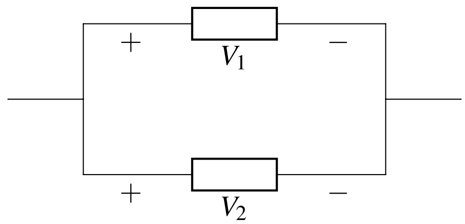
EECS 16A Designing Information Devices and Systems I
Summer 2020 Discussion 5D

1. Circuits Intuition Practice

(a) What does KVL tell you about V_1 and V_2 for any elements connected to the same pair of nodes?

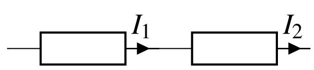


Answer: Going around the loop starting from the left node, and using KVL, we can write:

$$V_1 - V_2 = 0$$

The 2 elements are connected in **parallel**. This means they will have the same **voltage** across them.

(b) What does KCL tell you about I_1 and I_2 for any two elements connected to a node with nothing else connected to that node?

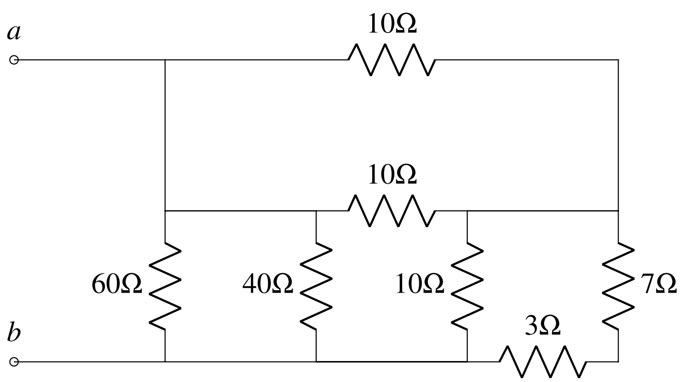


Answer: Using KCL at the center node the relationship for the currents are:

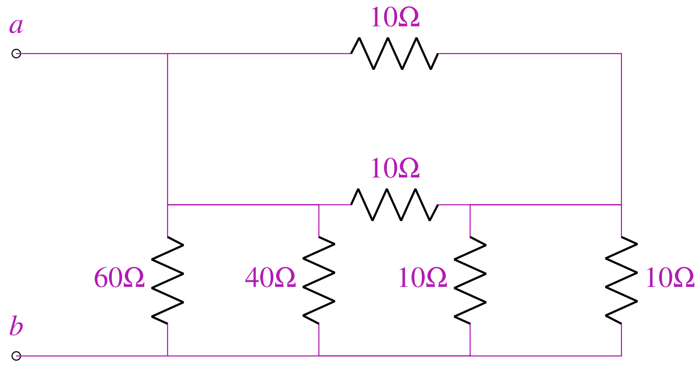
$$I_1 = I_2$$

For any two elements that are connected to a node with nothing else connected to them, the current values will be identical. The 2 elements are connected in **series**.

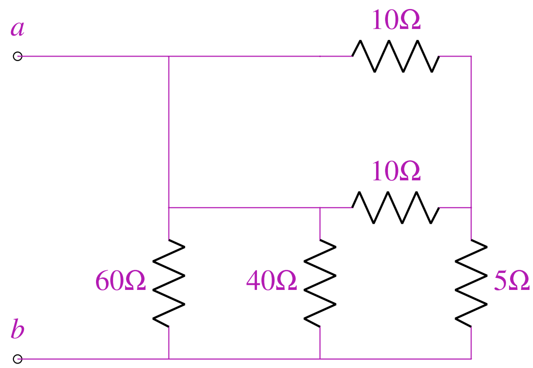
(c) Find R_{ab}, the equivalent resistance between terminals a and b. Give your answer as a number, or an expression involving no more than one use of $||$.



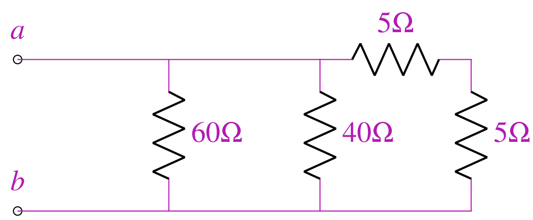
Answer: The 3Ω and 7Ω resistors are in series. Combining them, we get 10Ω.



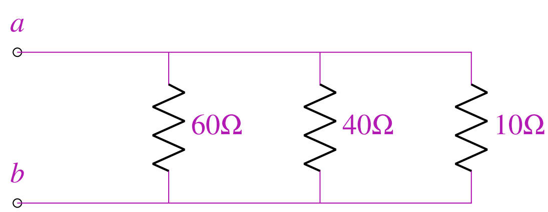
The two lower resistors with values of 10Ω are in parallel. Combining them, we get $10||10\Omega = 5\Omega$.



