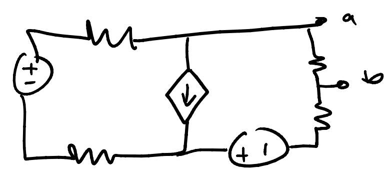
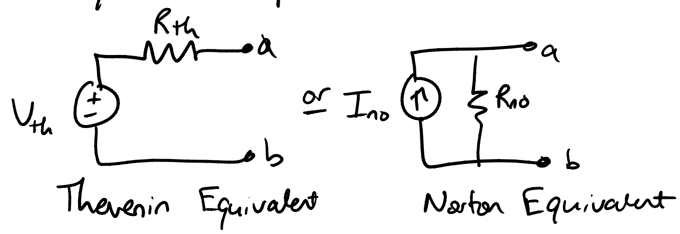
complex circuit

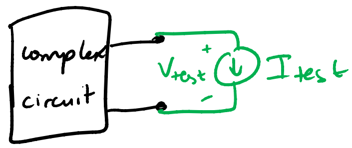


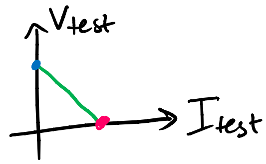
$\Rightarrow$

equivalent simpler circuit



* 2 circuits are equivalent if they have the same I-V relationship

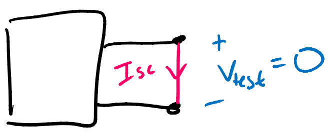


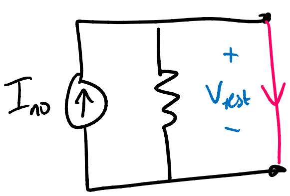


when $V_{test} = 0$

→ no voltage drop

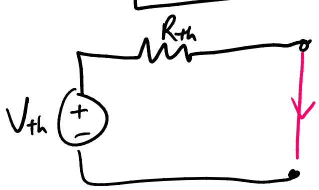
→ short circuit current $I_{test} = I_{sc}$





when $V_{test} = 0$

$I_{test} = I_{sc} = I_{no}$

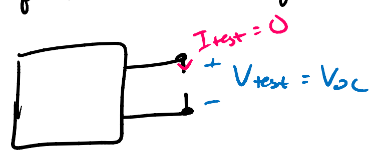


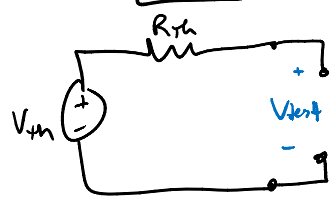
$$R_{th} = \frac{V_{th}}{I_{no}}$$

when $I_{test} = 0$

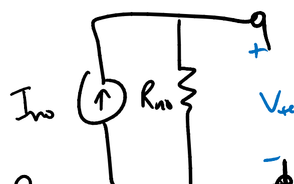
→ no current flowing

→ open circuit voltage $V_{test} = V_{oc}$





$V_{test} = V_{th} = V_{oc}$



$V_{test} = V_{th}$

$$R_{no} = \frac{V_{th}}{I_{no}}$$

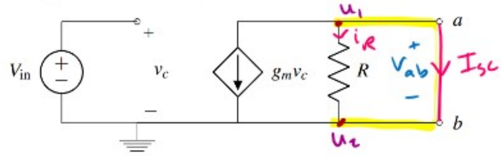
$$R_{th} = R_{no}$$

Finding Thevenin and Norton Equivalents

1) $V_{th} = V_{oc}$ – NVA or Superposition
2) $I_{no} = I_{sc}$ – NVA or Superposition
3) R_{th}, R_{no}
   a) $R_{th} = R_{no} = \frac{V_{th}}{I_{no}} = \frac{V_{oc}}{I_{sc}}$ (only works if $V_{th} \neq 0, I_{no} \neq 0$)
   b) turn off independent sources, find R_{eq} (only works if all sources turn off) * quickest
   c) turn off independent sources, excite output w/ test source $R_{th} = R_{no} = \frac{V_{test}}{I_{test}}$ * slowest (always works)

Sp 18 MT2 Q7

(b) (6 points) Find and draw the **Norton** equivalent circuit between terminals a and b in the circuit below. Clearly label the Norton equivalent current, I_{no}, and the Norton equivalent resistance R_{no}.



