EECS16A DIS 1C

Skills to take away / Practice

1. Computing Inverses using Gaussian Elimination
2. Describing transition matrix systems
   - a. Translating diagrams into equations
   - b. Translating equations into a matrix equation

$\underset{\substack{n\times n \\ \text{square} \\ \text{matrix}}}{A}$ has an inverse $\underset{\substack{n\times n \\ \text{square} \\ \text{matrix}}}{A^{-1}}$ if $AA^{-1} = \underset{n\times n}{I}$ and $A^{-1}A = \underset{n\times n}{I}$

I forgot to start the recording at the very beginning.

Here are some annotations for the second slide to help piece together what was said.

Up to this point in the course, you know how to solve problems of the form

$$\underset{\substack{n\times n\\ \text{known}}}{A}\,\underset{n\times 1}{\vec{x}} = \underset{\substack{n\times 1\\ \text{known}}}{\vec{b}}, \quad \vec{x} \text{ unknown}$$

(Use gaussian elimination)

If we want to solve for an inverse, $\overset{\text{known}}{A}\,\underset{\text{unknown}}{A^{-1}} = \overset{\text{known}}{I}$ almost looks like it.

Using matrix multiplication, where the columns of A^{-1} are $\vec{\alpha}_1, \vec{\alpha}_2, \ldots, \vec{\alpha}_n$

$$AA^{-1} = A\begin{bmatrix} | & | & & | \\ \vec{\alpha}_1 & \vec{\alpha}_2 & \cdots & \vec{\alpha}_n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & | & & | \\ A\vec{\alpha}_1 & A\vec{\alpha}_2 & A\vec{\alpha}_3 & \cdots & A\vec{\alpha}_n \\ | & | & | & & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & & 0 \\ 0 & 1 & \vdots & & \vdots \\ \vdots & 0 & & \ddots & 0 \\ 0 & 0 & & & 1 \end{bmatrix}$$

This is just a bunch of $A\vec{x} = \vec{b}$ problems! with $\vec{x} = \vec{\alpha}_i$ and $\vec{b} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \leftarrow$ ith spot

With GE, $[A \mid \vec{b}] \xrightarrow{GE} [I \mid \vec{x}]$ (if a unique solution exists)

So try $\left[A \;\middle|\; \begin{matrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{matrix}\right] \leftarrow$ ith spot $\xrightarrow{GE} [I \mid \vec{\alpha}_i]$ But! The columns never interact! only rows! (because of GE's row operations)

So stack them together: $\left[A \;\middle|\; \begin{matrix} 1 & 0 & & 0 \\ 0 & 1 & & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & & 1 \end{matrix}\right] \xrightarrow{GE} \left[I \;\middle|\; \begin{matrix} | & | & & | \\ \vec{\alpha}_1 & \vec{\alpha}_2 & \cdots & \vec{\alpha}_n \\ | & | & & | \end{matrix}\right]$ (If a unique solution to all exist)

$$[I \mid A^{-1}]$$

1 (a) Does the inverse of $\begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$ exist? If so, what is it?

$$AA^{-1} = I$$

$$A\underset{n}{\vec{x}} = \vec{b}$$

$$A^{-1} = \begin{bmatrix} | & | & & | \\ \vec{\alpha}_1 & \vec{\alpha}_2 & \cdots & \vec{\alpha}_n \\ | & | & & | \end{bmatrix}$$

$$A \begin{bmatrix} | & | & & | \\ \vec{\alpha}_1 & \vec{\alpha}_2 & \cdots & \vec{\alpha}_n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} | \\ A\vec{\alpha}_1 \\ | \end{matrix}} & \begin{matrix} | \\ A\vec{\alpha}_2 \\ | \end{matrix} & \cdots & \begin{matrix} | \\ A\vec{\alpha}_n \\ | \end{matrix} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ 0 & & & \\ \vdots & & & \\ 0 & & & \end{bmatrix}$$

$$A\vec{\alpha}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A\vec{\alpha}_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

$$\left[A \mid \vec{b} \right] \to \left[A \mid I \right] \xrightarrow{GE} \left[I \mid A^{-1} \right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix} \to \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 9 & 0 & 1 \end{array}\right] \xrightarrow[R_2/9 \to R_2]{} \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1/9 \end{array}\right]$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1/9 \end{bmatrix}}_{A^{-1}}$$

