Equivalence: two circuit are equivalent if they have the same I-V characteristics

* with respect to 2 nodes

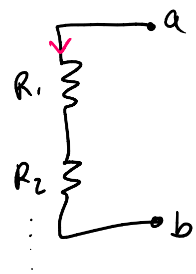
Series

- same current going through the elements

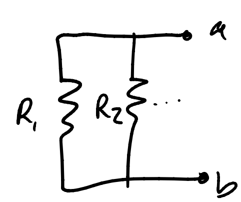
Parallel

- same nodes across the elements

Resistors

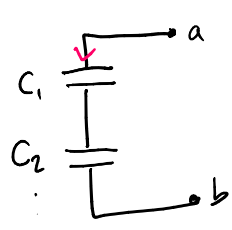


$\Rightarrow R_1+R_2+\cdots$

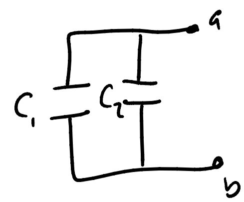


$\Rightarrow \dfrac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$

Capacitors



$\Rightarrow \dfrac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\cdots}$

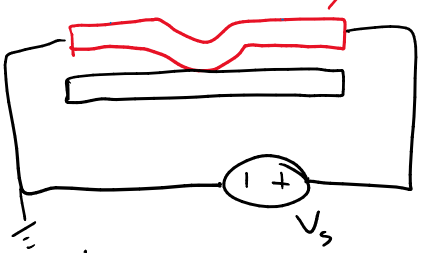


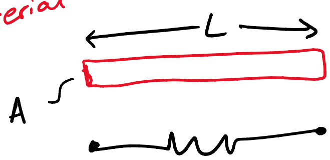
$\Rightarrow C_1+C_2+\cdots$

Resistive Touchscreen:

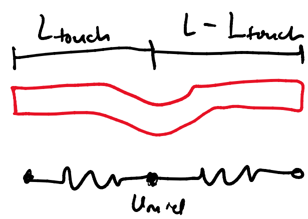
1D

bendy conductive material



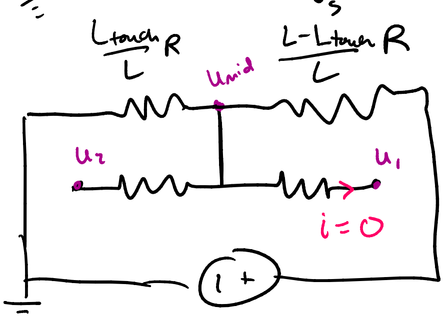


$R=\rho\frac{L}{A}$



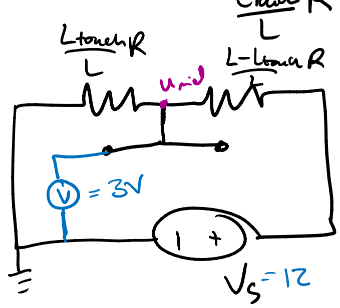
$\rho\frac{L_{touch}}{A}$ $\quad \rho\frac{(L-L_{touch})}{A}$

$\frac{L_{touch}}{L}R$ $\quad \frac{L-L_{touch}}{L}R$



$u_1=u_2=u_{mid}$

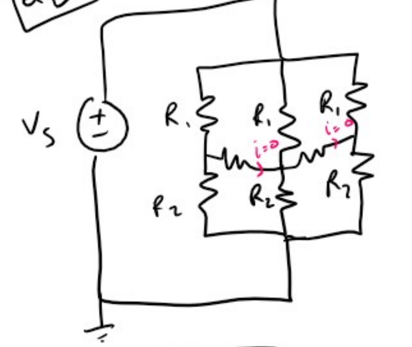
$\Rightarrow$

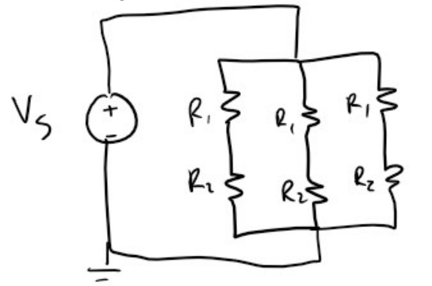


$$u_{mid}=\frac{\frac{L_{touch}}{L}R}{\frac{L_{touch}}{L}R+\frac{L-L_{touch}}{L}R}V_s$$

