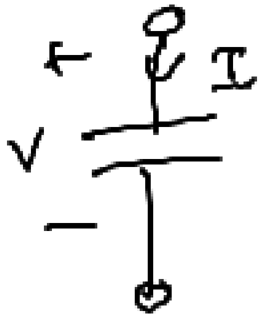
EECS16A DIS 4B

* Don't forget, there's a checkoff today

Today's topics

1. Capacitor Review (voltage, current, charge, energy)
2. Time dependent behavior of a charging capacitor
3. If time: Capacitor Equivalence derivations + practice
   ↳ Appeared in lecture yesterday



I-V relation / I-V characteristic / Branch-Branch relationship
I V

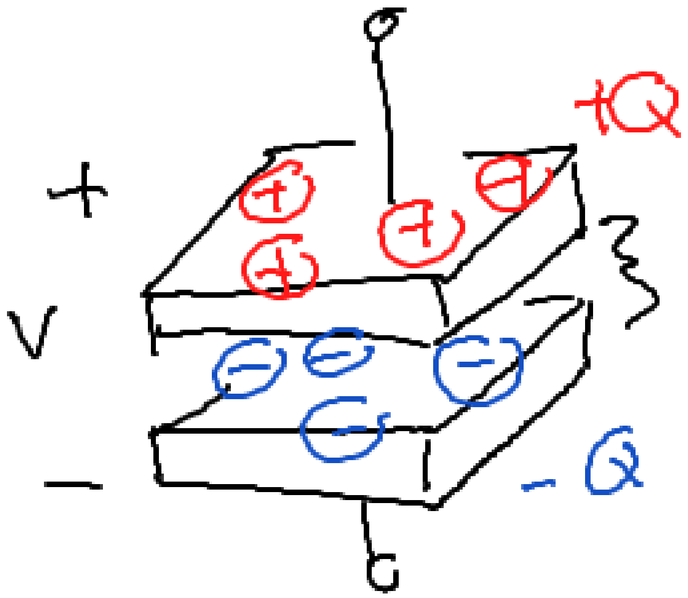
$$\boxed{Q = CV}$$

$$i = \frac{dQ}{dt}$$

Assuming a constant C

$$i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

$$\boxed{i = C\frac{dV}{dt}}$$



[F]

C - Capacitance [Farads]

Energy (stored in the capacitor)

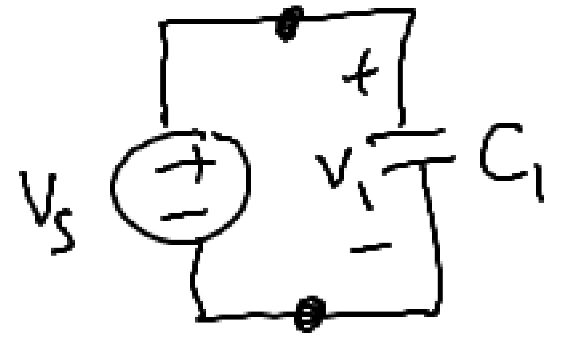
$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2}QV \quad [C]\left[\frac{J}{C}\right]$$

$\rightarrow [J]$

1 ⓐ



What is the charge + energy ?

$Q_1 = \underline{C_1} V_1$

$[F][V] = [C]$ ← coulombs, unit of charge

$V_s = 1V$

$C_1 = 1\mu F$

$V_1 = V_s$ ✓

$Q_1 = (1\mu F)(1V) = \boxed{1\mu C}$

Q: How do SI prefixes combine?

$\underset{10^{-3}}{\underline{\text{milli}}}[A] \times \text{milli}[B] = 10^{-3} \times 10^{-3}[AB] = 10^{-6}[AB] = \underset{\mu}{\underline{\text{micro}}}[AB]$

