EECS 16A DIS 6B

Today's topics
- Geometric interpretation of inner product: lengths and angles
- More cross correlation: Inner products as similarity

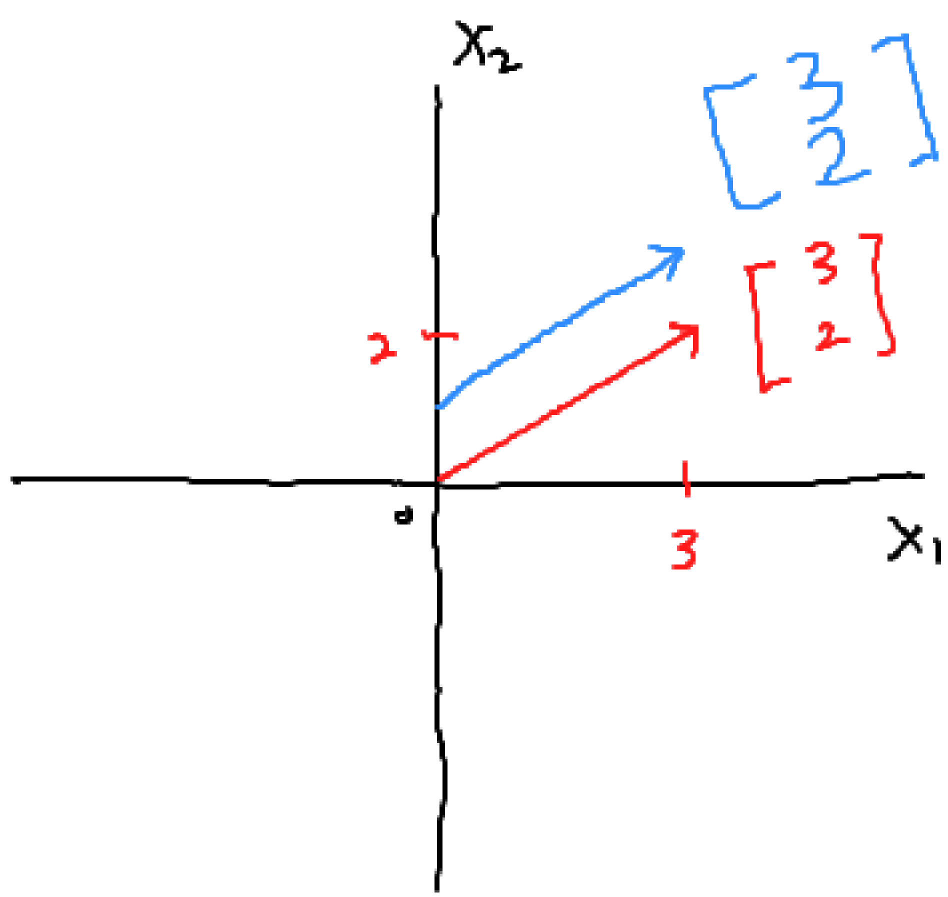
Definitions

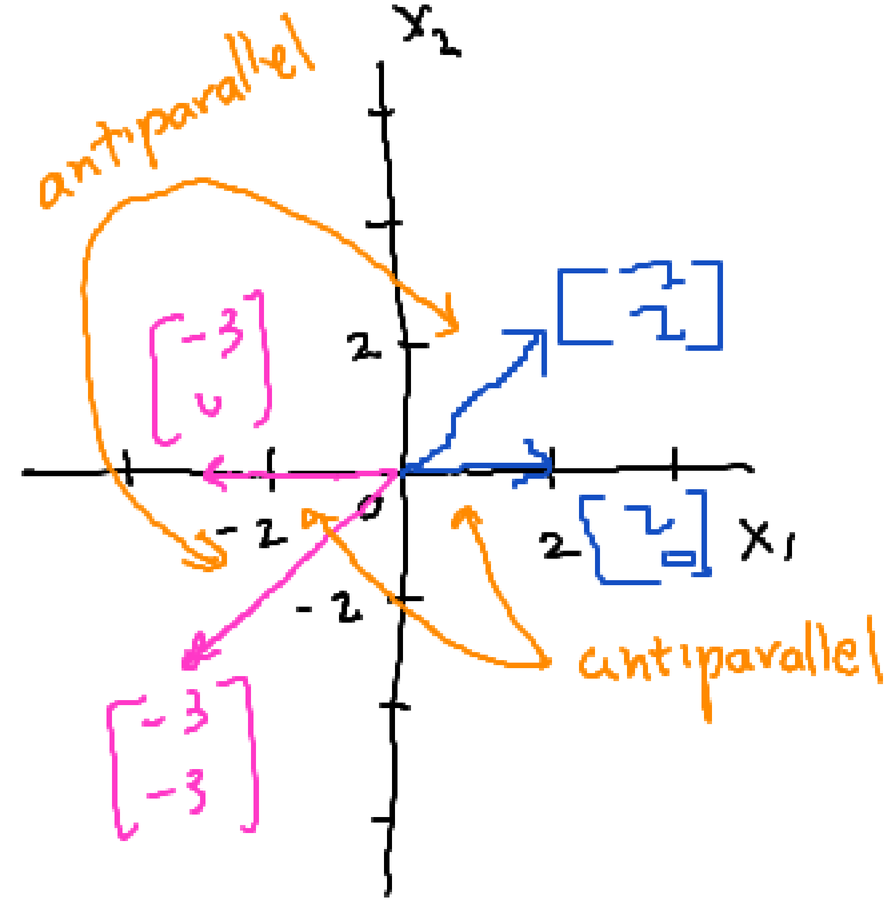
