EECS16A DIS1B

<u>Concepts from lecture</u>

Matrix - Matrix multiplication, operations & properties

<u>Skills / Things to take away</u>

① Expression for rotation matrix in $\mathbb{R}^2$

② How to go about matrix proofs

T_1 - rotates by 15° $\quad \vec{v} \in \mathbb{R}^2 \quad T_1, T_2 \rightarrow 2\times 2$ matrices

T_2 - rotates by 30°

$T_1\vec{v} = \vec{v}$ rotated by 15°

$T_2\vec{v} = \vec{v}$ rotated by 30°

$\underline{45^\circ}$? $\Rightarrow$?

60° ? $\Rightarrow$?

$T_2(T_1\vec{v}) = ?$

$T_1 T_2 \vec{v} = ?$

T_2 (vector $\vec{v}$ rotated by 15°)

$=$ vector $\vec{v}$ rotated by $15^\circ + 30^\circ$ ✓

T_1 (vector $\vec{v}$ rotated by 30°)

$=$ rotation of $\vec{v}$ by 45° ✓

$T_2 T_2 \vec{v} \Rightarrow 60^\circ$ ✗

$(T_1(T_1(T_1\vec{v}))) \Rightarrow 45^\circ$

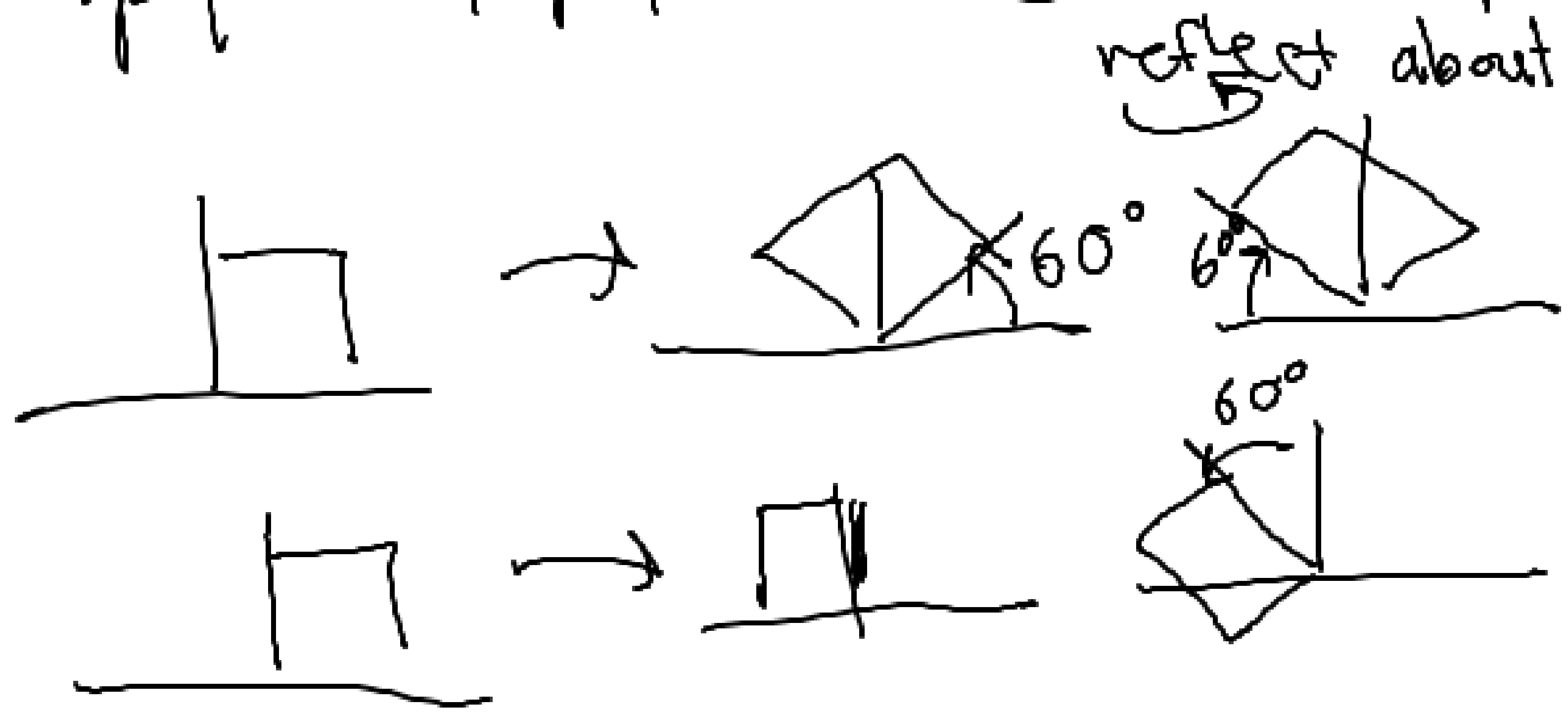
$$T_1(T_1(T_1\vec{v})) \overset{\checkmark}{=} (T_1 T_1 T_1)\vec{v}$$