$\text{milli}[A] \times [B] = 10^{-3}[AB] = \text{milli}[AB]$

$E = \frac{1}{2}CV^2 = \frac{1}{2}(1\mu F)(1V)^2 = \boxed{\frac{1}{2}\mu J}$

$[F][V]^2$

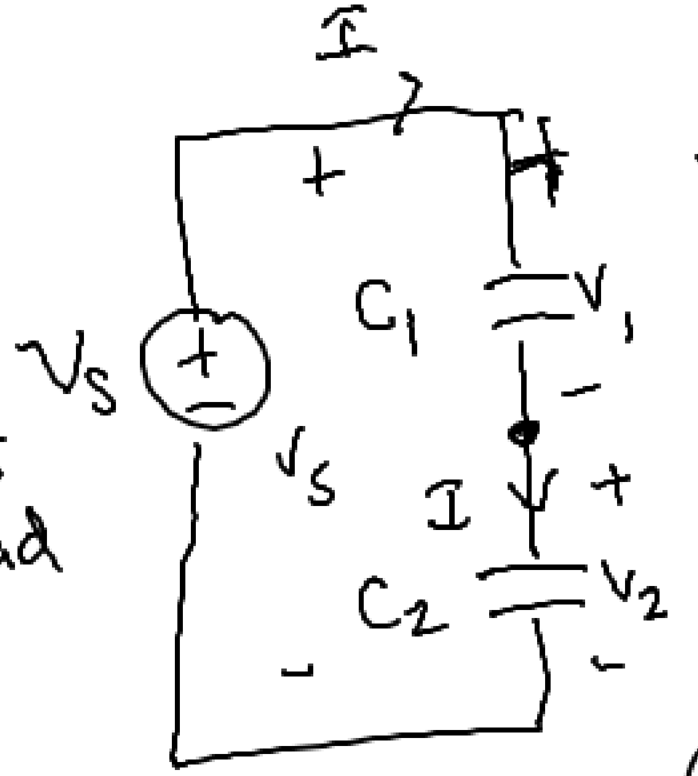
$[C][V]$

$[J]$

$[V] = \left[\frac{J}{C}\right]$

$\boxed{1}$ ⓑ

After capacitors have had time to charge



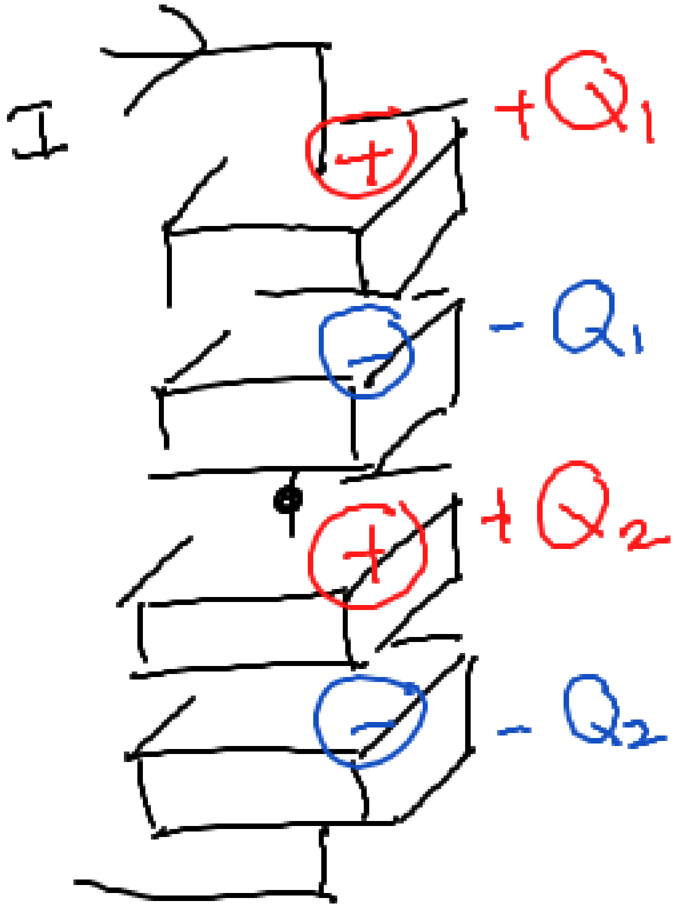
$V_s = 1V$

$C_1 = 1\mu F$

$C_2 = 3\mu F$

Find Q_1, Q_2 & E_1, E_2

(charge on each cap.) (energy stored in each cap.)



$\underline{Q_1 = Q_2} = Q$ (true even $\underline{\text{during}}$ charging)

① Method 1: look at both caps charges

② Method 2: Use equivalence

Method ①

$Q = CV$

$\Rightarrow V_s = V_1 + V_2$ (true after charging)

$= \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$

We're assuming both caps uncharged @ start

$V_s = \frac{Q}{C_1} + \frac{Q}{C_2}$

$Q = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} V_s = \frac{C_1 C_2}{C_1+C_2} V_s = \frac{1\underline{\mu F}\cdot 3\cancel{\mu F}}{1\cancel{\mu F} + 3\cancel{\mu F}} \cdot 1\underline{V} = \boxed{\frac{3}{4}\underline{\mu}\,\underline{C}} = Q_1 = Q_2$

Using method ② $\{\ Q = (C_1 || C_2) V_s$

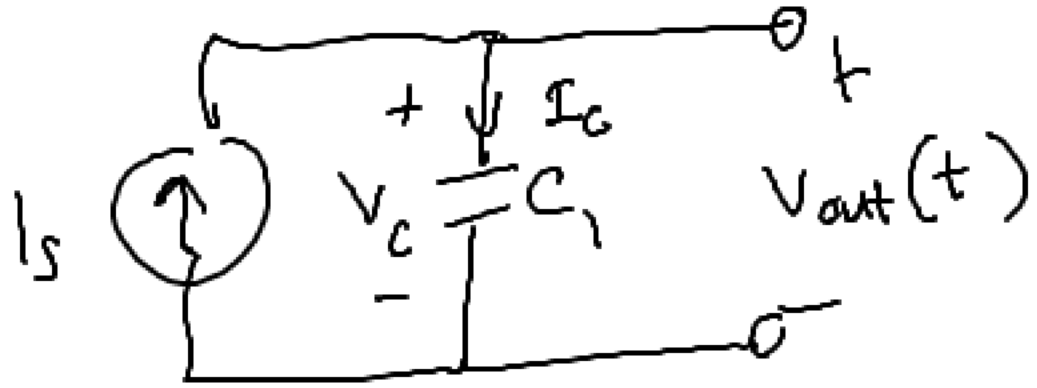
I didn't calc. V_1

$E_1 = \cancel{\frac{1}{2} Q_1 V_1} = \frac{1}{2}\frac{Q_1^2}{C_1} = \frac{1}{2}\frac{(\frac{3}{4}\mu C)^2}{1\mu F} = \frac{1}{2}\boxed{\frac{9}{16}\ \mu J}$