length/norm/magnitude of a vector: $\sqrt{\langle \vec{x}, \vec{x} \rangle} = \|\vec{x}\|$, sometimes $|\vec{x}|$

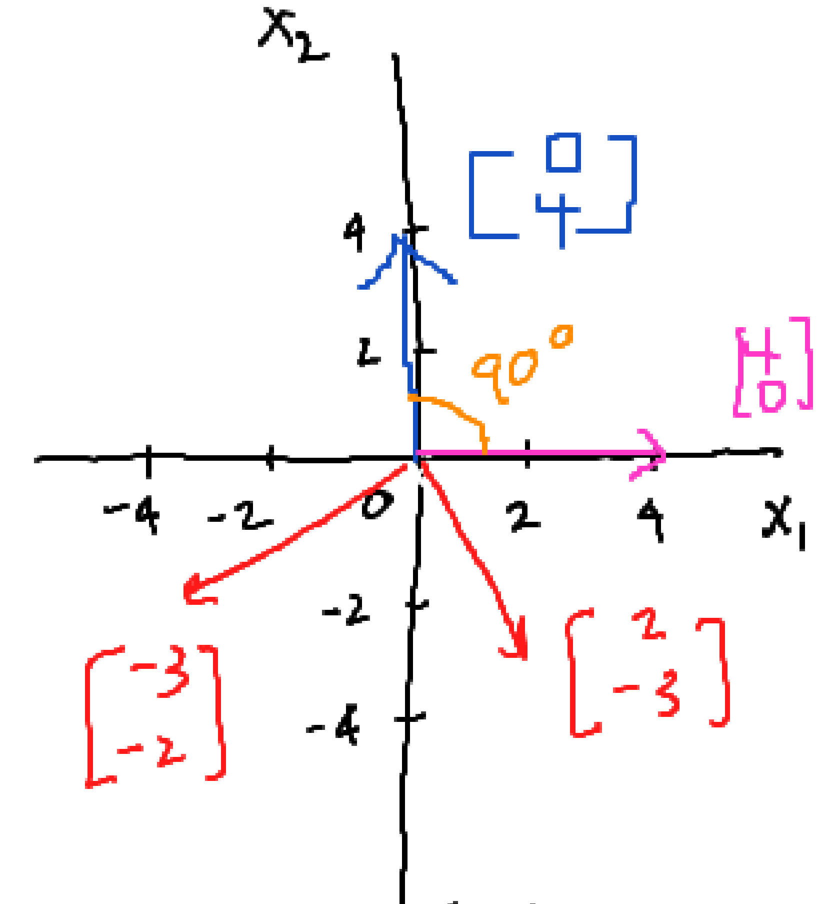
angle between two vectors: $\cos^{-1}\left(\dfrac{\langle \vec{x}, \vec{y} \rangle}{\|\vec{x}\| \, \|\vec{y}\|} \right)$