matrix-matrix mult. is associative

$$\overset{?}{=} T_1 T_1 \vec{v} T_1$$

is mult. commutative? $\Rightarrow$ $\boxed{\text{NO}}$

in ipython/python @ multiplies matrices together



$T_1 \vec{v}$

$2\times 2 \quad 2\times 1$

$\vec{v}\, T_1$

$2\times \cancel{1 \quad 2}\times 2$

$$\underline{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \leftarrow$$ check that this is the matrix that rotates by angle θ

$$\underline{\vec{v}} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

$\underline{R\vec{v}\,?}$

$$\vec{w} = \begin{bmatrix} \cos(\alpha+\theta) \\ \sin(\alpha+\theta) \end{bmatrix}$$

$$\underline{\vec{w} \overset{?}{=} R\vec{v}} \Leftarrow$$

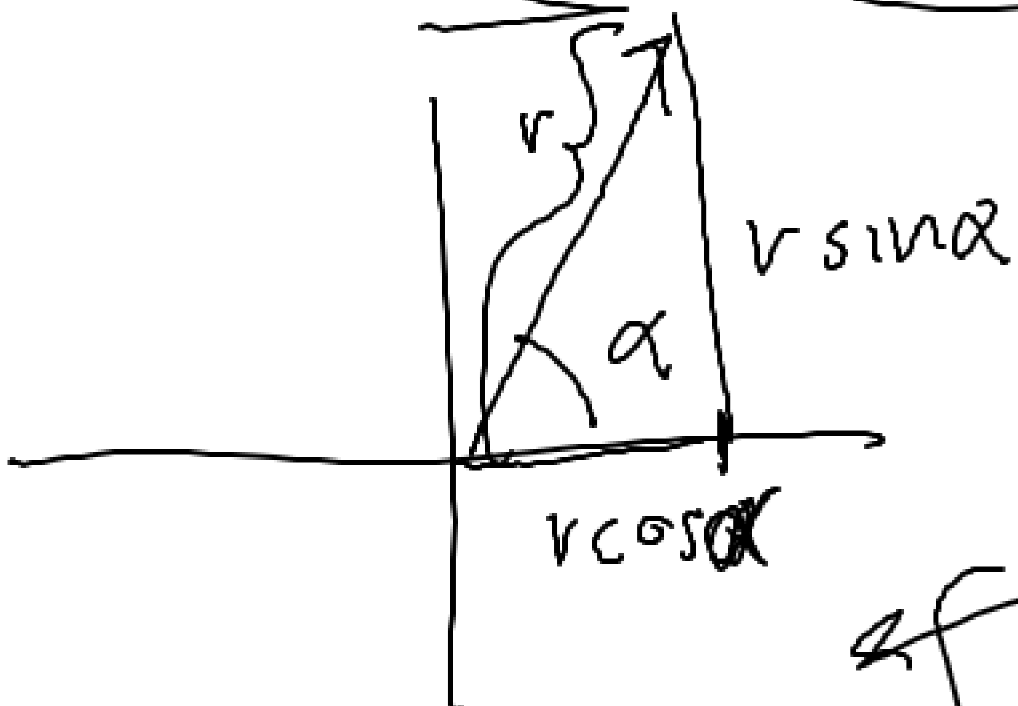
$$\begin{bmatrix} \cos(\alpha+\theta) \\ \sin(\alpha+\theta) \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\theta - \sin\alpha\sin\theta \\ \sin\alpha\cos\theta + \sin\theta\cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\theta \\ \sin\theta\cos\alpha \end{bmatrix} + \begin{bmatrix} -\sin\alpha\sin\theta \\ \sin\alpha\cos\theta \end{bmatrix} = R\vec{v} \checkmark$$

