage sign first but then the current has to abide by the passive sign convention.

Step 5: Write KCL equations for all nodes with unknown voltages (namely  $u_2, u_3$ ):

$$\begin{aligned} -I_{R_3} + I_{S_1} + I_{R_4} &= 0 \\ -I_{R_2} - I_{S_1} + I_{R_1} &= 0 \end{aligned}$$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$\begin{aligned} u_3 - 0 &= I_{R_1} R_1 \\ V_s - u_3 &= I_{R_2} R_2 \\ u_2 - 0 &= I_{R_4} R_4 \\ V_s - u_2 &= I_{R_3} R_3 \end{aligned}$$

Where we used the fact that  $u_1 = V_s$

Step 7: Substitute expressions found in 6 into the KCL equations from step 5,

$$\begin{aligned} -\frac{V_s - u_2}{R_3} + I_{S_1} + \frac{u_2}{R_4} &= 0 \\ -\frac{V_s - u_3}{R_2} - I_{S_1} + \frac{u_3}{R_1} &= 0 \end{aligned}$$

Notice that we can now directly solve for  $u_2$  and  $u_3$  and then substitute into the equations of Step 6 to find the currents through all resistors. The node voltages are given by:

$$\begin{aligned} u_2 &= \frac{V_s R_4 - I_{S_1} R_3 R_4}{R_3 + R_4} \\ u_3 &= \frac{V_s R_1 + I_{S_1} R_1 R_2}{R_1 + R_2} \end{aligned}$$

Finally, KCL in node  $V_1$  will give us the current through the voltage source,  $V_s$ .

(b)

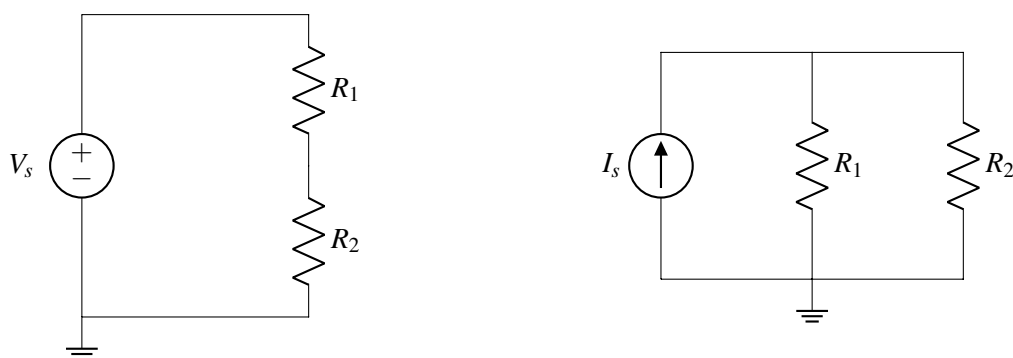
$$I_{R_3} = \frac{V_s - u_2}{R_3}$$

## 2. Current Divider

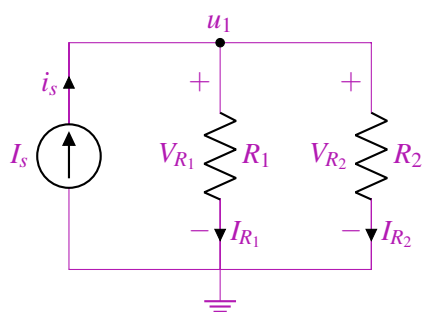
In lecture, we showed that for the circuit shown to the left, we can find the voltage across the resistor  $R_2$  using a simple formula:

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_s$$

Try to derive a similar formula for the circuit shown below to the right, relating the current  $I_{R_2}$  through  $R_2$  to the current through the current source  $I_s$ .

**Answer:**

Let us follow the procedure illustrated in lecture. Step 1 (selecting a ground node) has already been done for us, we will go through steps 2-4 in the following circuit diagram. As a reminder the steps are the following:  
 Step 2: Label all known voltages determined by voltage sources (none in this circuit),  
 Step 3: Label all remaining unknown nodes (denoted below  $u_1$ ),  
 Step 4: Label element voltages and currents ( $V_{R_1}, I_{R_1}$ , etc.)



**Note:** You can pick any direction you want for the currents but once you have done so you the voltages you choose have to abide by the passive sign convention!! Vice versa, you can pick the element voltage sign first but then the current has to abide by the passive sign convention.

Step 5: Write KCL equations for all nodes with unknown voltages (namely  $u_1$ ):

$$I_s - I_{R_1} - I_{R_2} = 0$$

Where we used the fact that  $i_s = I_s$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$u_1 - 0 = I_{R_1} R_1$$

$$u_1 - 0 = I_{R_2} R_2$$

Step 7: Substitute expressions found in 6 into the KCL equations from step 5,

$$I_s - \frac{u_1}{R_1} - \frac{u_1}{R_2} = 0$$

Notice that we can now directly solve for  $u_1$  and then substitute into the equations of Step 6 to find the currents through all resistors. The node voltages are given by:

$$u_1 = \frac{R_1 R_2}{R_1 + R_2} I_s$$

The current through the resistors are given by:

$$I_{R_1} = \frac{R_2}{R_1 + R_2} I_s$$

$$I_{R_2} = \frac{R_1}{R_1 + R_2} I_s$$

### 3. Practice: Bio-Molecule Detector

One application for electronics that has gained a lot of attention over the past several years is in so-called “bio-molecule” detection. The idea is to build a system that detects the presence of specific molecules and/or cells (e.g. specific viruses, proteins, etc.) in a biological sample; if this detection can be performed automatically and using relatively low-cost components, it can have a dramatic impact on a number of areas such as medical diagnosis, drug development, DNA sequencing, etc.