$E_2 = \frac{1}{2}\frac{Q_2^2}{C_2} = \frac{1}{2}\frac{(\frac{3}{4}\mu C)^2}{3\mu F} = \boxed{\frac{3}{32}\ \mu J}$

2 How capacitors behave (with time) when charged by a Constant Current source.

ⓐ



Find $V_{out}(t)$ in terms of C_1, I_s, t (time)

$$\begin{cases} Q = CV & ① \\ i = C\frac{dV}{dt} & ② \leftarrow \end{cases}$$

$I_c = I_s$ (KCL ⓐ top)

$$C_1 \frac{dV_c}{dt} = I_s \qquad V_c(t) = V_{out}(t)$$

$$\frac{dV_{out}}{dt} = \frac{I_s}{C_1}$$

$$\int_{V_{out}(0)}^{V_{out}(t)} dV_{out} = \int_{t=0}^{t=t} \frac{I_s}{C_1} dt$$

$$V_{out} \Big|_{V_{out}(0)}^{V_{out}(t)} = \frac{I_s}{C_1} t \Big|_0^t$$

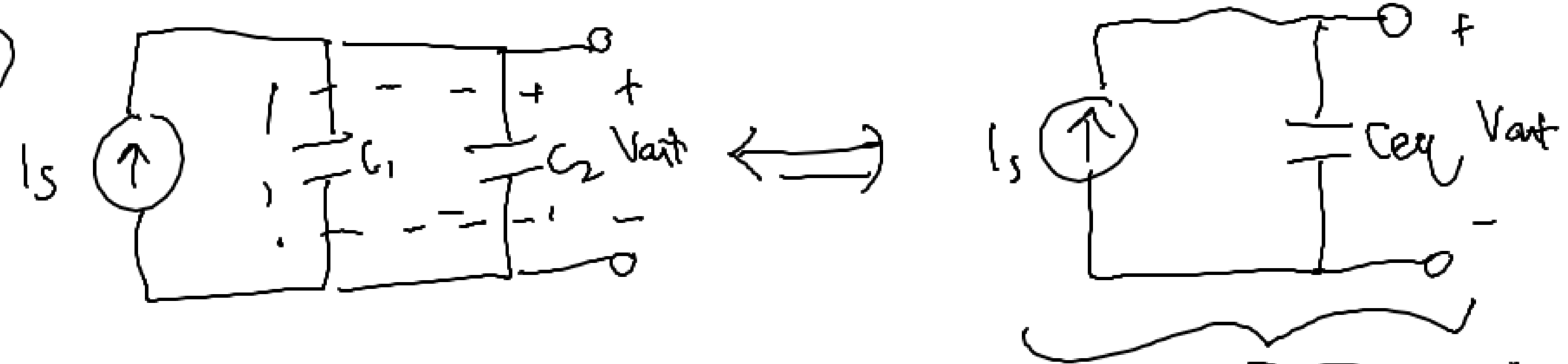
$$V_{out}(t) - V_{out}(0) = \frac{I_s}{C_1}(t - 0)$$

$$V_{out}(t) = \underbrace{V_{out}(0)}_{} + \underbrace{\frac{I_s}{C_1} t}_{} \Rightarrow \boxed{V_{out}(t) = \frac{I_s}{C_1} t}$$

$V_{out}(0) = 0V$

Constant current leads to constantly increasing (linearly) voltage

(b)



Using cap. equivalence

$$C_{eq} = C_1 + C_2$$

(parallel caps add)

$$V_{at}(t) = \frac{I_s}{C_{eq}} t = \boxed{\frac{I_s}{C_1 + C_2} t}$$

3 ⓐ Deriving Parallel equivalent capacitance

ⓑ Deriving series " "