$\underline{R(2\vec{v})} \Rightarrow$

$$= \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \underline{\cos\alpha} + \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \underline{\sin\alpha} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

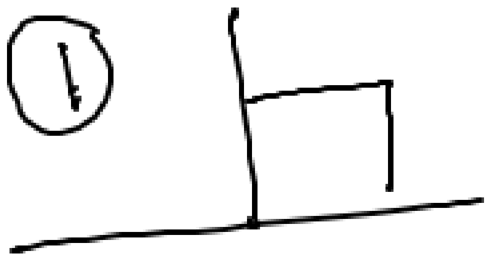
$$\vec{v} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$



$$\begin{bmatrix} r\cos\alpha \\ r\sin\alpha \end{bmatrix}$$

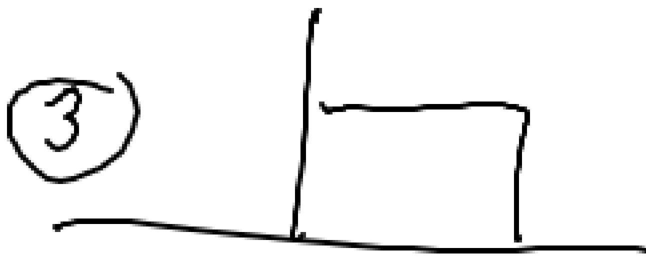
$$\begin{bmatrix} & & \\ & R & \\ & & \end{bmatrix} \begin{bmatrix} r\cos\alpha \\ r\sin\alpha \end{bmatrix}$$

$$r \begin{bmatrix} & & \\ & R & \\ & & \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

1  $\xrightarrow{\text{rotate by } 60^\circ}$ 2 

How do we get back to

$\theta = 60^\circ$

3 

$$\underline{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\underbrace{R(\theta = 300^\circ)\, R(\theta = 60^\circ)} \rightarrow \underline{R(\theta = 360^\circ)} \quad \underline{\text{No change}}$$

$$\underbrace{R(\theta = -60^\circ)\, R(\theta = 60^\circ)} \rightarrow R(\theta = 0^\circ) \quad \underline{\text{No change}}$$

$$\underline{M}\vec{v} = \vec{v} \qquad M = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{R_\theta\, R_{-\theta}} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} \stackrel{?}{=} \underline{I}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ \sin\theta\cos\theta - \cos\theta\sin\theta & 0 \end{bmatrix}$$