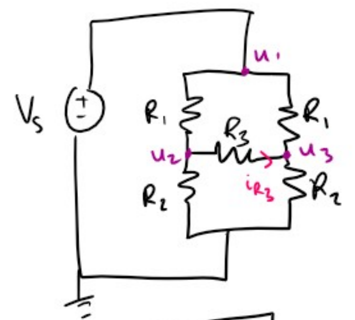
$$=\frac{L_{touch}}{L}V_s$$

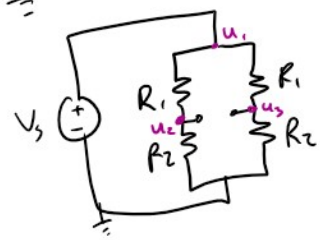
2D





simpler interesting circuit





could use NVA

or

make a guess

$u_2 = u_3$

$i_{R_3} = \frac{u_2 - u_3}{R_3} = 0 \rightarrow$ open circuit

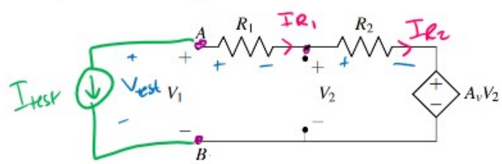
$u_2 = \frac{R_2}{R_1 + R_2} V_s \qquad u_3 = \frac{R_2}{R_1 + R_2} V_s$

$u_2 = u_3$ ✓ justifies our assumption

Final Fall 19 Q7

7. Fun With Circuits (14 points) (All subparts of this problem can be solved independently.)

In his spare time Professor Boser invents new circuits. The circuit schematic below shows his latest creation that uses a voltage controlled voltage source.



excite output w/ a test source (V_{test}, I_{test})

$R_{eq} = \frac{V_{test}}{I_{test}}$

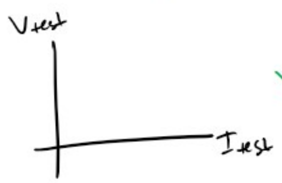
(a) **Equivalent Resistance** (8 points)

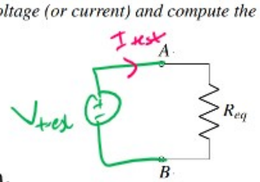
Analyze the circuit above and model it with an equivalent resistance R_{eq} as illustrated below. The I/V curves of the original and equivalent circuit should be identical. Use the following values: $A_v = 3$, $R_1 = 1\ k\Omega$, and $R_2 = 4\ k\Omega$. Show your calculations.

Recall that a resistor is just a model of the IV dependence of a circuit element; you may get a positive or a negative answer for your equivalent resistance.

Hint: Apply a test voltage (or current) and compute the resulting current (or voltage).

$V_{test} = u_A - u_B \qquad V_1 = u_A - u_B$





$I_{R_1} = I_{R_2} = I_{test} \qquad \& \ V_1 = V_{test}$

$\frac{V_1 - V_2}{R_1} = \frac{V_2 - A_v V_2}{R_2} \qquad \frac{V_1}{R_1} = V_2 \left(\frac{1 - A_v}{R_2} - \frac{1}{R_1} \right)$

$R_{eq} = \frac{V_{test}}{I_{test}}$

$V_2 = \frac{V_1}{\left(\frac{1 - A_v}{R_2} - \frac{1}{R_1} \right) R_1} = \frac{V_1 R_2}{R_1 (1 - A_v) + R_2} = \frac{V_{test} R_2}{R_1 (1 - A_v) + R_2}$

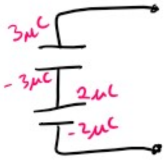
$I_{test} = \frac{V_{test} - V_2}{R_1} = \frac{V_{test}}{R_1} - \frac{1}{R_1} \frac{V_{test} R_2}{R_1(1 - A_v) + R_2} = \frac{V_{test}}{R_1} \left(1 - \frac{R_2}{R_1(1 - A_v) + R_2} \right) =$