orthogonal: two vectors, $\vec{x}, \vec{y}$ are orthogonal iff $\langle \vec{x}, \vec{y} \rangle = 0$

↳ captures idea of being perpendicular more generally







Parallel

two vectors that point in the same direction

$$\vec{v} = \alpha \vec{w} \qquad \alpha > 0$$

$$\left\langle \begin{bmatrix}3\\2\end{bmatrix}, \begin{bmatrix}3\\2\end{bmatrix} \right\rangle = \underline{13} > 0$$

$$\langle \vec{v}, \vec{w} \rangle = \langle \alpha \vec{w}, \vec{w} \rangle = \underline{\alpha \|\vec{w}\|^2} > 0$$

Anti-parallel

two vectors that point in opposite directions (along the same line but pointed away from each other)

$\begin{bmatrix}-3\\-3\end{bmatrix}$, $\begin{bmatrix}2\\2\end{bmatrix}$ are antiparallel

$\rightarrow \vec{v} = \alpha \vec{w} \qquad \alpha < 0$

$$\left\langle \begin{bmatrix}-3\\-3\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix} \right\rangle = -12$$

$$\langle \vec{v}, \vec{w} \rangle = \alpha \|\vec{w}\|^2 < 0$$

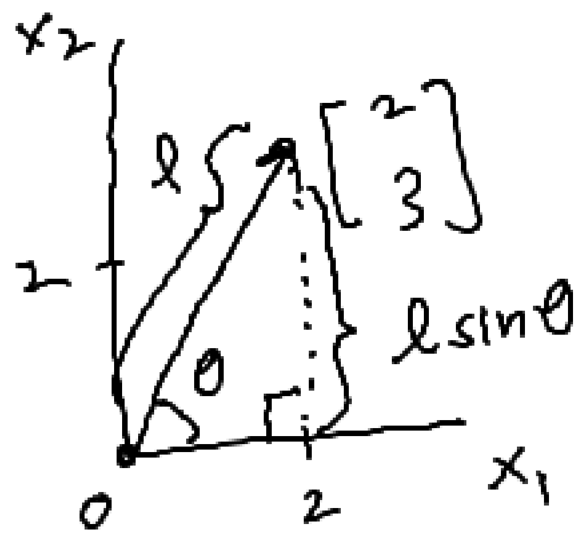
Perpendicular

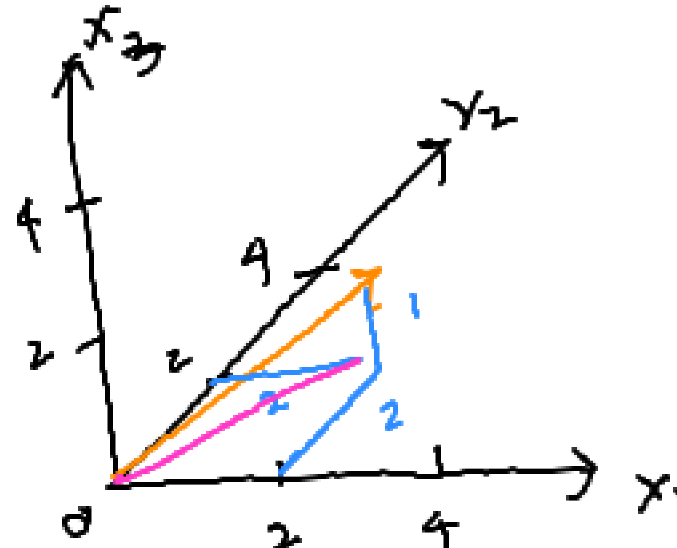
two vectors that have 90° between them

$$\left\langle \begin{bmatrix}0\\4\end{bmatrix}, \begin{bmatrix}4\\0\end{bmatrix} \right\rangle = 0$$

