EECS 16A Designing Information Devices and Systems I
Summer 2020 Discussion 0D

1. Solving Systems of Equations

(a) Systems of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following system of equations, state whether or not a solution exists. If a solution exists, list all of them.

i. $$\begin{cases} 49x + 7y = 49 \\ 42x + 6y = 42 \end{cases}$$

ii. $$\begin{cases} 5x + 3y = -21 \\ 2x + y = -9 \end{cases}$$

iii. $$\begin{cases} 49x + 7y = 60 \\ 42x + 6y = 30 \end{cases}$$

iv. $$\begin{cases} 2x + 2y + 4z = -1 \\ y + z = -2 \\ x + 2y + 3z = 2 \end{cases}$$

v. $$\begin{cases} 2x + 2y + 4z = 6 \\ y + z = 1 \\ x + 2y + 3z = 4 \end{cases}$$

vi. $$\begin{cases} x + y + z = 4 \\ 3z = 6 \\ y + z = 3 \end{cases}$$

(b) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow along as your TA walks through `dis0D.ipynb`.

2. Vectors Introduction to vectors and vector addition.

Definitions:
Vector: An ordered list of elements - for example:

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$

$\underline{\mathbb{R} \text{ or } \mathbb{R}^1}$: The set of all real numbers (i.e. the real line)
$\underline{\mathbb{R}^2}$: The set of all two-element vectors with real numbered entries (i.e. plane of 2×1 vectors) - for example:

$$\vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \in \mathbb{R}^2$$

$\underline{\mathbb{R}^3}$: The set of all three-element vectors with real numbered entries (i.e. 3-space of 3×1 vectors) - for example:

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3$$

$\underline{\mathbb{R}^n}$: The set of all n-element vectors with real numbered entries (i.e. n-space of $n \times 1$ vectors)

(a) Are the following vectors in $\mathbb{R}^2$?

   i. $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$

   ii. $\begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$

(b) Graphically show the vectors:

   i. $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

   ii. $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(c) Graphically show the vector sum and check your answer algebraically:

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

3. Matrix Multiplication

Consider the following matrices:

$$\mathbf{A_1} = \begin{bmatrix} 1 & 4 \end{bmatrix} \quad \mathbf{B_1} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

For each matrix multiplication problem, if the product exists, find the product by hand. Otherwise, explain why the product does not exist.

(a) $\mathbf{A_1B_1}$

(b) $\mathbf{AB}$

(c) $\mathbf{BA}$

(d) $\mathbf{AC}$

(e) **DC**

(f) **CD** (Write down the dimensions of the product if it exists. For practice, you can compute the product on your own)

(g) **EF** (Practice on your own)

(h) **FE** (Practice on your own)