$= \frac{V_{test}}{\cancel{R_1}} \left(\frac{\cancel{R_1} (1 - A_v)}{R_1 (1 - A_v) + R_2} \right) = \frac{(1 - A_v) V_{test}}{R_1 (1 - A_v) + R_2}$

$R_{eq} = \frac{\cancel{V_{test}}}{\frac{(1 - A_v) \cancel{V_{test}}}{R_1 (1 - A_v) + R_2}} = \frac{R_1 (1 - A_v) + R_2}{1 - A_v}$

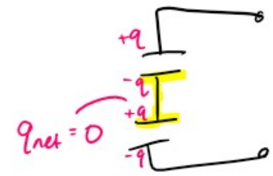
$\boxed{R_{eq} = R_1 + \frac{R_2}{1 - A_v}}$

Capacitors in Series



* only in series if they have the same charge

← not in series



← in series

MT2 Sp20 Q11

(a) Assuming that the floating gate node (V_{FG}) has no net charge ($Q_T = 0$). Find V_{FG} in terms of V_{in}, C_G, and C_S, and then plug in values from the table.

(b) Now let's assume there is a net charge of $Q_T = -20.0\text{pC}$ on the floating gate node. Find V_{FG} as a function of C_G, C_S, V_{in}, and Q_T and then plug in the numerical values. (*Hint: Use conservation of charge.*)

11. Flash Memory (16 points)

Solid state drives depend on charge to store information. In several integrated circuit applications, the charge is stored on a floating node of a transistor. A cartoon of such a transistor and its corresponding circuit model are shown below. *Note: You do not need to understand transistors in order to do this problem.*

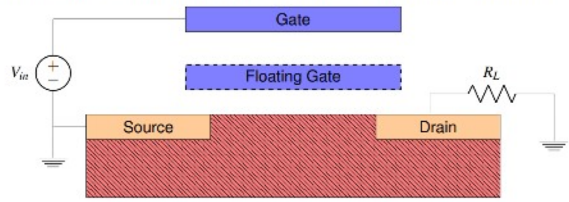


Figure 3: Transistor schematic for flash memory.

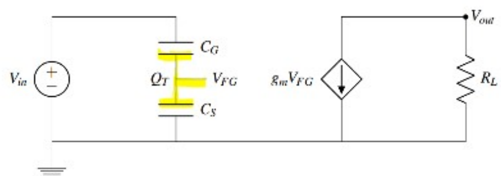


Figure 4: Corresponding circuit model for flash memory transistor from Figure 3.

Depending on the amount of charge on the floating node, the transistor is either storing a "0" bit or a "1" bit. In order to write a bit, electrons are added or removed from the "floating gate node" (labelled by V_{FG} in Figure 4), which is capacitively coupled to the gate and the source of the transistor as shown in Figure 3. Therefore, during transistor operation, there can be a net charge Q_T at the node V_{FG}.[1]

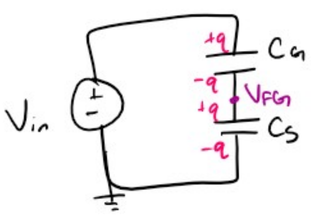
In each part of the problem, the values of each parameter do not change, and are given in the table below:

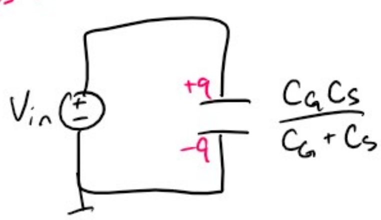
Component Parameters	
V_{in}	2.0V
C_G	35.0pF
C_S	3.0pF
g_m	300.0μA V^{-1}
R_L	35.0kΩ

(a) Assuming that the floating gate node (V_{FG}) has no net charge ($Q_T = 0$). Find V_{FG} in terms of V_{in}, C_G, and C_S, and then plug in values from the table.