$AA^{-1} = I$? $\begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 5 & 4 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \quad R_1/5$$

$$\left[\begin{array}{cc|cc} 1 & 4/5 & 1/5 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \quad R_2 - R_1$$

$$\left[\begin{array}{cc|cc} 1 & 4/5 & 1/5 & 0 \\ 0 & 1/5 & -1/5 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 4/5 & 1/5 & 0 \\ 0 & 1 & -1 & 5 \end{array}\right] \quad 5R_2$$

$$R_1 - \frac{4}{5}R_2 \to \left[\begin{array}{cc|cc} 1 & 0 & 1 & -4 \\ 0 & 1 & -1 & 5 \end{array}\right]$$

$$\boxed{\begin{matrix} 1 & -4 \\ -1 & 5 \end{matrix}}\ A^{-1}$$

if col. are lin. dep.
$\Rightarrow$ (Matrix)$^{-1}$ doesn't exist

$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

If an inverse doesn't exist: after GE

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & \uparrow & \uparrow & \uparrow & \uparrow \end{array}\right]$$

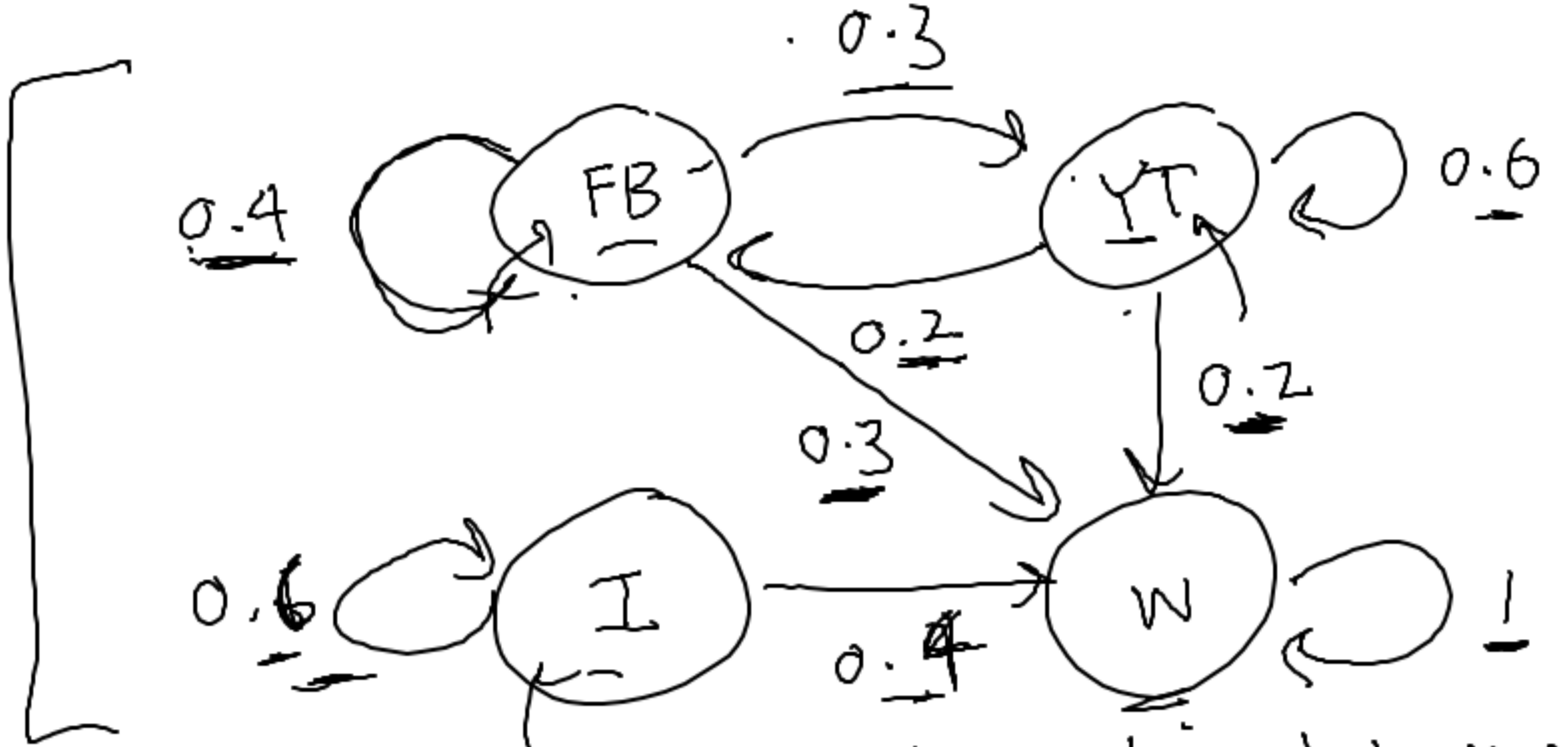
one of these is nonzero.

