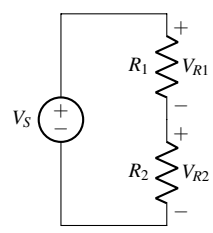
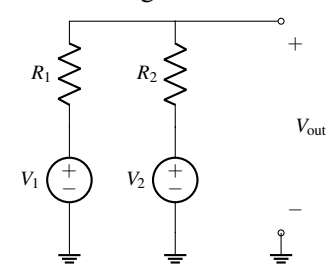
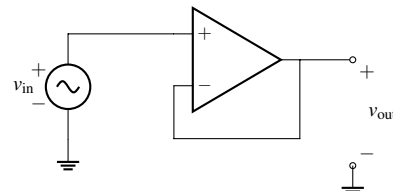
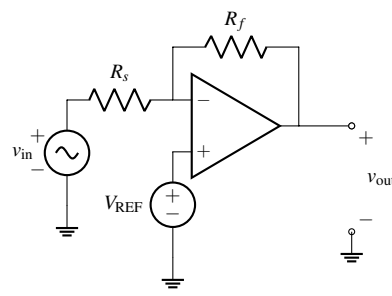
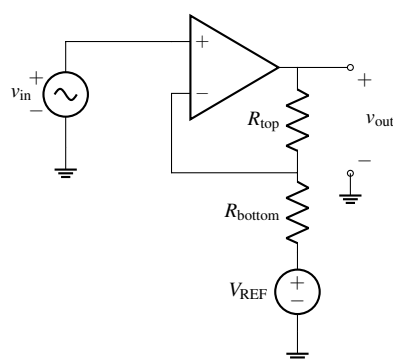
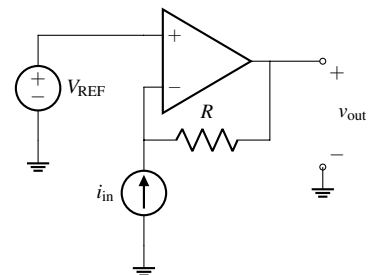


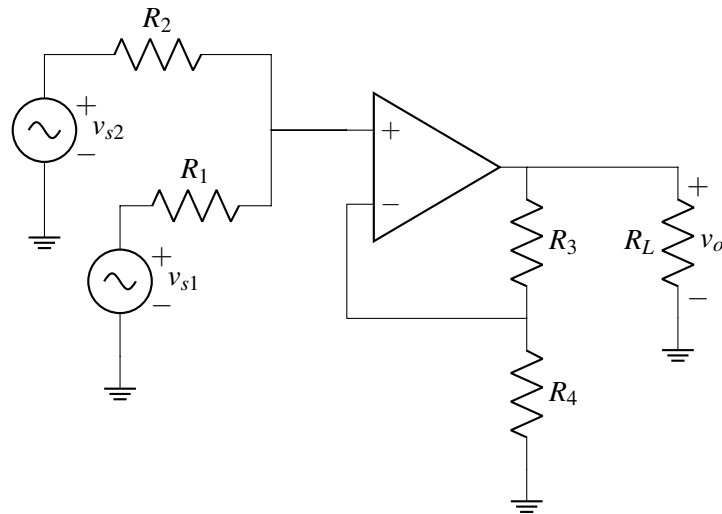
EECS 16A Designing Information Devices and Systems I

Summer 2020 Discussion 5A

For Reference: Example Circuits

<p style="text-align: center;">Voltage Divider</p>  $V_{R2} = V_S \left(\frac{R_2}{R_1 + R_2} \right)$	<p style="text-align: center;">Voltage Summer</p>  $V_{out} = V_1 \left(\frac{R_2}{R_1 + R_2} \right) + V_2 \left(\frac{R_1}{R_1 + R_2} \right)$	<p style="text-align: center;">Unity Gain Buffer</p>  $\frac{v_{out}}{v_{in}} = 1$
<p style="text-align: center;">Inverting Amplifier</p>  $v_{out} = v_{in} \left(-\frac{R_f}{R_s} \right) + V_{REF} \left(\frac{R_f}{R_s} + 1 \right)$	<p style="text-align: center;">Non-inverting Amplifier</p>  $v_{out} = v_{in} \left(1 + \frac{R_{top}}{R_{bottom}} \right) - V_{REF} \left(\frac{R_{top}}{R_{bottom}} \right)$	<p style="text-align: center;">Transresistance Amplifier</p>  $v_{out} = i_{in} (-R) + V_{REF}$