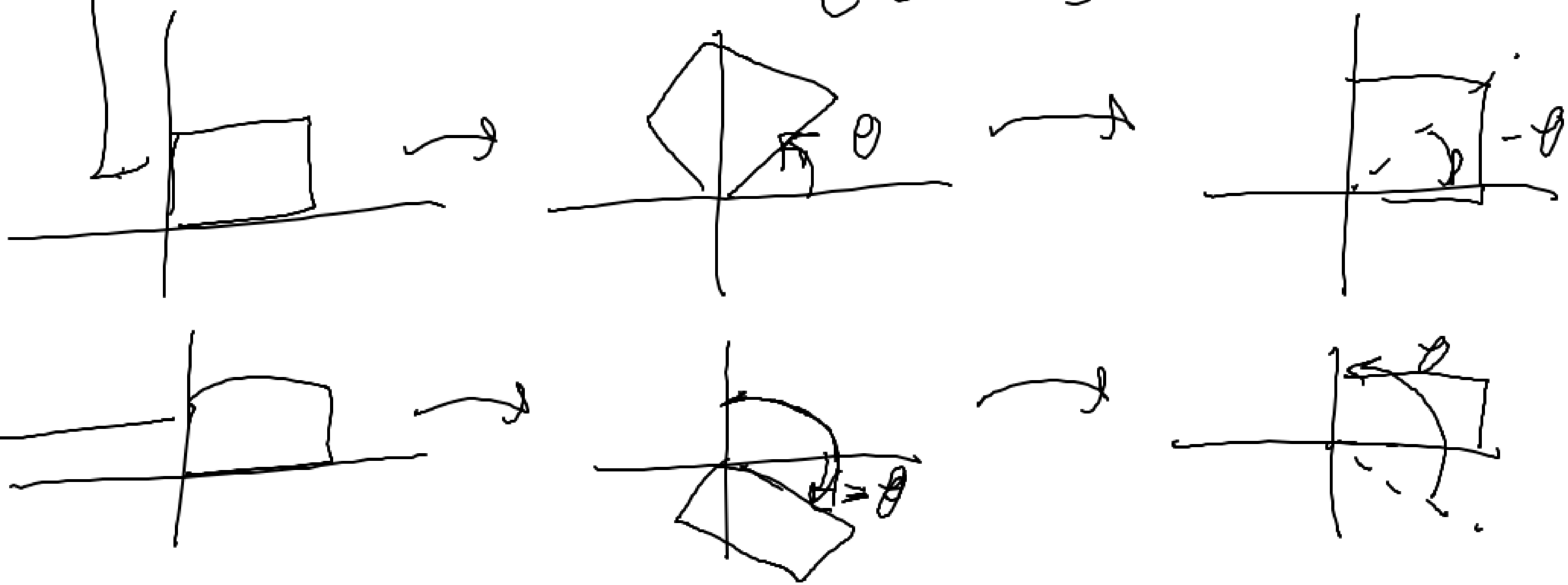
$\cos(\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$-\theta$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ✓$$

$$R_{-\theta} R_\theta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \overset{?}{\neq} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \checkmark$$

$$\boxed{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \checkmark$$

Reflection matrix about y axis

$$A(\vec{v}_1+\vec{v}_2) \overset{?}{=} A\vec{v}_1 + A\vec{v}_2$$

Start by naming rows, columns, entries of matrices and vector

A 2x2

$\vec{v}_1, \vec{v}_2 \in \mathbb{R}^2$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

2x1

$$\vec{v}_1 = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} v_{\cancel{12}} \\ v_{22} \end{bmatrix}$$

$$A\Bigg(\underbrace{\begin{bmatrix} v_{11}+v_{\cancel{12}} \\ v_{21}+v_{22} \end{bmatrix}}_{\text{column}}\Bigg) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} v_{11}+v_{12} \\ v_{21}+v_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(v_{11}+v_{12}) + a_{12}(v_{21}+v_{22}) \\ \\ \end{bmatrix}$$

$$A\vec{v}_1 + A\vec{v}_2 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} v_{11} \\ v_{\cancel{21}} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix}$$

$$\begin{bmatrix} (a_{11} \cdot v_{11} + a_{12} \cdot v_{21}) + (a_{11} v_{12} + a_{12} \cdot v_{22}) \\ \cancel{} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} (v_{11} + v_{12}) + a_{12} (v_{21} + v_{22}) \\ \end{bmatrix} \qquad A(\vec{v}_1 + \vec{v}_2)$$