EECS16A DIS 1D

Today's Discussion

1. More on inverses
2. Using transition matrix systems to understand A^T vs. A^{-1}
3. Bases

Definitions

Transpose (of a matrix)

$\underset{m\times n}{A} \rightarrow \underset{n\times m}{A^T}$ (A transpose) ← read

$$A = \begin{bmatrix} | & | & & | \\ \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \\ | & | & & | \end{bmatrix} \quad A^T = \begin{bmatrix} - & \vec{a}_1 & \rightarrow \\ - & \vec{a}_2 & \rightarrow \\ & \vdots & \\ - & \vec{a}_n & \rightarrow \end{bmatrix}$$

e.g.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \end{bmatrix} \longrightarrow A^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 3 \end{bmatrix}$$

Basis (of a vector space V)

↳ Set of vectors $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis if and only if

1. set is linearly independent
2. set spans V, i.e. $\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} = V$

$$(ABC)^{-1} \overset{?}{=} C^{-1}B^{-1}A^{-1}$$

$$ABC(ABC)^{-1} = I$$

$$ABC(C^{-1}B^{-1}A^{-1}) \overset{?}{=} I$$

$$AB(CC^{-1})B^{-1}A^{-1} = ABIB^{-1}A^{-1}$$

$$= A\underline{BB^{-1}}A^{-1}$$

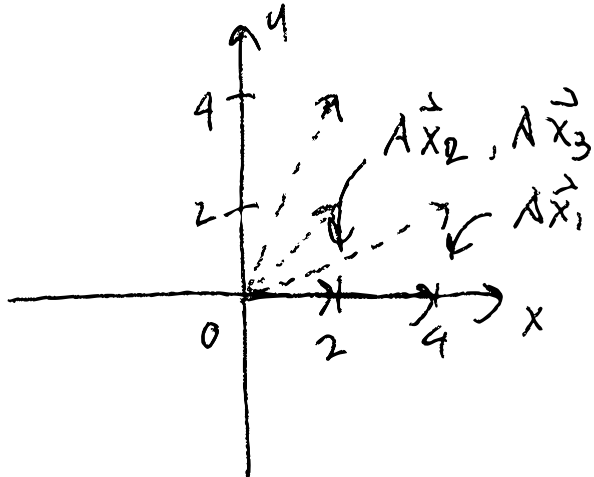
$$= AIA^{-1}$$

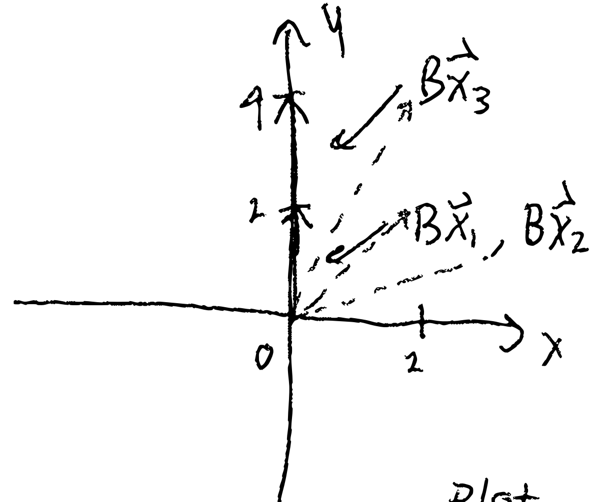
$$= AA^{-1} = I$$

$AA^{-1} = I \quad A^{-1}A = I$

$C^{-1}B^{-1}A^{-1}ABC = I$ ✓

Proofs: Try using definitions or properties you know

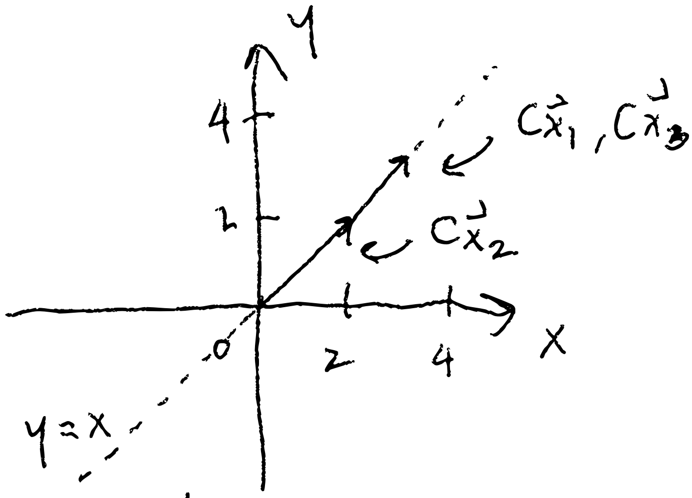




$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{Plot } A\vec{x}_1,\ A\vec{x}_2,\ A\vec{x}_3$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{Plot } B\vec{x}_1,\ B\vec{x}_2,\ B\vec{x}_3$$

$$\vec{x}_1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \vec{x}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \vec{x}_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$