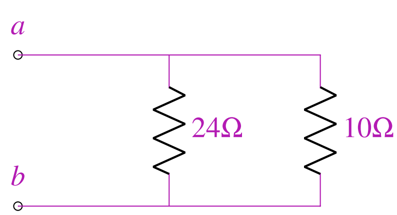
The upper two resistors with values of 10Ω are also in parallel. We get another 5Ω resistor.



Now, we can combine the two 5Ω resistors into one 10Ω resistor.

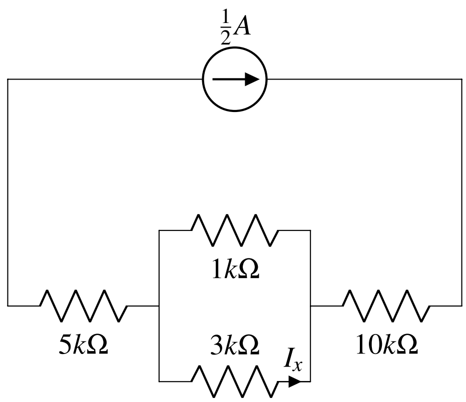


The equivalent of the 60Ω and 40Ω resistors in parallel are $R_{eq} = \frac{60\Omega \cdot 40\Omega}{60\Omega + 40\Omega} = 24\Omega$.

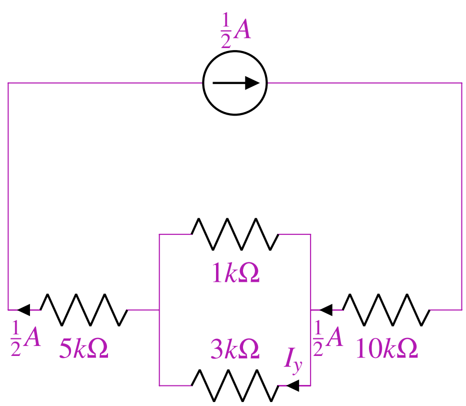


The final equivalent resistance R_{ab} can be written as $R_{ab} = 24\Omega||10\Omega$, or as $R_{ab} = \frac{24 \cdot 10}{24+10}\Omega = \frac{120}{17}\Omega \approx 7.058\Omega$.

(d) Find I_x. (Hint: Can you see the current divider?)



Answer: By KCL at the right node of $10k\Omega$, the current that goes through $10k\Omega$ is the same as the current that comes from the source. By KCL at the left node of $10k\Omega$, the current that goes through $10k\Omega$ is the same as the sum of currents that go through $1k\Omega$ and $3k\Omega$. We can consider the resistors $1k\Omega$ and $3k\Omega$ as a current divider, since the current that goes through $5k\Omega$ is also $\frac{1}{2}A$.



The current I_y will therefore be:

$$I_y = \frac{1k\Omega}{1k\Omega + 3k\Omega}\frac{1}{2}A = \frac{1}{4} \cdot \frac{1}{2}A = \frac{1}{8}A$$

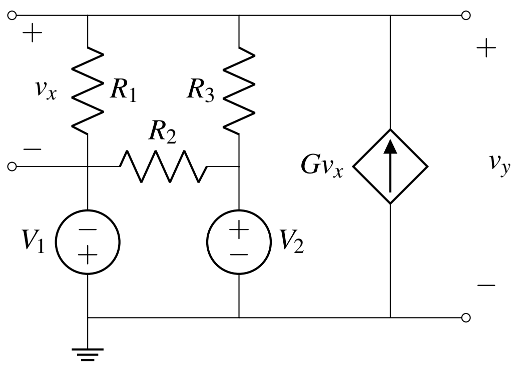
To be consistent with the original current direction labeled, $I_x = -I_y = -\frac{1}{8}A$

2. Take Node of the Voltage Sources

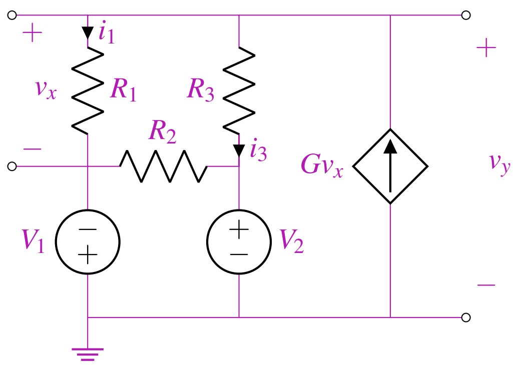
Use nodal analysis to solve for the voltages v_x and v_y. Use the following values for numerical calculations. **Note the polarity on the voltage sources.**