Does the inverse of $\begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$ exist?

If so what is it? If not why not?

$$\begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

columns are the same. Lin. dep.



Population of students on each website

$t \to$ time (integer)

$X_{FB}[t]$ → # of students @ FB

$X_{YT}[t]$

$X_I[t]$

$X_W(t)$

$$\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.3 & 0.6 & & 0 \\ 0 & 0 & 0.6 & 0 \\ 0.3 & 0.2 & 0.4 & 1 \end{bmatrix}$$

Can we express a relationship between $X_{FB}[t+1]$, $X_{YT}[t+1]$, $X_I[t+1]$, $X_W[t+1]$

$$\begin{cases} X_{FB}[t+1] = 0.4\, X_{FB}[t] + 0.2\, X_{YT}[t] + 0 \\ X_{YT}[t+1] = 0.3\, X_{FB}[t] + 0.6\, X_{YT}[t] + 0\, X_I(t) + 0 \\ X_I[t+1] = 0\, X_{FB}[t] + 0\, X_{YT}[t] + 0.6\, X_I[t] + 0 \\ X_W[t+1] = 0.3\, X_{FB}[t] + 0.2\, X_{YT}[t] + 0.4\, X_I[t] + 1 \end{cases}$$

$$\vec{x}[t+1] = \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.3 & 0.6 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0.3 & 0.2 & 0.4 & 1 \end{bmatrix} \vec{x}[t]$$

$$\vec{x}[t] = \begin{bmatrix} x_{FB}[t] \\ x_{YT}[t] \\ x_I[t] \\ x_W[t] \end{bmatrix}$$

$$\vec{x}[t] = \begin{bmatrix} 700 \\ 450 \\ 200 \\ 150 \end{bmatrix}$$

1500 total

$$\vec{x}[t+1] = \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.3 & 0.6 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0.3 & 0.2 & 0.4 & 1 \end{bmatrix} \begin{bmatrix} 700 \\ 450 \\ 200 \\ 150 \end{bmatrix} = \begin{bmatrix} 370 \\ 480 \\ 120 \\ 530 \end{bmatrix}$$

1500 students

\# of students didn't change

$x_{FB} \rightarrow$ Decrease

$x_{YT} \rightarrow$ Decrease

$x_I \rightarrow$ Decrease

$x_W \rightarrow$ Increase

$$\vec{x}(t \text{ really large}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1500 \end{bmatrix} ?$$

this system is conservative

$$\vec{x}[t+1] = T\vec{x}[t]$$

$$\vec{x}[t+2] = T\vec{x}[t+1] = T^2\vec{x}[t]$$

$$\vec{x}[t+3] = T\vec{x}[t+2] = T^3\vec{x}[t]$$

$$\vec{x}[t+100] = T^{100}\vec{x}[t]$$

$$\vec{x}'[t] = \begin{bmatrix} x_{FB} \\ x_{YT} \\ x_{I} \end{bmatrix}$$

non conservative

$$\vec{x}[t+1] = \underset{3\times 3}{T'}\,\vec{x}[t]$$

$$\vec{x}\left[\begin{matrix}\text{far}\\ \text{in}\\ \text{the}\\ \text{future}\end{matrix}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

goes out from work

amount that goes to work