EECS 16A Designing Information Devices and Systems I
Summer 2020 Discussion 2B

1. **Mechanical Eigenvalues and Eigenvectors**

   In each part, find the eigenvalues of the matrix **M** and the associated eigenvectors. State if the inverse of **M** exists.

   (a) $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

   (b) $\mathbf{M} = \begin{bmatrix} -2 & 4 \\ -4 & 8 \end{bmatrix}$

   (c) $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

   (d) $\mathbf{M} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

   (e) **(PRACTICE)** $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

2. **Eigenvalues and Special Matrices – Visualization**

   An eigenvector $\vec{v}$ belonging to a square matrix **A** is a nonzero vector that satisfies

   $$\mathbf{A}\vec{v} = \lambda\vec{v}$$

   where λ is a a scalar known as the **eigenvalue** corresponding to eigenvector $\vec{v}$.

   The following parts don't require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

   (a) Does the identity matrix in $\mathbb{R}^n$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

   (b) Does a diagonal matrix $\begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix}$ in $\mathbb{R}^n$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

   (c) Does a rotation matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (d) Does a reflection matrix in $\mathbb{R}^{2\times 2}$, where the reflection is around any line passing through the origin, have any eigenvalues $\lambda \in \mathbb{R}$?

   (e) If a matrix **M** has an eigenvalue $\lambda = 0$, what does this say about its null space? What does this say about the solutions of the system of linear equations $\mathbf{M}\vec{x} = \vec{b}$?

   (f) **(Practice)** Does the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?

3. Steady State Reservoir Levels

We have 3 reservoirs: A, B and C. The pumps system between the reservoirs is depicted in Figure 1.

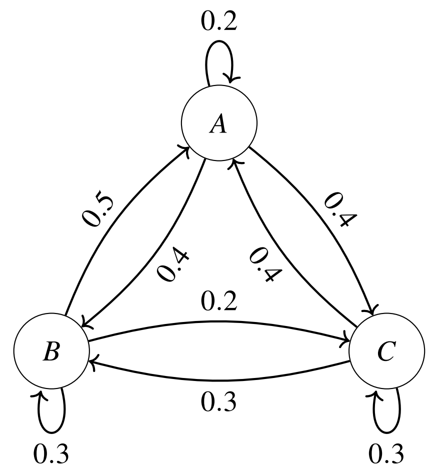

Figure 1: Reservoir pumps system.

(a) Write out the transition matrix $\mathbf{T}$ representing the pumps system.

(b) You are told that $\lambda_1 = 1$, $\lambda_2 = \frac{-\sqrt{2}-1}{10}$, $\lambda_3 = \frac{\sqrt{2}-1}{10}$ are the eigenvalues of $\mathbf{T}$. Find a steady state vector $\vec{x}$, i.e. a vector such that $T\vec{x} = \vec{x}$.