$$\left\langle \begin{bmatrix}2\\-3\end{bmatrix}, \begin{bmatrix}-3\\-2\end{bmatrix} \right\rangle = 0$$

$$\boxed{\text{Orthogonal: } \langle \vec{x}, \vec{y} \rangle = 0}$$



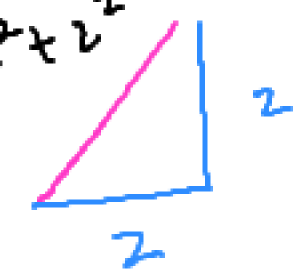


$$\ell = \sqrt{2^2 + 3^2}$$
$$= \sqrt{13}$$

$$\sqrt{\left\langle \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\rangle}$$

$$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$\sqrt{2^2+2^2}$



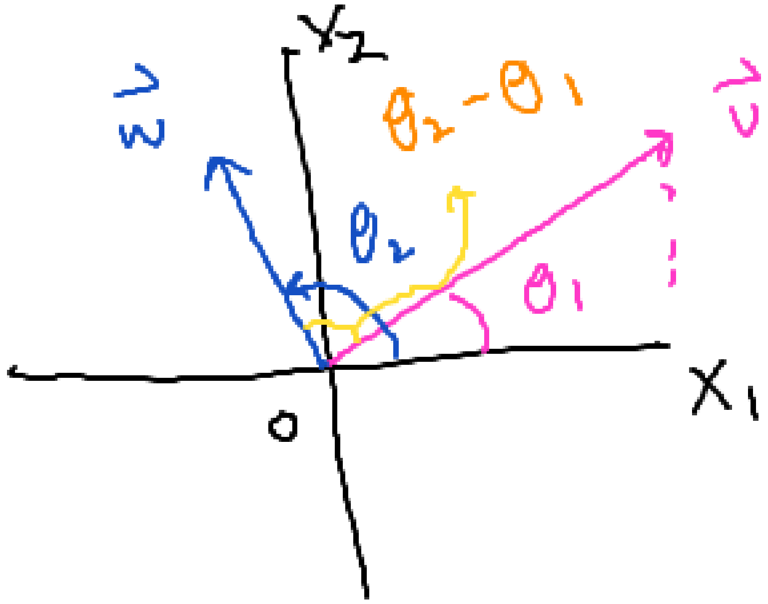
$\rightarrow \sqrt{2^2+2^2+1^2}$

$\sqrt{2^2+2^2}$

$$\sqrt{\left\langle \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\rangle}$$

orange vector length: 3

lengths from pythagorean thm are consistent with length defined by inner product



$\langle \vec{v}, \vec{w} \rangle$ = length of $\vec{v}$ · length of $\vec{w}$ · cosine of angle between $\vec{v}, \vec{w}$ ($\theta_1 - \theta_2$ or $\theta_2 - \theta_1$)

$$\theta_{\vec{v},\vec{w}} = \cos^{-1}\left(\frac{\langle \vec{v}, \vec{w} \rangle}{\|\vec{v}\| \, \|\vec{w}\|} \right)$$

gives angles from 0 to π

$$\vec{v} = \begin{bmatrix} \|\vec{v}\| \cos\theta_1 \\ \|\vec{v}\| \sin\theta_1 \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} \|\vec{w}\| \cos\theta_2 \\ \|\vec{w}\| \sin\theta_2 \end{bmatrix}$$

{ Note: $\vec{x}^T \vec{y}$ also called the dot product (standard inner product)

$$\langle \vec{v}, \vec{w} \rangle = \|\vec{v}\| \, \|\vec{w}\| \cos\theta_1 \cos\theta_2 + \|\vec{v}\| \, \|\vec{w}\| \sin\theta_1 \sin\theta_2$$