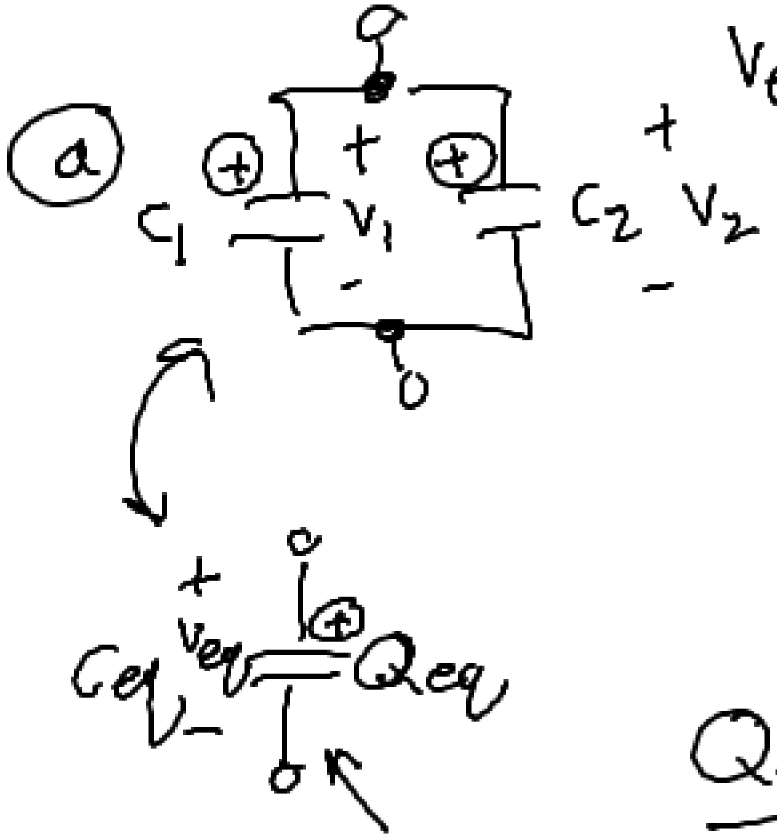
ⓒ Practicing using equivalence

$i = C\frac{dV}{dt}$

in lecture this was used

$Q = CV$

ⓐ



$V_{eq} = V_1 = V_2$ (parallel)

Q_1, Q_2

$Q_{eq} = \underline{Q_1 + Q_2}$

$V_{eq} \sim Q_{eq}$?

$\underline{Q_{eq}} = C_1 V_{eq} + C_2 V_{eq}$

$Q_1 = C_1 V_1 = C_1 V_{eq}$

$Q_2 = C_2 V_1 = C_2 V_{eq}$

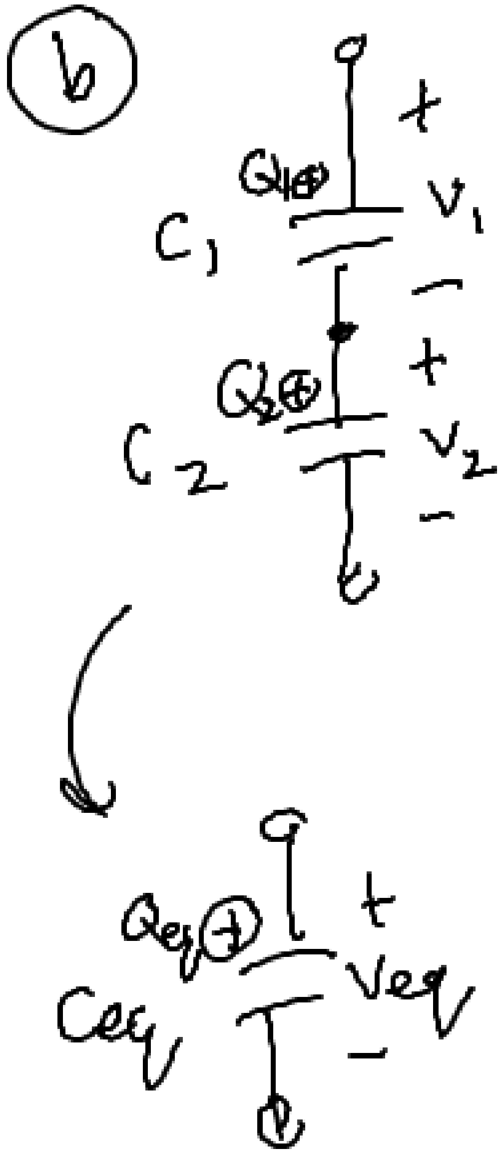
$$\overset{i_1}{\downarrow} \qquad \overset{i_2}{\downarrow}$$

$i_{eq} = C_1\frac{dV_1}{dt} + C_2\frac{dV_2}{dt}$

$Q_{eq} = \underbrace{C_{eq}} V_{eq} = \underbrace{(C_1 + C_2)} V_{eq}$

$\boxed{C_{eq} = C_1 + C_2}$ (in parallel)

ⓑ



Series eq. C : Assumption: both start uncharged
both start with the same amount of charge

$Q_{eq} = Q_1 = Q_2$

(series, same current dumps same charge)

$V_{eq} = V_1 + V_2$

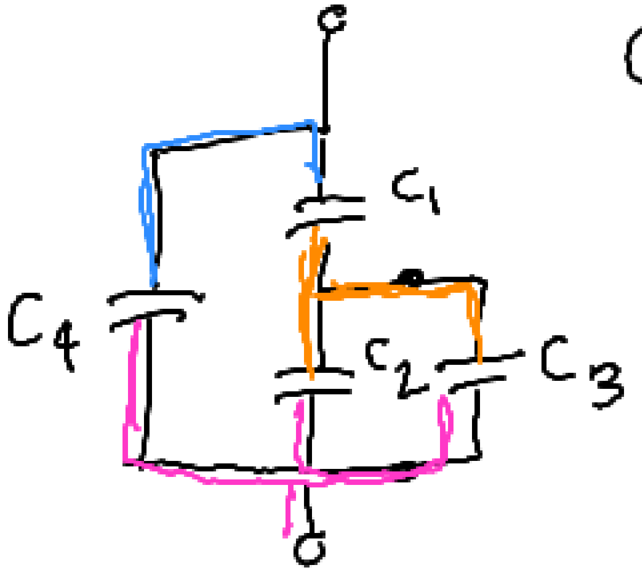
$\underline{V_{eq}} = \frac{Q_{eq}}{C_1} + \frac{Q_{eq}}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q_{eq}$

