DISOC

Concepts from lecture that'll come up

1. Gaussian Elimination
2. RREF (Row reduced echelon form)
3. Linear equations

Skills / Things to take away

1. 2 methods for how to know if something is a linear function
2. How to write solutions when using Gaussian Elimination
3. How to tell how many solutions there are when using Gaussian elimination

Linear function

$f(x)$

$f(x_1, x_2)$

$f(x_1, x_2, \ldots, x_n)$

① For any two pairs of $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$

$$f(x_1+y_1, x_2+y_2, \ldots, x_n+y_n) = f(x_1, x_2, \ldots, x_n) + f(y_1, y_2, \ldots, y_n)$$

"Additivity" "Superposition"

② Any real number α, and any $(x_1, x_2, \ldots, x_n)$

$$f(\alpha x_1, \alpha x_2, \ldots, \alpha x_n) = \alpha f(x_1, x_2, \ldots, x_n)$$

" Homogeneity"

$$\underbrace{f(x_1, x_2, \ldots, x_n) = b}_{\text{linear equation}}$$

linear equation :
linear function set equal to a constant value

Linear function

$$f(x_1, \ldots, x_n) = \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$$

Affine function

linear function + some constant

$$f(x_1, \ldots, x_n) = \alpha_1 x_1 + \cdots + \alpha_n x_n + b$$

All linearfunctions are affine. But all Affine functions are nonlinear

1 ⓐ $f(x_1, x_2) = 3x_1 + 4x_2$

$f(x_1, x_2) = 3x_1 + 4x_2$

$f(y_1, y_2) = 3y_1 + 4y_2$

$$f(x_1+y_1, x_2+y_2) = 3(x_1+y_1) + 4(x_2+y_2)$$
$$= 3x_1 + 3y_1 + 4x_2 + 4y_2$$
$$= (3x_1 + 4x_2) + (3y_1 + 4y_2)$$
$$= f(x_1, x_2) + f(y_1, y_2)$$

(f is additive)

$f(x_1, x_2) = 3x_1 + 4x_2$, α is a real #

$$f(\alpha x_1, \alpha x_2) = 3\alpha x_1 + 4\alpha x_2$$
$$= \alpha \underbrace{(3x_1 + 4x_2)}$$
$$= \alpha f(x_1, x_2)$$

(f is homogeneous)

$\Rightarrow$ $\boxed{\text{f is linear}}$

$$f(x_1, x_2, \dots, x_n) = \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n$$

1 ⓑ $f(x_1, x_2) = e^{x_2} + x_1^2$

$\alpha = 0$

$$f(\alpha x_1, \alpha x_2) = f(0, 0)$$
$$= e^0 + 0^2$$
$$= 1 \overset{?}{\neq} \alpha \cancel{f(x_1, x_2)}$$

f is not homogeneous

$\boxed{\text{f is nonlinear}}$

1 ⓒ $f(x_1, x_2) = x_2 - x_1 + 3$

$\alpha = 0$ $\quad f(\alpha x_1, \alpha x_2) \overset{?}{=} \alpha f(x_1, x_2)$

$$f(0, 0) = 0 - 0 + 3$$
$$= 3$$
$$\overset{?}{=} \alpha \cancel{f(x_1, x_2)}$$

$3 \neq 0$

$\boxed{\text{f is nonlinear}}$

2 (b)

$$\begin{bmatrix}1&1&2\\0&1&1\\2&1&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}2\\0\\4\end{bmatrix}$$

$$x_1+x_2+2x_3=2$$
$$x_2+x_3=0$$
$$2x_1+x_2+3x_3=4$$

$$\left[\begin{array}{ccc|c}1&1&2&2\\0&1&1&0\\2&1&3&4\end{array}\right]$$

$R_3-2R_1\to R_3$

$$\left[\begin{array}{ccc|c}1&1&2&2\\0&1&1&0\\0&-1&-1&0\end{array}\right]$$

$R_3+R_2\to R_3$

$$\left[\begin{array}{ccc|c}1&1&2&2\\0&1&1&0\\0&0&0&0\end{array}\right]$$

$R_1-R_2\to R_1$

$$\left[\begin{array}{ccc|c}1&0&1&2\\0&1&1&0\\0&0&0&0\end{array}\right]$$

"RREF"