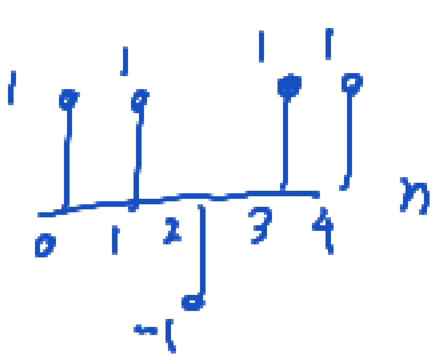
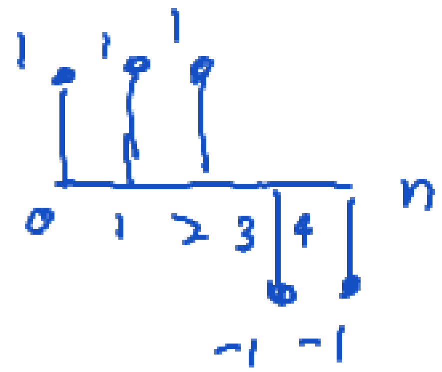
$$= \|\vec{v}\| \, \|\vec{w}\| \left(\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \right)$$

$$= \|\vec{v}\| \, \|\vec{w}\| \cos(\theta_1 - \theta_2)$$

$$\langle \vec{v}, \vec{w} \rangle = \|\vec{v}\| \, \|\vec{w}\| \cos(\theta_2 - \theta_1) \Leftarrow$$

Two signals we can possibly receive $s_1[n]$, $s_2(n)$

Single received signal on cellphone $r[n]$

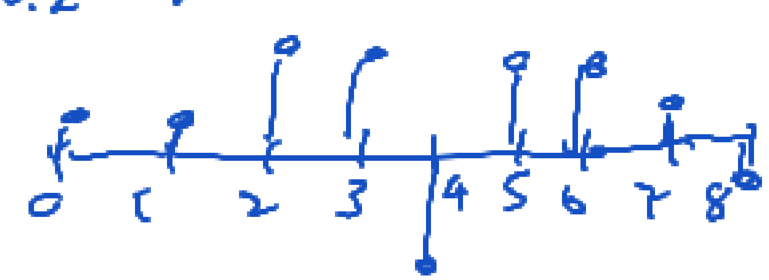
$s_1[n]$   $s_2[n]$

$$\text{corr}_r(s_i)[k] = \sum_{n=-\infty}^{\infty} r[n]\, s_i[n-k]$$

$$= \langle \underline{r[n]}, \underline{s_i[n-k]} \rangle$$

trying to see which part of $r[n]$ is most like $s_i[n-k]$

n	0	1	2	3	4	5	6	7	8
$r[n]$	0.2	0.2	1	1	-1.2	1	1	0.2	-0.2

↳ 

$$\text{corr}_r(s_1)[2] = \langle r[n], s_1[n-2] \rangle$$

n	0	1	2	3	4	5	6	7	8	9	10
$r[n]$	0.2	0.2	$\boxed{1}$	$\boxed{1}$	$\boxed{-1.2}$	$\boxed{1}$	$\boxed{1}$	0.2	-0.2	0	0
$s_1[n-2]$	0	0	$\boxed{1}$	$\boxed{1}$	$\boxed{-1}$	$\boxed{1}$	$\boxed{1}$	0	0	0	0

$$\boxed{\text{corr}_r(s_1)[2] = 5.2}$$

$$\text{corr}_r(s_1)[1] = \langle r[n], s_1[n-1] \rangle$$

$$= 1(0.2) + (1)(1) + (-1)(1) + (1)(-1.2) + (1)(1)$$

$$= 0.2 + 1 - 1 - 1.2 + 1$$

$$= 0$$

when signals look similar correlation values are large (in magnitude)