① find $I_{no} = I_{sc}$

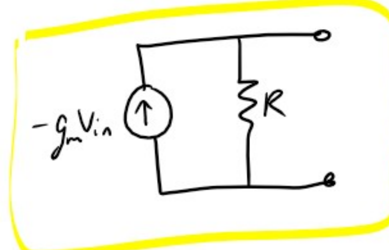
$u_1 = u_2$ $\quad V_{ab} = u_1 - u_2 = i_R R = 0$

$\rightarrow i_R = 0$

KCL @ u_1: $g_m V_c + \cancel{i_R}^{0} + I_{sc} = 0$ $\quad \boxed{I_{sc} = -g_m V_{in}}$

② find R_{no} → which methods can we use? a) b) c)

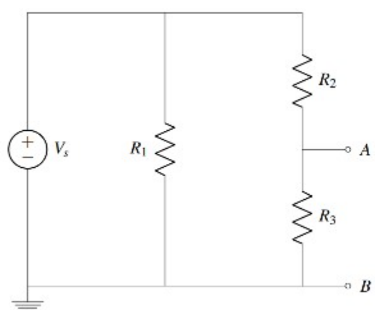
$R_{eq} = R = R_{no}$



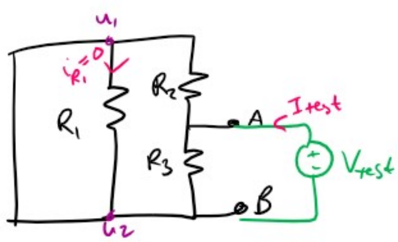
Sp 20 MT2 Q9

9. Thévenin Equivalence (12 points)

(a) Find the Thévenin resistance R_{th} of the circuit shown below, with respect to its terminals A and B. Assume that $R_1 = 4R$, $R_2 = R$ and $R_3 = 9R$.



could use a) b) & c)
but let's use b) because it's the easiest



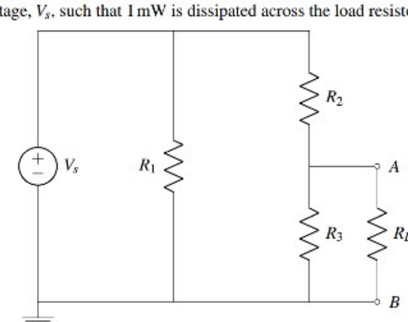
$u_1 = u_2$

$V_{R_1} = u_1 - u_2 = i_{R_1} R_1 = 0$

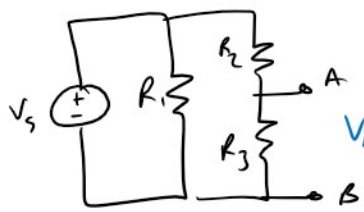
$i_{R_1} = 0$

$$\boxed{R_{th} = R_2 \| R_3}$$

(b) Now a load resistor, $R_L = 9R$, is connected across terminals A and B as shown in the circuit below. Find the supply voltage, V_s, such that 1 mW is dissipated across the load resistor. Let $R = 36\text{k}\Omega$.

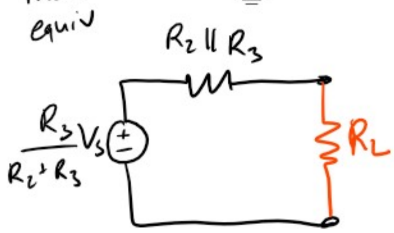


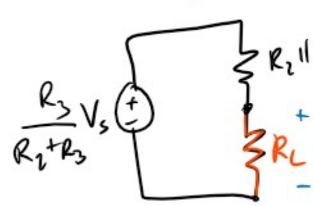
Want to find Thevenin equiv → find V_{th}



$$V_{AB} = V_{OC} = \frac{R_3}{R_2 + R_3} V_s$$

Thevenin equiv

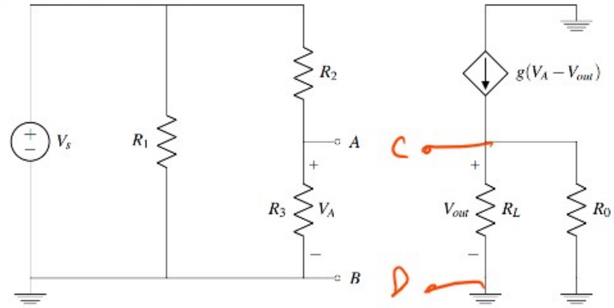




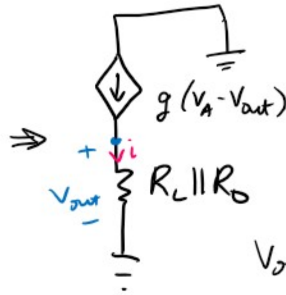
$$V_{R_L} = \frac{R_L}{R_{th} + R_L} V_{th} = \frac{R_L}{R_2 \| R_3 + R_L} \frac{R_3}{R_2 + R_3} V_s$$