$Q_T = 0$

series ✓

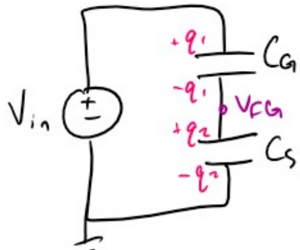




$$q = \frac{C_G C_S}{C_G + C_S} V_{in}$$

$$V_{FG} = \frac{q}{C_S} = \frac{C_G}{C_G + C_S} V_{in}$$

(b) Now let's assume there is a net charge of $Q_T = -20.0\text{pC}$ on the floating gate node. Find V_{FG} as a function of C_G, C_S, V_{in}, and Q_T and then plug in the numerical values. (*Hint: Use conservation of charge.*)



$Q_T = -q_1 + q_2$ ← conservation of charge

$q_1 = C_G (V_{in} - V_{FG})$

$q_2 = C_S V_{FG}$

$$Q_T = -C_G (V_{in} - V_{FG}) + C_S V_{FG}$$

$$V_{FG} = \frac{Q_T + C_G V_{in}}{C_G + C_S}$$

* Didn't get to this problem but wrote out the solutions

MT2 Fa19 Q5

5. Next-Phone (15 points)

You have been hired by "Next-Phone", a promising startup that has developed a 3D printer to produce individually customized smartphones.

Only one problem remains: designing accurate position sensing for the printhead. "No problem," you tell your new boss, "I'll take care of that!"

Figure 5.1 shows your design. The printhead is supported by two rollers that move the head in the x direction. Each roller runs on two conductive tracks with resistivities ρ_1 and ρ_2, respectively, length L, and cross-sectional area A. The rollers are made of metal electrically connecting the strips. The printhead is an insulator (i.e. nonconductive material) so it can be modeled as an open-circuit. Roller 1 and Roller 2 are disconnected by the non-conductive Print Head. You connect two voltage sources of voltage V_s as shown in Figure 5.1. You then measure voltage $V_{AB} = V_A - V_B$ to sense position x.

(a) Draw an equivalent circuit diagram of Figure 5.1 consisting of sources, resistors, etc.

(b) Derive algebraic expressions for $V_{AA'} = V_A - V_{A'}$ and $V_{BB'} = V_B - V_{B'}$ as a function of x_{pos}.

(c) Find the value of voltage $V_{AB}(x_{pos}) = V_A - V_B$ for $V_s = 10\text{V}$, $\rho_1 = 1\,\Omega\text{m}$, $\rho_2 = 2\,\Omega\text{m}$, $L = 200\text{mm}$, $A = 1\text{cm}^2$, and $x_{pos} = 50\text{mm}$.

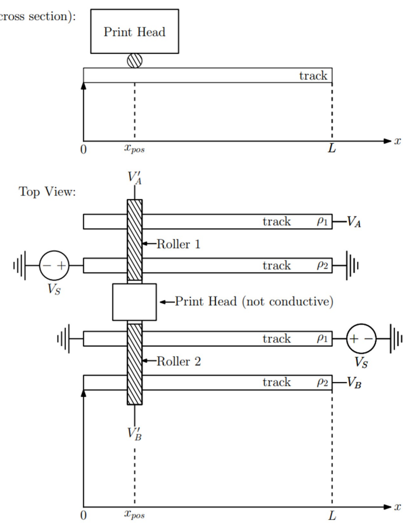
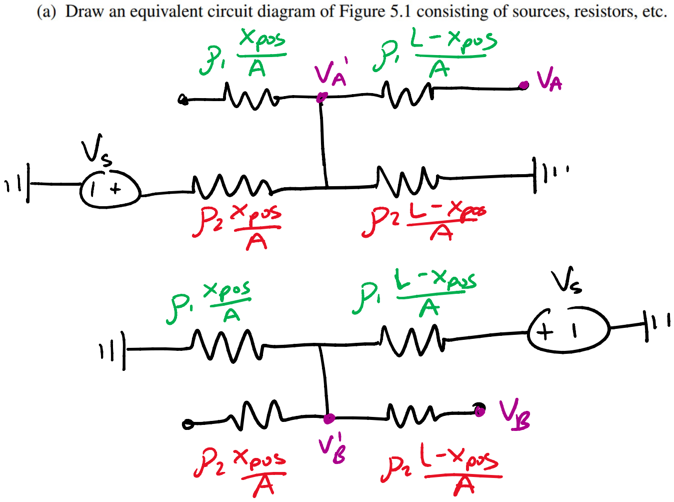