infinitely many solutions

$$\begin{cases}x_1+t=2\\x_2+t=0\\x_3=t\end{cases}$$

$x_1=2-t$
$x_2=-t$
$x_3=t$

x_1, x_2 basic

$x_3=t$ free

$[0\ 0\ 0\ |\ 1\ 1]$

system is inconsistent, no solution

2 (a)

$$\left[\begin{array}{ccc|c}2&0&4&6\\0&1&2&-3\\1&2&0&3\end{array}\right]$$

$R_1/2\to R_1$

$$\left[\begin{array}{ccc|c}1&0&2&3\\0&1&2&-3\\1&2&0&3\end{array}\right]$$

$R_3-R_1\to R_3$

$$\left[\begin{array}{ccc|c}1&0&2&3\\0&1&2&-3\\0&2&-2&0\end{array}\right]$$

$R_3-2R_2\to R_3$

$$\left[\begin{array}{ccc|c}1&0&2&3\\0&1&2&-3\\0&0&-6&6\end{array}\right]$$

$R_3/{-6}\to R_3$

$$\left[\begin{array}{ccc|c}1&0&2&3\\0&1&2&-3\\0&0&1&-1\end{array}\right]$$

$R_2-2R_3\to R_2$
$R_1-2R_3\to R_1$

$$\left[\begin{array}{ccc|c}1&0&0&5\\0&1&0&-1\\0&0&1&-1\end{array}\right]$$

$x_1=5$
$x_2=-1$
$x_3=-1$

2 (c)

No solutions!

$$\left[\begin{array}{ccc|c}1&4&2&2\\1&2&8&0\\1&3&5&3\end{array}\right]$$

$$\left[\begin{array}{ccc|c}0&0&0&0\end{array}\right]$$

non zero

$0\neq$ non zero

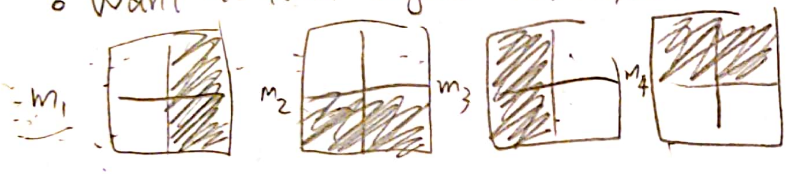
inconsistent, no solution

Bright Cave

Four caves

Nava Kody

- Can't see individual intensity (only total) $m_1, m_2, \ldots$
- Want to find light $x_1, x_2, \ldots$

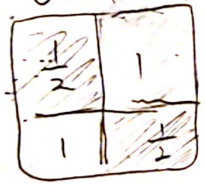


$$\begin{cases} m_1 = x_1 + x_3 \\ m_2 = x_1 + x_2 \\ m_3 = x_2 + x_4 \\ m_4 = x_3 + x_4 \end{cases}$$

(a) Equation for each

(b) Uniquely determinable? Why/Why not?

(c) Uniquely determinable now? Why/why not?



- Row reduce new
- Can use 4, but which you use are important

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & m_1 \\ 1 & 1 & 0 & 0 & m_2 \\ 0 & 1 & 0 & 1 & m_3 \\ 0 & 0 & 1 & 1 & m_4 \end{array}\right]$$

$m_1 + m_3 = x_1 + \cdots + x_4$

$m_2 + m_4 = x_1 + \cdots + x_4$

$$\frac{1}{2}x_1 + x_2 + x_3 + \frac{1}{2}x_4 = m_5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & m_1 \\ 1 & 1 & 0 & 0 & m_2 \\ 0 & 1 & 0 & 1 & m_3 \\ 0 & 0 & 1 & 1 & m_4 \\ \frac{1}{2} & 1 & 1 & \frac{1}{2} & m_5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Extra:

Gaussian Elimination → ① Elimination (REF)
② Back substitution (RREF)

① Row operations

② RREF

significance: gives us a way to write solutions cleanly (for unique or many solutions)

③ Terminating conditions

$[0 \; \cdots \; 0 \mid 3]$

inconsistent

consistent

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Unique solution

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

leading entry in every column

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & \boxed{0} & 1 & -3 \end{array}\right]$$

Many solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & \boxed{0} & 0 \end{array}\right]$$

No leading entry in every column

Examples (if time limited)

ⓐ Solution

ⓑ Many

ⓒ None

↳ How to parameterize