$$P_{R_L} = I_{R_L} V_{R_L} = \frac{V_{R_L}^2}{R_L}$$ → solve for V_s

(c) We modify the circuit as shown below:



$\Rightarrow$



$$V_A = \frac{R_3}{R_2 + R_3} V_s$$

$$V_{out} = i\,(R_L \| R_0)$$

$$= g(V_A - V_{out})(R_L \| R_0)$$

$$V_{out} + gV_{out}(R_L \| R_0) = g\,V_A\,(R_L \| R_0)$$

$$V_{out}(1 + g(R_L \| R_0)) = g\,V_A\,(R_L \| R_0)$$

Find a symbolic expression for V_{out} as a function of V_s.

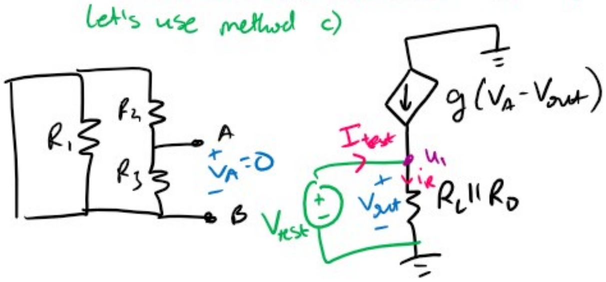
$$V_{out} = \frac{g\,(R_L \| R_0) V_A}{(1 + g\,(R_L \| R_0))}$$

$$\boxed{V_{out} = \frac{R_3}{R_2 + R_3} \frac{g\,(R_L \| R_0)}{1 + g\,(R_L \| R_0)} V_s}$$

let's also find R_{th} w.r.t. C and D

which methods can we use? not b)

let's use method c)



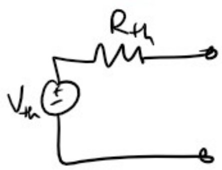
KCL @ u_1: $I_{test} + g(\cancel{V_A}^{0} - V_{out}) - i_R = 0$

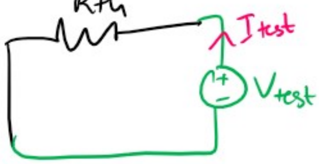
$$I_{test} = i_R + gV_{out} = i_R + gV_{test}$$

$$= \frac{V_{test}}{R_L \| R_0} - gV_{test} = \left(\frac{1}{R_L \| R_0} + g\right) V_{test}$$

$* \; i_R = \frac{V_{out}}{R_L \| R_0}$

$V_{out} = V_{test}$





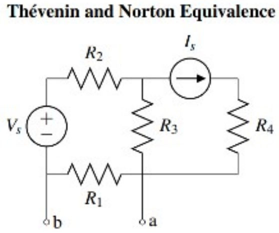
$$R_{th} = \frac{V_{test}}{I_{test}} = \frac{\cancel{V_{test}}}{\left(\frac{1}{R_L \| R_0} + g\right)\cancel{V_{test}}}$$