Figure 5.1: 3D printhead position sensor (top and side views)

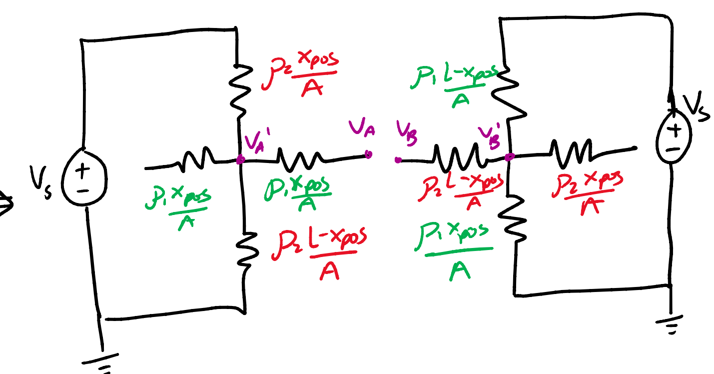
of total resistor

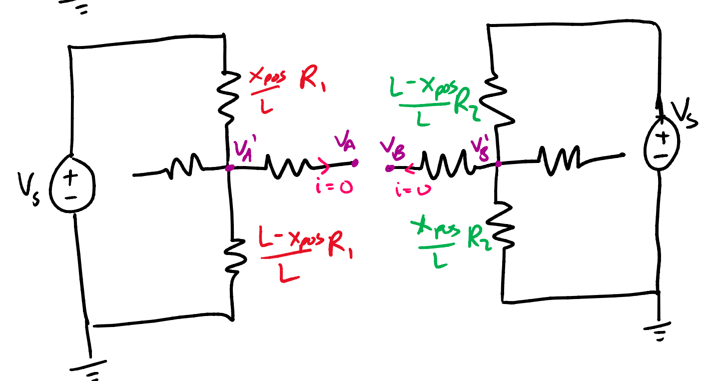
$$R_1 = \rho_1 \frac{L}{A} \qquad R_2 = \rho_2 \frac{L}{A}$$

(a) Draw an equivalent circuit diagram of Figure 5.1 consisting of sources, resistors, etc.



Same as $\Rightarrow$





(b) Derive algebraic expressions for $V_{AA'} = V_A - V_{A'}$ and $V_{BB'} = V_B - V_{B'}$ as a function of x_{pos}.

no current going through those resistors because open circuit

$V_A = V_{A'}$

$V_B = V_{B'}$

$$V_{AA'} = V_A - V_{A'} = 0 \qquad V_{BB'} = V_B - V_{B'} = 0$$

(c) Find the value of voltage $V_{AB}(x_{pos}) = V_A - V_B$ for $V_s = 10\text{V}$, $\rho_1 = 1\,\Omega\text{m}$, $\rho_2 = 2\,\Omega\text{m}$, $L = 200\text{mm}$, $A = 1\text{cm}^2$, and $x_{pos} = 50\text{mm}$.

$$V_A = \frac{\frac{L-x_{pos}}{L} R_1}{\frac{x_{pos}}{L} R_1 + \frac{L-x_{pos}}{L} R_1} V_s = \frac{L-x_{pos}}{L} V_s$$

$$V_B = \frac{\frac{x_{pos}}{L} R_2}{\frac{L-x_{pos}}{L} R_2 + \frac{x_{pos}}{L} R_2} V_s = \frac{x_{pos}}{L} V_s$$

$$V_{AB} = \left(\frac{L-x_{pos}}{L} - \frac{x_{pos}}{L}\right) V_s = \frac{L-2x}{L} V_s = 5\text{V}$$