EECS 16A Designing Information Devices and Systems I
Summer 2020 Discussion 6A

Reference: Inner products

Let $\vec{x}$, $\vec{y}$, and $\vec{z}$ be vectors in real vector space $\mathbb{V}$. A mapping $\langle \cdot, \cdot \rangle$ is said to be an inner product on $\mathbb{V}$ if it satisfies the following three properties:

(a) Symmetry: $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$

(b) Linearity: $\langle \vec{x}, \vec{y} + \vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$ and $\langle c\vec{x}, \vec{y} \rangle = c\langle \vec{x}, \vec{y} \rangle$

(c) Positive-definiteness: $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

We define the norm of $\vec{x}$ as $\|\vec{x}\| = \sqrt{\langle \vec{x}, \vec{x} \rangle}$.

Cross-correlation:

The cross-correlation between two signals $r[n]$ and $s[n]$ is defined as follows:

$$\text{corr}_r(s)[k] = \sum_{i=-\infty}^{\infty} r[i]s[i-k].$$

1. Mechanical Inner Products

For the following pairs of vectors, find the Euclidean inner product $\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y}$.

(a)

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

2. Inner Product Properties

Demonstrate the following properties of inner products for any vectors in $\mathbb{R}^2$, assuming we are working with the Euclidean inner product and norm.

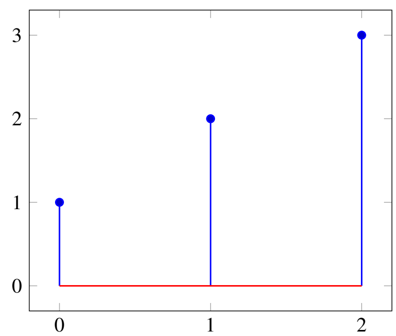
(a) Symmetry

(b) Linearity

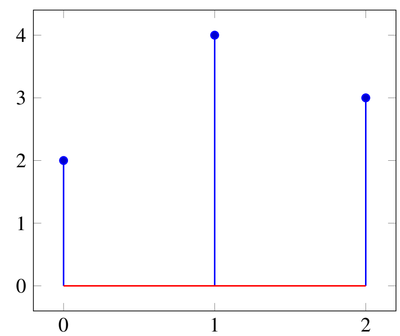
3. Correlation

We are given the following two signals, $s_1[n]$ and $s_2[n]$ respectively.

Signal 1



Signal 2



Find the cross correlations, $\text{corr}_{s_1}(s_2)$ and $\text{corr}_{s_2}(s_1)$ for signals $s_[n]$ and $s_2[n]$. Recall

$$\text{corr}_x(y)[k] = \sum_{i=-\infty}^{\infty} x[i]y[i-k].$$

$\text{corr}_{\vec{s}_1}(\vec{s}_2)[k]$

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n+2]$							
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n+1]$							
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n]$							
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n-1]$							
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n-2]$							
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	+	+	+	+	+	+	=

$\text{corr}_{\vec{s}_2}(\vec{s}_1)[k]$

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n+2]$							
$\langle \vec{s}_2, \vec{s}_1[n+2] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n+1]$							
$\langle \vec{s}_2, \vec{s}_1[n+1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n]$							
$\langle \vec{s}_2, \vec{s}_1[n] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n-1]$							
$\langle \vec{s}_2, \vec{s}_1[n-1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n-2]$							
$\langle \vec{s}_2, \vec{s}_1[n-2] \rangle$	+	+	+	+	+	+	=

