EECS 16A Designing Information Devices and Systems I
Summer 2020 Discussion 2A

1. Identifying a Subspace: Proof

Is the set

$$V = \left\{ \vec{v} \;\middle|\; \vec{v} = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } c, d \in \mathbb{R} \right\}$$

a subspace of $\mathbb{R}^3$? Why/why not?

2. Exploring Column Spaces and Null Spaces

- The **column space** is the **span** of the column vectors of the matrix.
- The **null space** is the set of input vectors that output the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of **A**? What is its dimension?
ii. What is the null space of **A**? What is its dimension?
iii. Are the column spaces of the row reduced matrix **A** and the original matrix **A** the same?
iv. Do the columns of **A** form a basis for $\mathbb{R}^2$? Why or why not?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 1 & 1 & 3 & -3 \end{bmatrix}$

3. Mechanical Determinants

(a) Compute the determinant of $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(b) Compute the determinant of $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

Reference Definitions: Matrices and Linear (In)Dependence The following statements are equivalent for an $n \times n$ matrix $\mathbf{A}$, meaning, if one is true then all are true:

(a) $\mathbf{A}$ is invertible
(b) $\Leftrightarrow$ The equation $\mathbf{A}\vec{x} = \vec{b}$ has a unique solution for any $\vec{b}$
(c) $\Leftrightarrow$ $\mathbf{A}$ has linearly independent columns
(d) $\Leftrightarrow$ $\mathbf{A}$ has a trivial nullspace
(e) $\Leftrightarrow$ the determinant of $\mathbf{A} \neq 0$.

In class have shown/proven that:

(a) $\mathbf{A}$ is invertible $\Longrightarrow$ the equation $\mathbf{A}\vec{x} = \vec{b}$ has a unique solution for any $\vec{b}$.
(b) $\mathbf{A}$ is invertible $\Longrightarrow$ $\mathbf{A}$ has linearly independent columns
(c) $\mathbf{A}$ is invertible $\Longrightarrow$ $\mathbf{A}$ has a trivial nullspace.

We have not yet shown/proven the implications in the other direction.