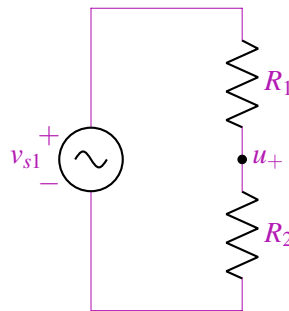
1. Multiple Inputs To One Op-Amp



(a) For the circuit above, find an expression for v_o . (Hint: Use superposition.)

Answer:

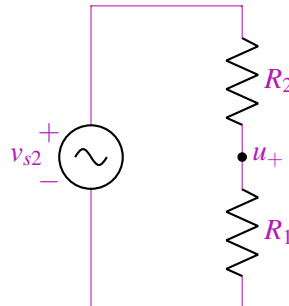
Let's call the potential at the positive input of the op-amp u_+ . Using superposition, we first turn off v_{s2} and find u_+ . The circuit then looks like:



We recognize the above circuit as a voltage divider. Thus,

$$u_{+,v_{s1}} = \frac{R_2}{R_1 + R_2} v_{s1}$$

By symmetry, we expect v_{s2} to have a similar circuit and expression. The circuit for v_{s2} looks like:



The expression for u_+ with v_{s2} is then:

$$u_{+,v_{s2}} = \frac{R_1}{R_1 + R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$u_+ = \frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}$$

With u_+ determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.

From the negative feedback rule, $u_+ = u_-$. Using voltage dividers, we can express u_- in terms of v_o :

$$u_- = \frac{R_4}{R_3 + R_4} v_o$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) u_- = \left(1 + \frac{R_3}{R_4}\right) u_+$$

Now, to find the final output, we can set u_+ to our earlier expression.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right)$$

- (b) How could you use this circuit to find the sum of different signals?

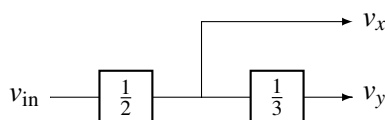
Answer:

The circuit already finds the weighted sum of two inputs. By setting $R_1 = R_2$ and $R_3 = R_4$, we can take the exact sum of two inputs.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right) = (1 + 1) \left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right) = v_{s1} + v_{s2}$$

2. Modular Circuits

In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations in order to elucidate some of the important properties and uses of op-amps in negative feedback. We have noticed that voltage dividers are not compose-able, so we will use op-amps instead. We would like to implement the block diagram shown below:

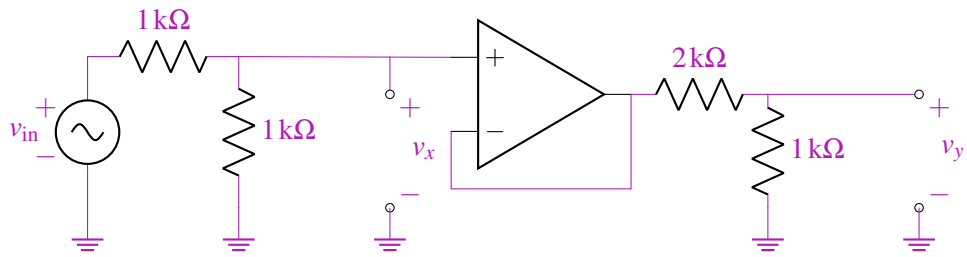


In other words, we want to implement a circuit with two outputs v_x and v_y , where $v_x = \frac{1}{2} v_{in}$ and $v_y = \frac{1}{3} v_x$.