$$\boxed{R_{th} = \frac{1}{\frac{1}{R_L \| R_0} + g}}$$

* Didn't get to this problem but wrote out the solutions

Fall 16 MT2 Q5

5. Thévenin and Norton Equivalence (10 points)

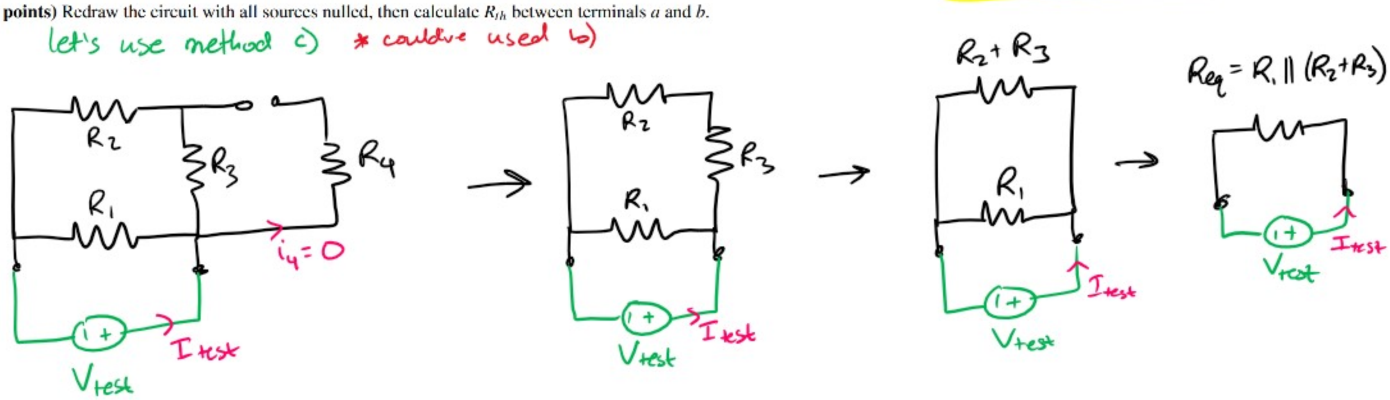


$$I_{test} = \frac{V_{test}}{R_{eq}} \qquad R_{th} = \frac{V_{test}}{I_{test}} = R_{eq}$$

$$\boxed{R_{th} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}}$$

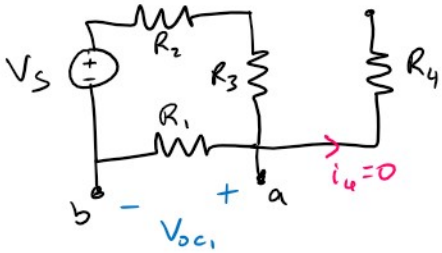
(a) **(4 points)** Redraw the circuit with all sources nulled, then calculate R_{th} between terminals a and b.

let's use method c) * could've used b)

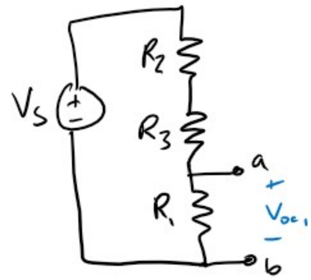


(b) **(6 points)** Find the Thévenin voltage between the terminals a and b. Hint: superposition may be useful.

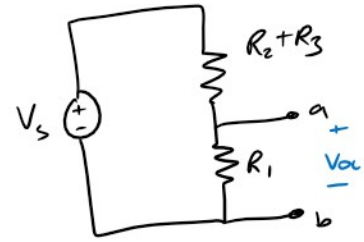
only V_s on



$\Rightarrow$

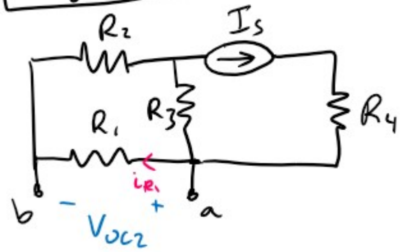


$\Rightarrow$



$$V_{oc_1} = \frac{R_1}{R_1+R_2+R_3} V_s$$

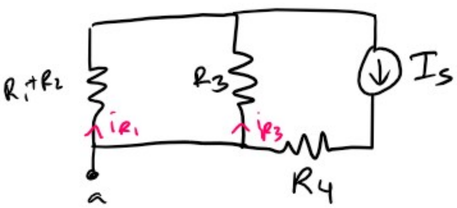
only I_s on



* Recall current divider:

$$I_{R_1} = \frac{R_2}{R_1+R_2} I_s$$

$$I_{R_2} = \frac{R_1}{R_1+R_2} I_s$$



$$i_{R_1} = \frac{R_3}{R_1+R_2+R_3} I_s$$

$$V_{oc_2} = i_{R_1} R_1 = \frac{R_3 R_1}{R_1+R_2+R_3} I_s$$

$$V_{oc} = V_{oc_1} + V_{oc_2} = \boxed{\frac{R_1}{R_1+R_2+R_3} V_s + \frac{R_3 R_1}{R_1+R_2+R_3} I_s = V_{th}}$$