In this problem, we’ll look at how some of the techniques we learned about in the touchscreen module can be applied to realize a hypothetical bio-molecule detector. (Real bio-molecule detection systems involve quite a bit more complexity than what we’ll include here, but in many designs the same basic principles apply.)

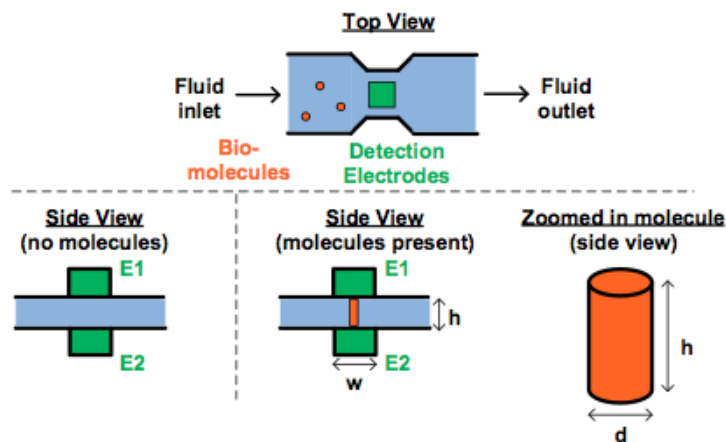


Figure 1: Bio-molecule detector.

As shown in Figure 1, the detector works by flowing a liquid that may or may not contain the biomolecules through a region in the device that has electrodes on the top and bottom of the liquid channel. The electrodes (E1/E2 in Figure 1) are chemically “functionalized” (using e.g. some appropriately designed antibodies), so that if the specific bio-molecule of interest is present in the fluid sample, one or more of the molecules will get physically trapped between the two electrodes (bottom right of Figure 1). After all of the fluid has been cleared out of the device (i.e., so that if there are no bio-molecules present, there is only air in between the two electrodes E1/E2), we can then figure out whether or not one or more bio-molecules were trapped by measuring the resistance between the two electrodes.

- (a) Let’s first assume that we want to detect the presence of a bio-molecule by measuring resistance. If no bio-molecule is present, what should be the resistance between E1/E2? As shown in Figure 1, if each bio-molecule is a cylinder with diameter  $d = 10\text{nm}$ , height  $h = 100\text{nm}$ , and has a resistivity

$\rho = 100 \mu\Omega\text{m}$ , what would be the resistance between E1 and E2 if only a single bio-molecule has been trapped? Note that you can assume that the trapped molecule is exactly vertically oriented when it is trapped – i.e., the top and bottom faces of the molecule are both aligned with surfaces of the electrodes.

**Answer:**

If no bio-molecule is present between E1/E2, we are left with just air between the electrodes, so for all intents and purposes, the resistance between them should be *infinite*.

If there is a bio-molecule trapped between E1/E2, the resistance  $R$  should be:

$$R_{\text{molecule}} = \frac{\rho \cdot l}{A} = \frac{100 \times 10^{-6} \Omega\text{m} \cdot 100 \times 10^{-9} \text{m}}{\pi \cdot \left(\frac{1}{2} \cdot 10 \times 10^{-9} \text{m}\right)^2} = 127.3 \text{k}\Omega$$

- (b) Using the same numbers for  $d$ ,  $h$ , and  $\rho$  as part (a), as a function of the number of trapped bio-molecules  $N_{\text{molecules}}$ , what is the resistance between E1 and E2? (Note that you can assume that  $N_{\text{molecules}}$  is small enough that all of the molecules fit within the electrode area and that all of the molecules are still trapped in an exactly vertical orientation.)

**Answer:**

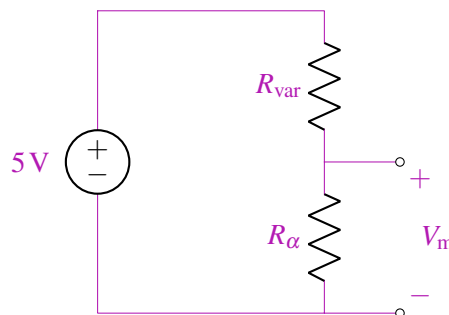
By having many bio-molecules in the detector, the cross-sectional area becomes the sum of the areas of all of the molecules. Therefore, the net resistance between E1/E2 is:

$$R = \frac{127.3 \text{k}\Omega}{N_{\text{molecules}}}$$

- (c) Given your answers to parts (a) and (b), design a circuit that will output a voltage greater than 2.5 V if more than 5 molecules are trapped.

**Answer:**

Design idea: use the relationship of the resistors in a voltage divider and the respective drops across them.



We use a voltage divider with a 5 V source and resistors  $R_{\alpha}$  and  $R_{\text{var}}$  in series.  $R_{\text{var}}$  is the resistance across the molecules, and  $R_{\alpha}$  is used to compare against  $R_{\text{var}}$ . We measure across  $R_{\alpha}$ . We want the measured voltage  $V_m > 2.5 \text{V}$  when we have more than 5 molecules, so we want:

$$R_{\alpha} > R_{\text{var}}(6)$$

$$R_{\alpha} > \frac{127.3 \text{k}\Omega}{6}$$

When we have 5 or less molecules, we want  $V_m < 2.5 \text{V}$  Therefore,

$$R_{\alpha} < R_{\text{var}}(5)$$

$$R_{\alpha} < \frac{127.3 \text{k}\Omega}{5}$$

We can then set  $R_{\alpha}$  to  $\frac{127.3 \text{k}\Omega}{5.5} = 23.15 \text{k}\Omega$ .