EECS 16A Designing Information Devices and Systems I
Summer 2020 Discussion 2C

1. Steady and Unsteady States

(a) You're given the matrix $\mathbf{M}$:

$$\mathbf{M} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

Which generates the next state of a physical system from its previous state: $\vec{x}[k+1] = \mathbf{M}\vec{x}[k]$. ($\vec{x}$ could describe either people or water.) Find the eigenspaces associated with the following eigenvalues:

i. span($\vec{v}_1$), associated with $\lambda_1 = 1$

ii. span($\vec{v}_2$), associated with $\lambda_2 = 2$

iii. span($\vec{v}_3$), associated with $\lambda_3 = \frac{1}{2}$

(b) Define $\vec{x} = \alpha\vec{v}_1 + \beta\vec{v}_2 + \gamma\vec{v}_3$, a linear combination of the eigenvectors. For each of the cases in the table, determine if

$$\lim_{n\to\infty} \mathbf{M}^n\vec{x}$$

converges. If it does, what does it converge to?

α	β	γ	Converges?	$\lim_{n\to\infty} \mathbf{M}^n\vec{x}$
0	0	$\neq 0$		
0	$\neq 0$	0		
0	$\neq 0$	$\neq 0$		
$\neq 0$	0	0		
$\neq 0$	0	$\neq 0$		
$\neq 0$	$\neq 0$	0		
$\neq 0$	$\neq 0$	$\neq 0$		

2. Polynomials as a Vector Space

Let $\mathbb{P}_2$ be the set of polynomials of degree of at most two (that is, $p(t) = at^2 + bt + c$).

(a) Give a basis for $\mathbb{P}_2$.

(b) Consider the linear transformations

$$T_1(f(t)) = 2f(t)$$
$$T_2(f(t)) = f'(t)$$

For each, find the transformation matrix with respect to the basis from part (a).

(c) Suppose that $\{x_0, x_1, x_2\}$ form a basis for $\mathbb{P}_2$ and that the following polynomials have the corresponding coordinates in this basis.

$$\begin{aligned}(1,1,1) &\Rightarrow 2t^2+3t\\(1,0,-1) &\Rightarrow t+1\\(0,2,0) &\Rightarrow 4t+2\end{aligned}$$

Find the basis vectors x_0, x_1, x_2.

