
EECS 16A Designing Information Devices and Systems I

Summer 2020 Discussion 1B

1. Visualizing Matrices as Operations

This problem is going to help you visualize matrices as operations. For example, when we multiply a vector by a “rotation matrix,” we will see it “rotate” in the true sense here. Similarly, when we multiply a vector by a “reflection matrix,” we will see it be “reflected.” The way we will see this is by applying the operation to all the vertices of a polygon and seeing how the polygon changes.

Your TA will now show you how a unit square can be rotated, scaled, or reflected using matrices!

Part 1: Rotation Matrices as Rotations

- (a) We are given matrices \mathbf{T}_1 and \mathbf{T}_2 , and we are told that they will rotate the unit square by 15° and 30° , respectively. Design a procedure to rotate the unit square by 45° using only \mathbf{T}_1 and \mathbf{T}_2 , and plot the result in the IPython notebook. How would you rotate the square by 60° ?
- (b) Try to rotate the unit square by 60° using only one matrix. What does this matrix look like?
- (c) \mathbf{T}_1 , \mathbf{T}_2 , and the matrix you used in part (b) are called “rotation matrices.” They rotate any vector by an angle θ . Show that a rotation matrix has the following form:

$$\mathbf{R} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

where θ is the angle of rotation. To do this consider rotating the unit vector $\begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$ by θ degrees using the matrix \mathbf{R} .

(Definition: A vector, $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \end{bmatrix}$, is a unit vector if $\sqrt{v_1^2 + v_2^2 + \dots} = 1$.)

(Hint: Use your trigonometric identities!)

- (d) Now, we want to get back the original unit square from the rotated square in part (b). What matrix should we use to do this? (**Note:** Don’t use inverses! Answer this question using your intuition, we will visit inverses very soon in lecture!)
- (e) Use part (d) to obtain the “inverse” rotation matrix for a matrix that rotates a vector by θ . Multiply the inverse rotation matrix with the rotation matrix and vice-versa. What do you get?
- (f) What are the matrices that reflect a vector about the (i) x -axis, (ii) y -axis, and (iii) $x = y$

Part 2: Commutativity of Operations

A natural question to ask is the following: Does the *order* in which you apply these operations matter? Follow your TA to obtain the answers to the following questions!

- (a) Let’s see what happens to the unit square when we rotate the square by 60° and then reflect it along the y -axis.

- (b) Now, let's see what happens to the unit square when we first reflect the square along the y -axis and then rotate it by 60° .
- (c) Try to do steps (a) and (b) by multiplying the reflection and rotation matrices together (in the correct order for each case). What does this tell you?
- (d) If you reflected the unit square twice (along any pair of axes), do you think the order in which you applied the reflections would matter? Why/why not?

Part 3: Distributivity of Operations

- (a) The distributivity property of matrix-vector multiplication holds for any vectors and matrices. Show for general $\mathbf{A} \in \mathbb{R}^{2 \times 2}$ and $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^2$ that $\mathbf{A}(\vec{v}_1 + \vec{v}_2) = \mathbf{A}\vec{v}_1 + \mathbf{A}\vec{v}_2$.