- (a) Using an ideal op-amp in negative feedback, modify the design of one of the two voltage divider circuits you built (i.e. the $\frac{1}{2}$ block or the $\frac{1}{3}$ block), so that the originally intended relationships between v_x and v_{in} as well as v_y and v_x are realized by the resulting overall circuit (where each block is replaced by its individual implementation). Is this configuration enough by itself to attach loads at v_x and v_y ?

Answer:

Use a voltage buffer. Note that this configuration's outputs would change with the addition of a load. As a follow-up, think about ways to make the outputs agnostic to the loads attached. If we used the latter half of the circuit as a fractional divider block, we would need to buffer the output.



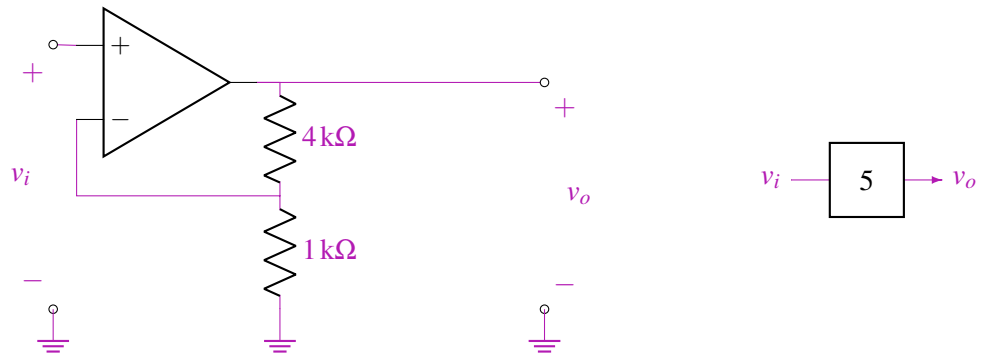
(b) Now let's assume that we want to expand our toolbox of circuits that implement mathematical operations. In particular, design blocks that implement:

- i. $v_o = 5v_i$
- ii. $v_o = -2v_i$
- iii. $v_o = v_{i_1} + v_{i_2}$

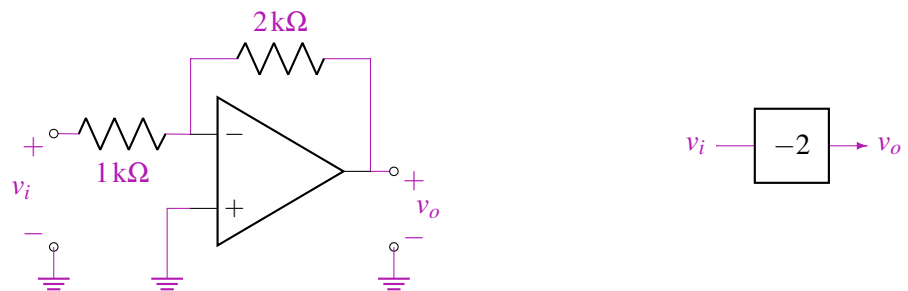
Pay careful attention to the way you design these blocks, so that connecting any one block to any other block does not modify the intended functionality of any of the blocks.

Answer:

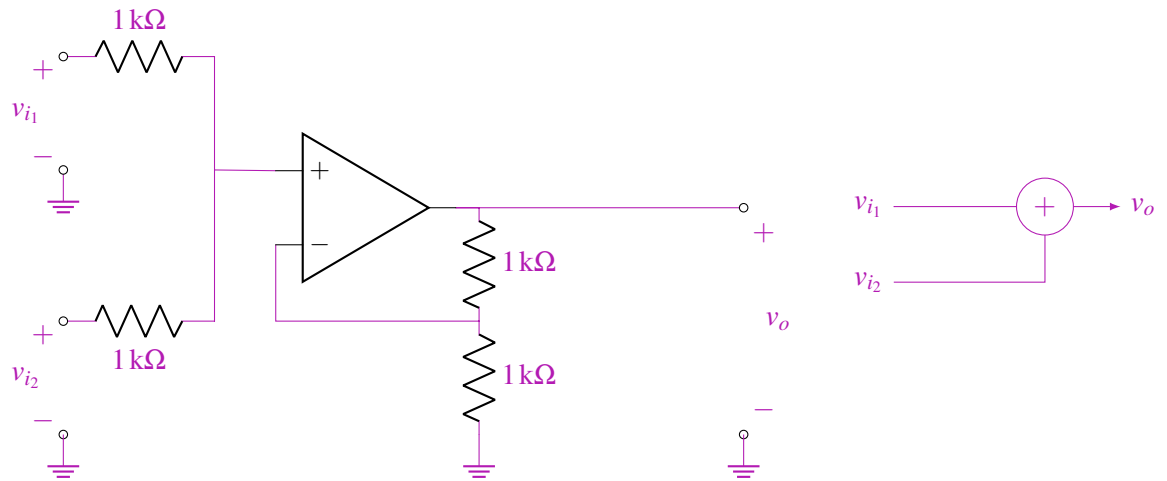
i.



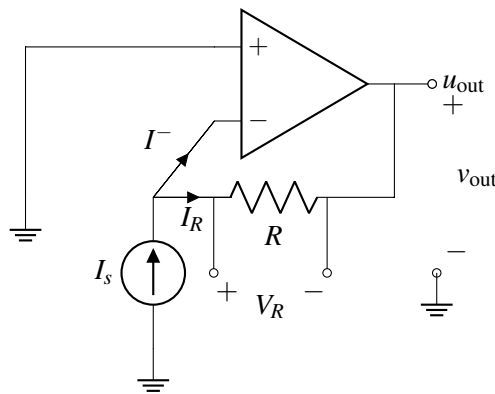
ii.



iii.



3. Practice: A Trans-Resistance Amplifier



(a) Calculate v_{out} as a function of I_s and R .

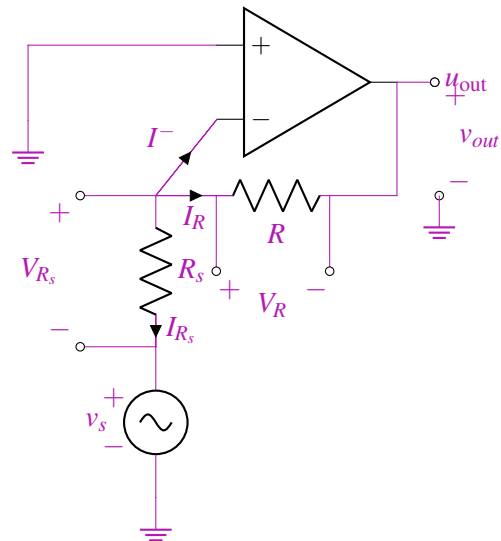
Answer:

$I^- = 0$, so $I_s = I_R$. Thus, $V_R = I_s R$.

Because the op-amp is in negative feedback, we also know that $u^+ = u^- = 0V$. Therefore, $v_{out} = u^- - V_R = -I_s R$.

(b) Implement the same behavior as the above circuit, but replace the current source with a voltage source and a resistor.

Answer:



We know that $u^- = 0$ V because of the rules of op-amps in negative feedback. Applying KCL, we see that $I_{R_s} + I_R = 0 \Rightarrow I_{R_s} = -I_R$. In addition, applying Ohm's Law, $I_{R_s} = -\frac{v_s}{R_s}$, so $I_R = \frac{v_s}{R_s}$. Hence, $v_{out} = -V_R = -\frac{v_s}{R_s}R = -\frac{R}{R_s}v_s$. This is the **inverting amplifier**.