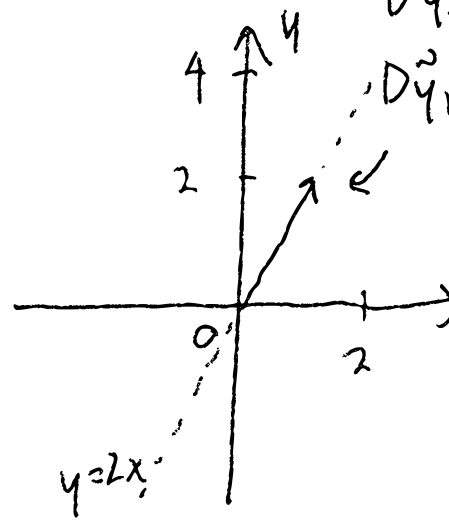
$$C = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

plot
$C\vec{x}_1$, $C\vec{x}_2$, $C\vec{x}_3$

$$\vec{x}_1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \vec{x}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \vec{x}_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$D\vec{y}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$D\vec{y}_1 = \begin{bmatrix} 3/5 \\ 6/5 \end{bmatrix} + \begin{bmatrix} 2/5 \\ 4/5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



$$D = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & 4/5 \end{bmatrix}$$

$$\vec{y}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \vec{y}_2 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

plot $D\vec{y}_1$, $D\vec{y}_2$

$$\left[\begin{array}{c|c} \underset{n \times n}{A} & \underset{n \times n}{I} \end{array}\right] \xrightarrow{G.E.} \left[\begin{array}{c|c} \underset{n \times n}{I} & \underset{n \times n}{A^{-1}} \end{array}\right]$$

If A^{-1} exists

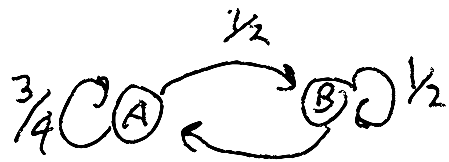
If not, you'll get a set of inconsistent eqns.

$$\begin{cases} x_A(t+1) = \underline{\quad} x_A(t) + \underline{\quad} x_B(t) \\ x_B(t+1) = \underline{\quad} x_A(t) + \underline{\quad} x_B(t) \end{cases} \rightarrow \begin{bmatrix} \; \\ \; \end{bmatrix} = \begin{bmatrix} \; & \; \\ \; & \; \end{bmatrix} \begin{bmatrix} \; \\ \; \end{bmatrix}$$

2 (a) $x_A[t+1] = \boxed{}\, x_A[t] + \boxed{}\, x_B[t]$

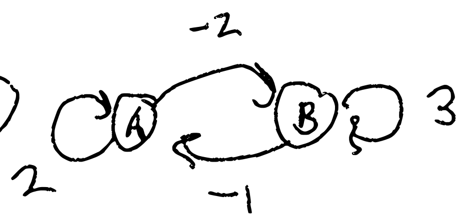
$x_B[t+1] = \boxed{}\, x_A[t] + \boxed{}\, x_B[t]$

$$S = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix}$$

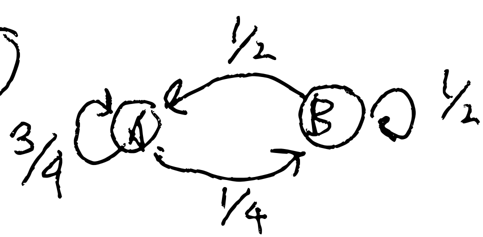


(b) $S^{-1} = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$ $\quad \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

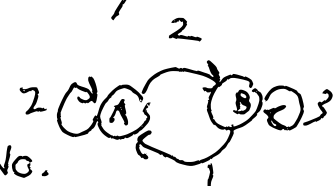
(c)



(d)



$$T = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = S^T \qquad S^T \neq S^{-1}?$$



$\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}_X$

Q: Can we reverse arrows & flip sign? No.

$V_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ $V_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ $V_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

Is V_1 LI? (linearly independent?)

<u>Yes</u>

$\mathrm{Span}(V_1) \overset{?}{\neq} \mathbb{R}^3$

<u>No</u>

$\boxed{\text{Not a basis}}$

V_2 LI?

<u>Yes</u>

$\mathrm{Span}(V_2) \overset{?}{=} \mathbb{R}^3$

<u>Yes</u>

$\boxed{\text{Is a basis}}$

V_3 LI?

<u>No</u>

$\mathrm{Span}(V_3) \overset{?}{\neq} \mathbb{R}^3$

<u>No</u>

$\boxed{\text{Is not a basis}}$