$Q_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} V_{eq} = \underbrace{\frac{C_1 C_2}{C_1 + C_2}}_{C_1 \| C_2} V_{eq} \longrightarrow \boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \text{ (series eq.)}}$

$\underbrace{A \| B = \frac{AB}{A+B}}_{\text{operation, not shape of circuit}}$

Extra: Not covered, but here for your use/learning/checking

3 ⓒ Practice using series and parallel equivalence to reduce to a single equivalent



① Check series & parallel for pairs

✗ C_1 & C_2? No, neither (bc of C_3, not series, bc of C_4 not parallel)

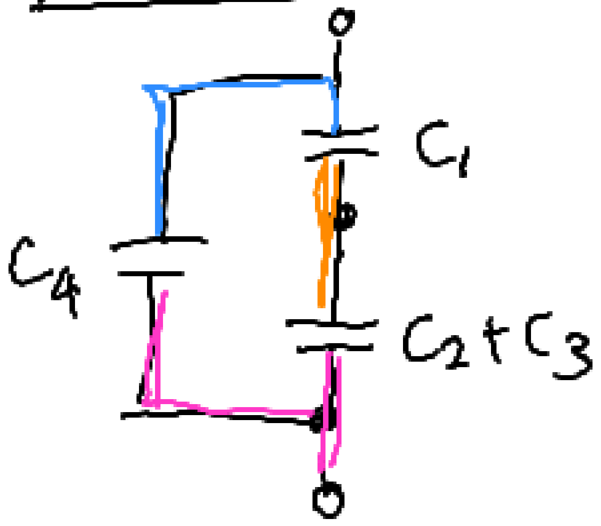
✗ C_4 & C_2? also Neither

○ C_2 & C_3? Yes! Parallel

② Calculate value

$C_{eq} = C_2 + C_3$

③ Redraw with substitution: The equivalent should be connected to the same pair of nodes (●, ●)

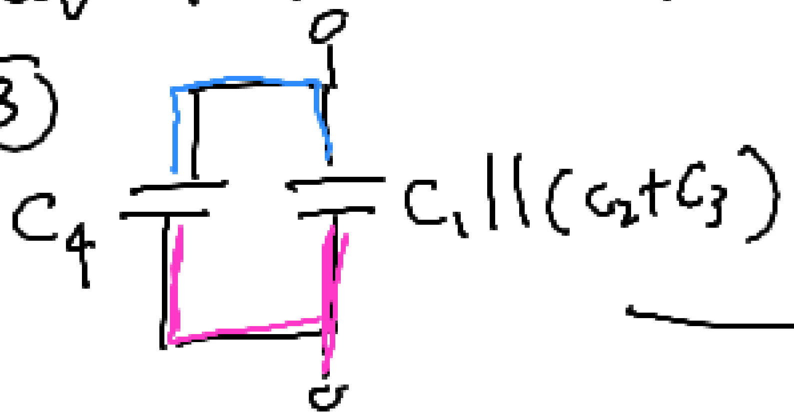


④ Iterate!

① C_1 & $C_2 + C_3$ series

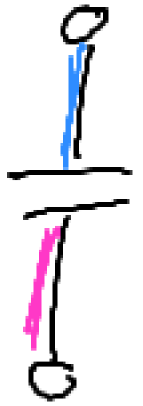
② $C_{eq} = C_1 || (C_2 + C_3)$ (parallel operator doesn't distribute)

③



(●, ●)

④



$$\boxed{C_{eq} = C_4 + C_1 || (C_2 + C_3)}$$

Extra: not covered during discussion but here for your use/learning

Q: Why is equivalence useful?

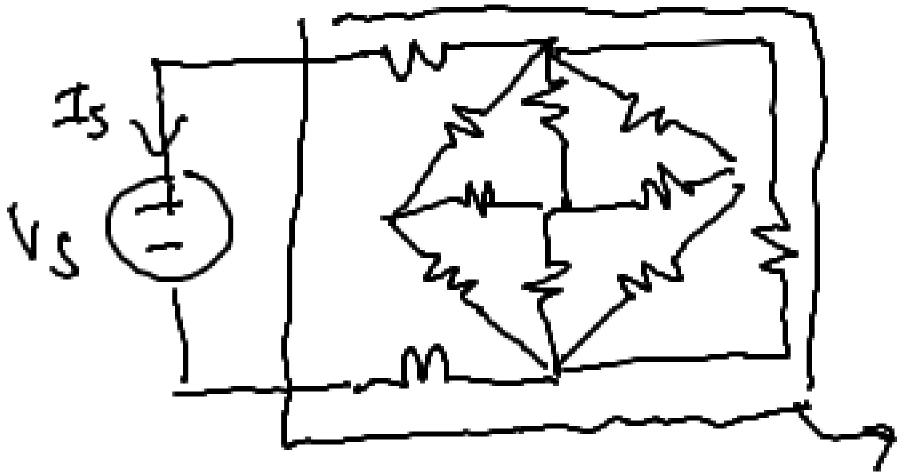
So far, we have learned of 4 kinds of equivalence

