EECS16A DIS6C

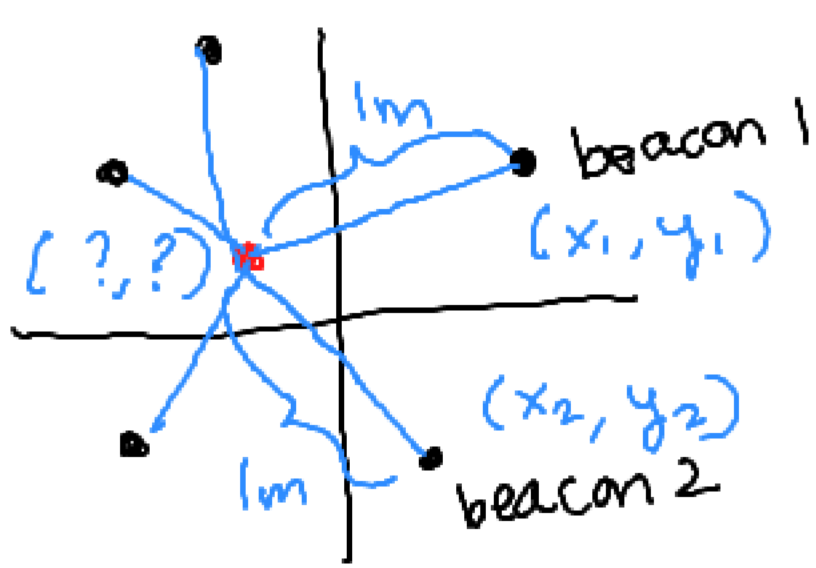
Today's topics

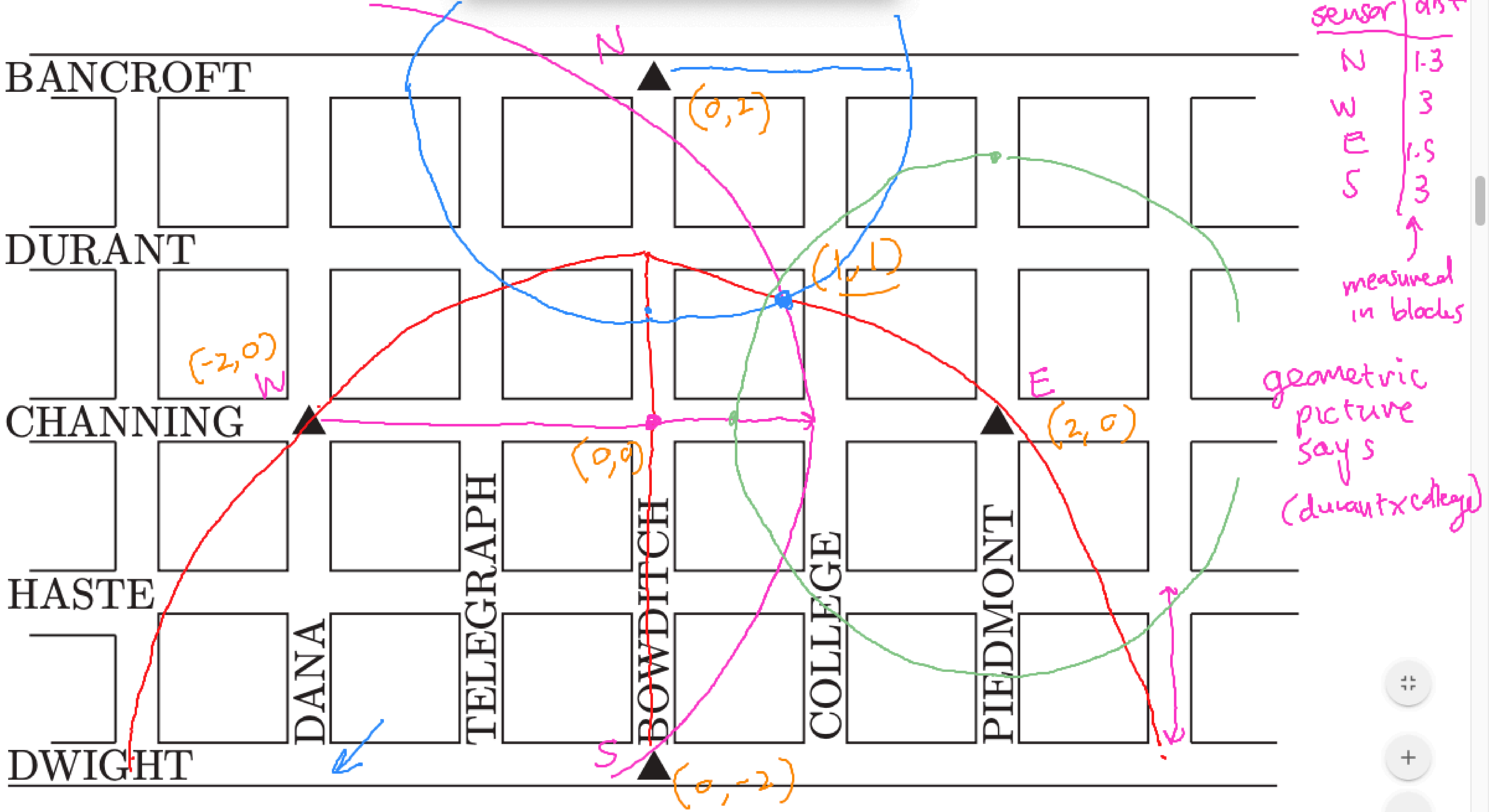
① Trilateration – a problem that motivates the new techniques you'll see