Aside: S refers to the unit "Siemens" which is equal to 1/Ω (essentially measures conductance instead of resistance). Don't let this unit scare you.

$$\begin{array}{ll} V_1 = 5\,\text{V} & R_1 = 10\,\Omega \\ V_2 = 5\,\text{V} & R_2 = 50\,\Omega \\ G = \frac{1}{4}\text{S} & R_3 = 40\,\Omega \end{array}$$



Answer:



Applying KCL at the node v_y:

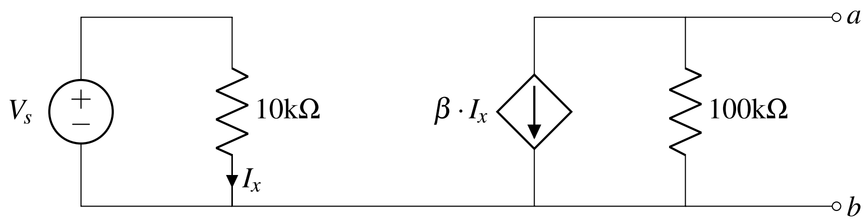
$$i_1 + i_3 - Gv_x = 0\,\text{A}$$
$$\frac{v_y + 5\,\text{V}}{10\,\Omega} + \frac{v_y - 5\,\text{V}}{40\,\Omega} - (\frac{1}{4}\text{S})(v_y + 5\,\text{V}) = 0\,\text{A}$$
$$4(v_y + 5\,\text{V}) + (v_y - 5\,\text{V}) - 10(v_y + 5\,\text{V}) = 0\,\text{V}$$
$$4v_y + 20\,\text{V} + v_y - 5\,\text{V} - 10v_y - 50\,\text{V} = 0\,\text{V}$$
$$-5v_y - 35\,\text{V} = 0\,\text{V}$$
$$v_y = -7\,\text{V}$$
$$v_x = v_y + 5\,\text{V} = -7\,\text{V} + 5\,\text{V} = -2\,\text{V}$$

Finally, we arrive at:

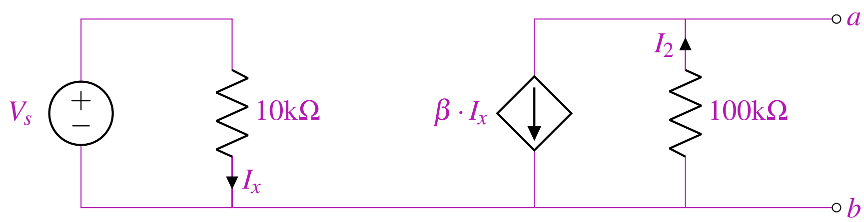
$$v_x = -2\,\text{V}$$
$$v_y = -7\,\text{V}$$

3. Practice: Equivalence

Find the Thévenin equivalent of the following circuit across the terminals a and b (in terms of V_s and β). Note that the current source is dependent on the current I_x.



Answer:



We start by calculating the open circuit voltage V_{th}. To calculate the open circuit, we start on the left, and work our way to the right. We begin by calculating I_x.

$$I_x = \frac{V_s}{10\,\text{k}\Omega}$$

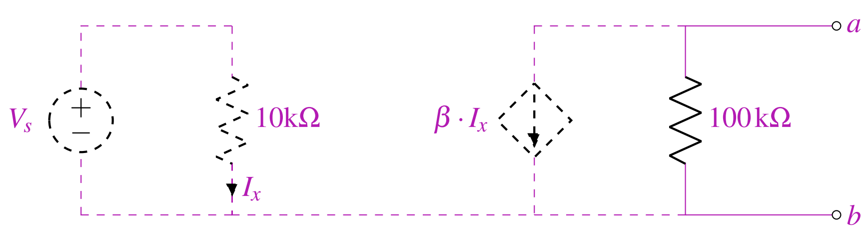
Now, knowing I_x, we can find the current through the dependent source.

$$I_2 = \beta \frac{V_s}{10\,\text{k}\Omega}$$

Knowing I_2, we can find the votlage across the resistor:

$$V_{ab} = -I_2 \cdot 100\,\text{k}\Omega = -\beta 100\,\text{k}\Omega \frac{V_s}{10\,\text{k}\Omega} = -10\beta V_s$$

Next, we need to turn off the voltage source. There's no current through the resistor, so the current source is not sourcing any current. Then, a model of the circuit is shown below:



Thus the Thévenin resistance is $100\,\text{k}\Omega$.