- Norton
- Thevenin
- Resistor
- Capacitor

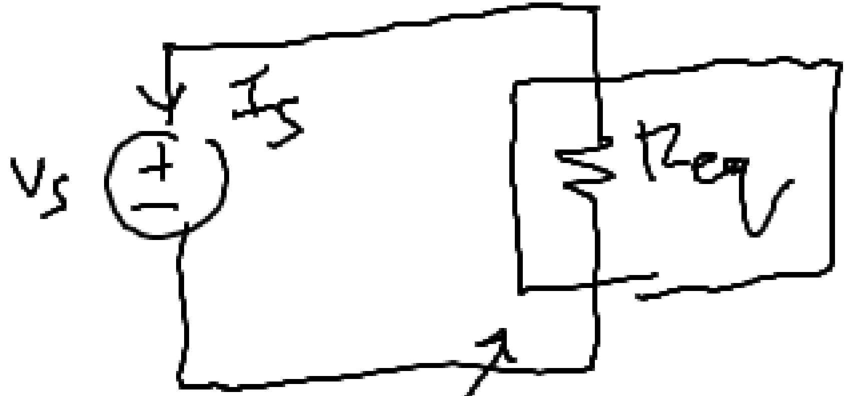
A: Equivalence is actually useful to analysis (finding voltages + currents)

Q: How?

Hard problem: Find I_s



Easier problem: Find I_s



Calculate R_{eq}

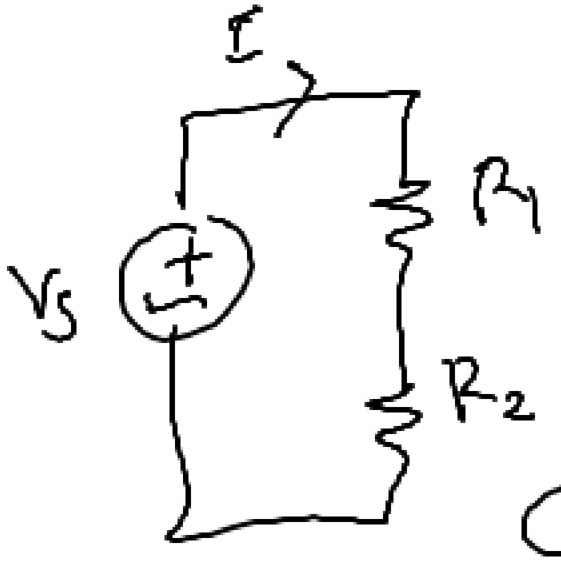
A: It converts hard problems into easier problems with the same behavior

Q: How to use it then? Here is a rough procedure

A:
① Reduce some subpart of a circuit to its equivalent (redraw the circuit)
② Calculate a voltage/current in the easier circuit
③ Go back to unreduced circuit / rewind the simplication you made
④ Use new quantity you know to iterate, apply value

Examples:

Deriving Voltage divider using equivalence

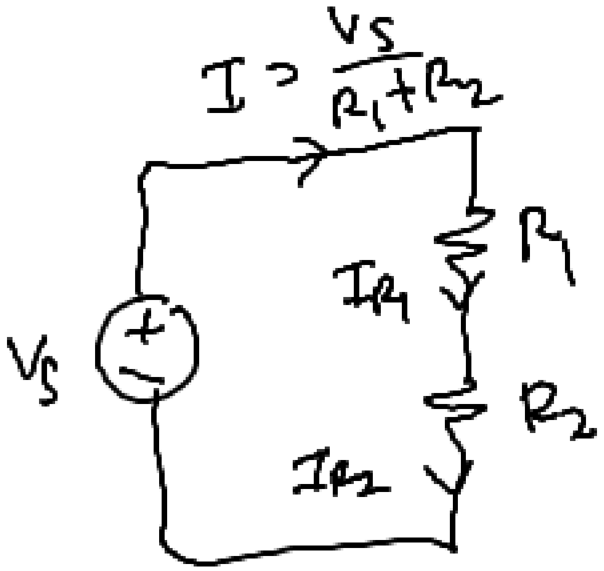


① Reduce

② Calculate

$$I = \frac{V_s}{R_1+R_2}$$

③ Rewind



④ apply

$$I_{R_1} = I$$

$$I_{R_2} = I$$

$$\Rightarrow V_{R_1} = R_1 I = R_1\left(\frac{V_s}{R_1+R_2}\right)$$

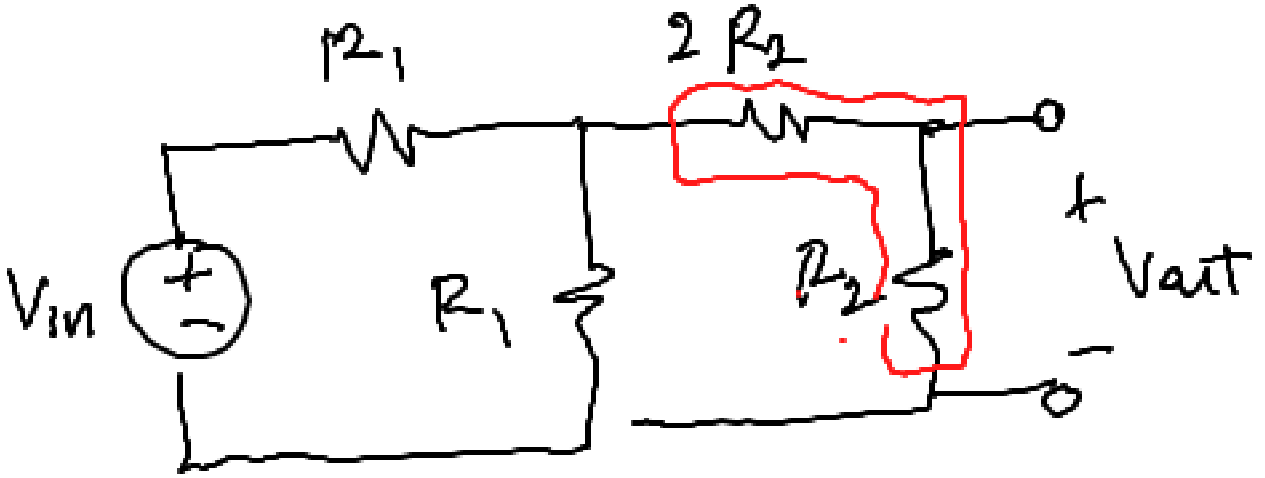
$$V_{R_2} = R_2 I = R_2\left(\frac{V_s}{R_1+R_2}\right)$$

} Tada! Very succinct!

Examples:

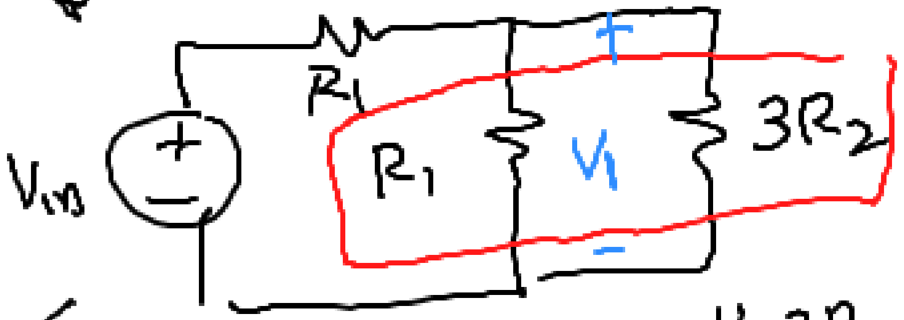
DIS 3C

Analyzing composed voltage dividers



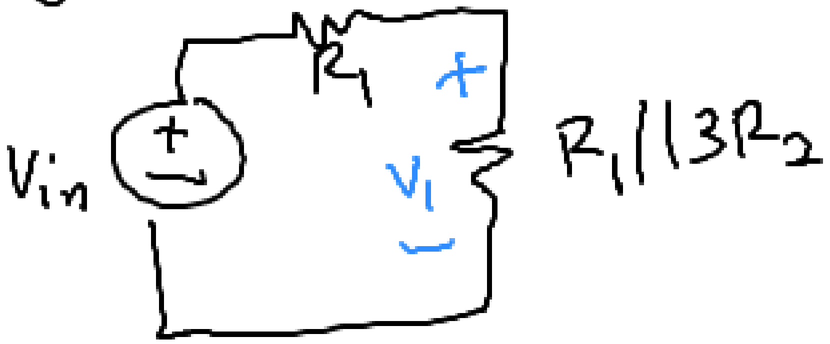
Objective: find V_{out} in terms of V_{in}