② Projection (of a vector onto another vector)
    Derivation

Trilateration

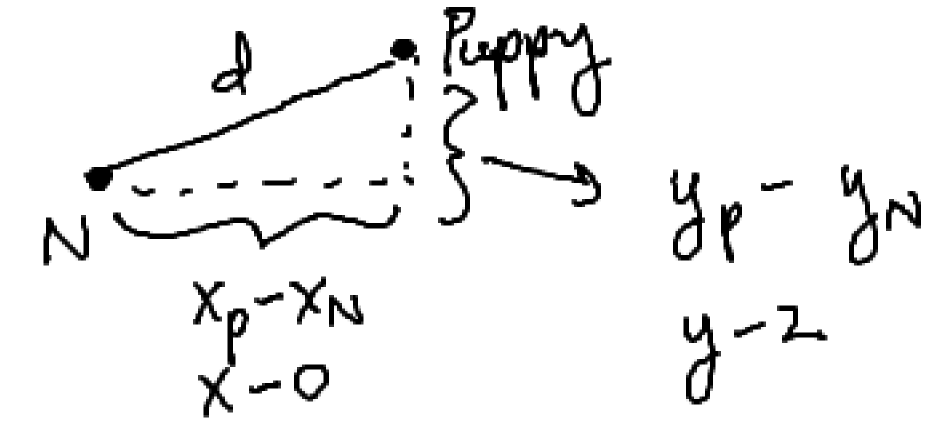
- few known locations (beacons) → coordinates
- distances from these known locations to an unknown locations
- Problem: find the coordinates of the unknown location




BANCROFT
DURANT
CHANNING
HASTE
DWIGHT
DANA
TELEGRAPH
BOWDITCH
COLLEGE
PIEDMONT
N
W
E
S
(0,2)
(1,1)
(-2,0)
(0,0)
(2,0)
(0,-2)
sensor | dist
N | 1.3
W | 3
E | 1.5
S | 3
measured in blocks
geometric picture says (durant x college)

N $(0,2)$ puppy (x,y)

What is the distance?



N: $d^2 = (x-0)^2 + (y-2)^2$ $d = 1.3$

W: $d^2 = (x+2)^2 + y^2$ ✓ $d = 3$

E: $d^2 = (x-2)^2 + y^2$ $d = 1.5$

S: $d^2 = x^2 + (y+2)^2$ $d = 3$

W $(-2,0)$

$ax_1 + bx_2 + \dots = z$

never x_1^2 or x_2^2...

→ This system of equations is NONLINEAR (x^2, y^2)