① Reduce: $2R_2$ & R_2 in series



V_{out} no longer here! thats okay though, will see later

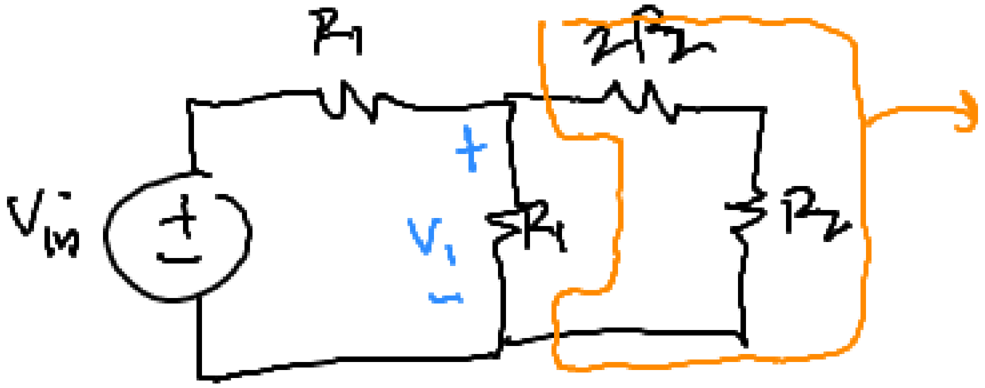
① Reduce: $R_1 || 3R_2$

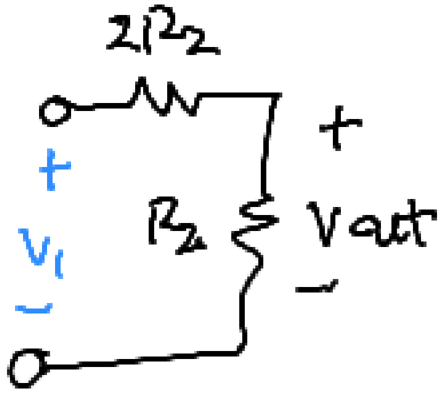


② Calculate! Use voltage division to find V_1

$$V_1 = \frac{R_1 || 3R_2}{R_1 + R_1||3R_2} V_{in}$$

③ Rewind! x2 → ④ Apply: Use voltage divider again





$$V_{out} = \frac{R_2}{2R_2+R_2} V_1$$
$$= \frac{1}{3} V_1$$
$$= \cdots$$

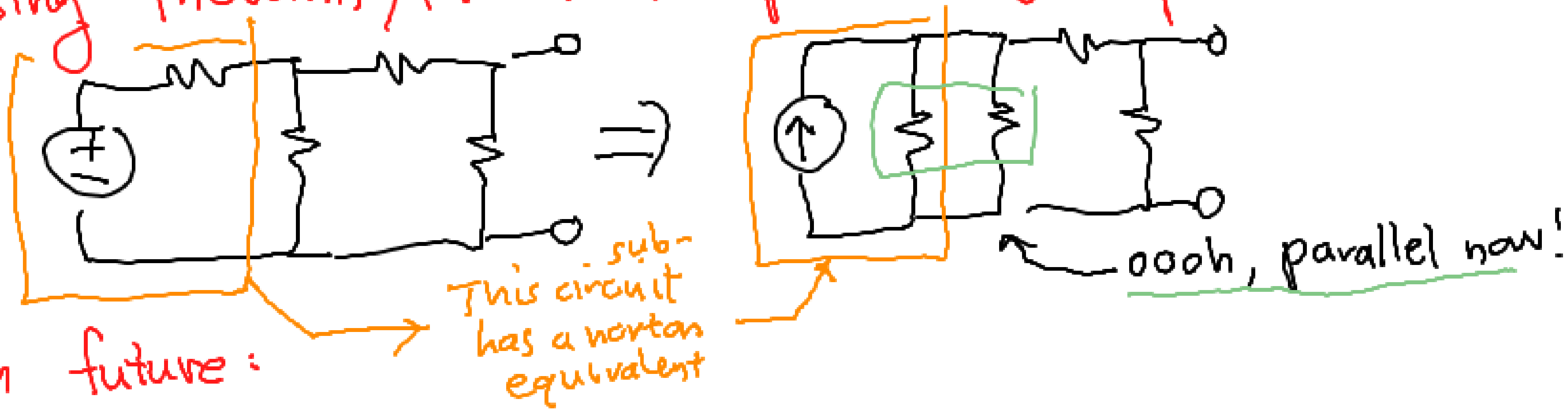
Final answer:

$$V_{out} = \frac{1}{3} \frac{R_1 || 3R_2}{R_1 + R_1||3R_2} V_{in}$$

Note: compute $R_1||3R_2$, compare to previous notes

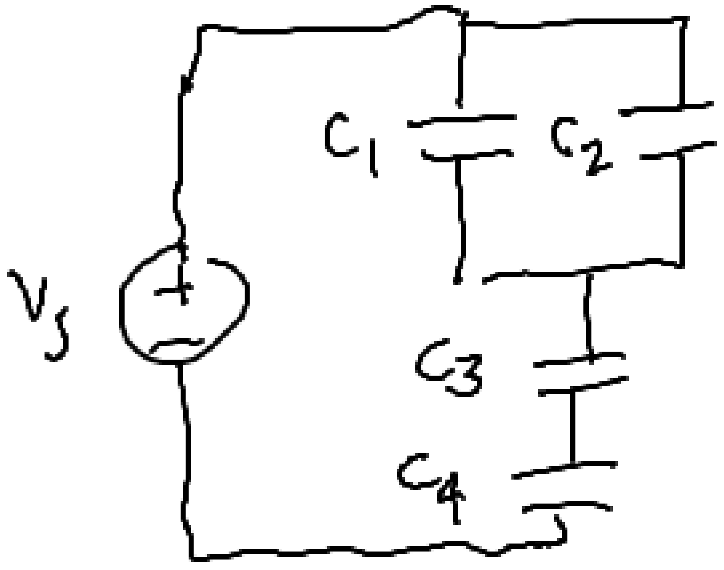
Exercise: Try using Thevenin/Norton on previous example

Hint:



Thing to look for in future:

Example:



Calculate Q on

C_1, C_2, C_3, C_4