N: $1.69 = x^2 + y^2 - 4y + 4$

W: $9 = x^2 + 4x + 4 + y^2$

E: $2.25 = x^2 - 4x + 4 + y^2$

S: $9 = x^2 + y^2 + 4y + 4$

use these. N-E, W-E

N-E: $-0.56 = -4y + 4x$

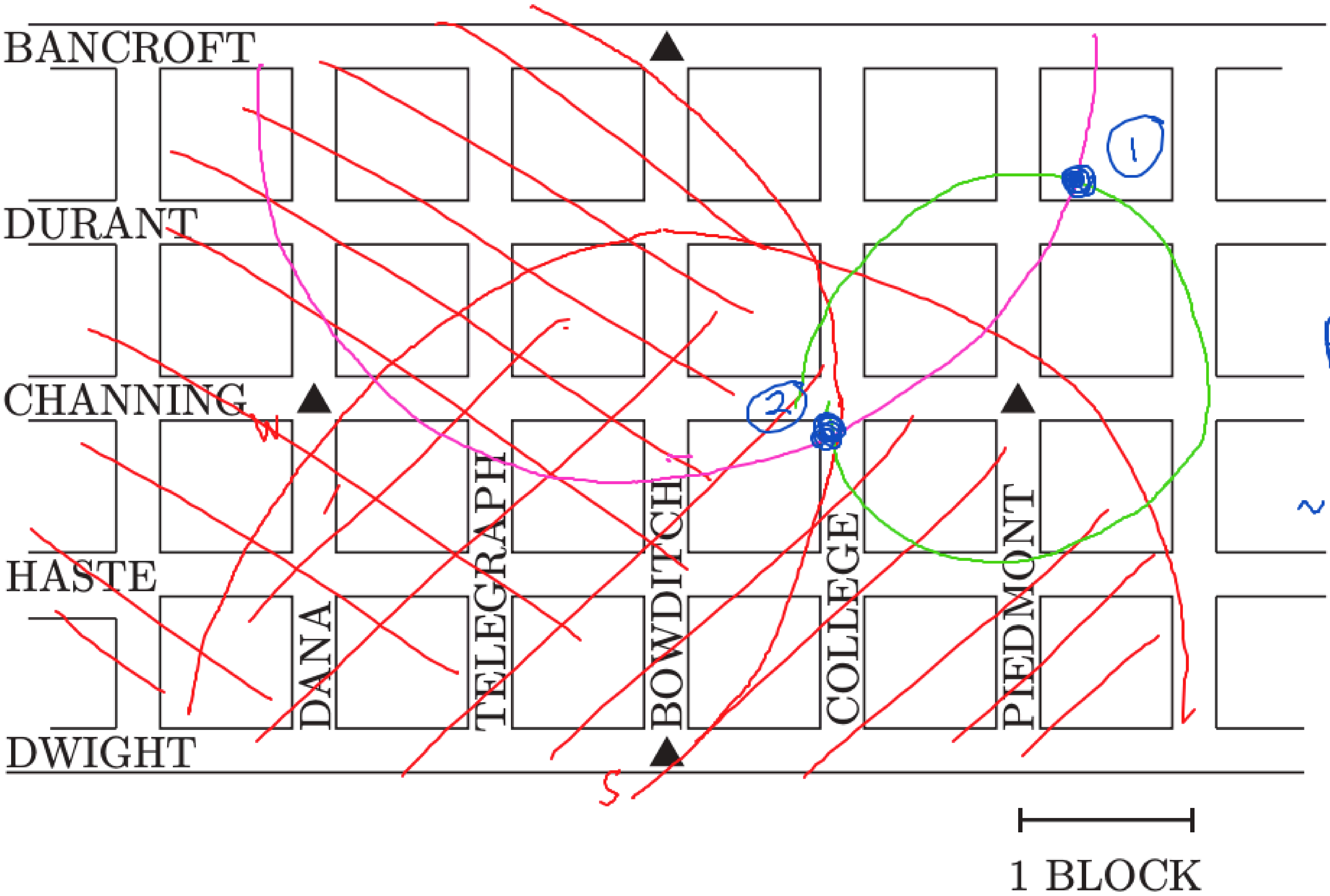
W-E: $6.75 = 8x$

$x = \frac{6.75}{8} = 0.84375$

$y = 0.98375$

$(x,y) \sim (1,1)$

↳ Cancel out nonlinear terms by "sacrificing" one equation



Sensor	dist
N	2.2
W	OOR
E	1.1
S	OOR

OOR
d > 3

piedmont
x
durant

~ (2, 1)

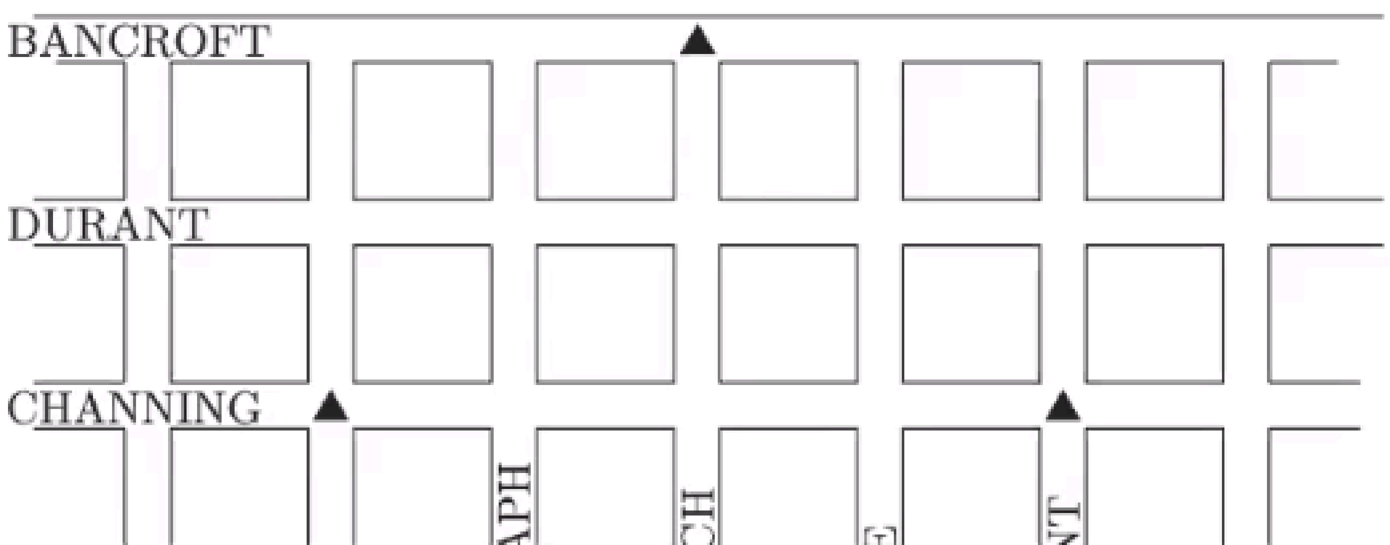
SOUTH OF DWIGHT

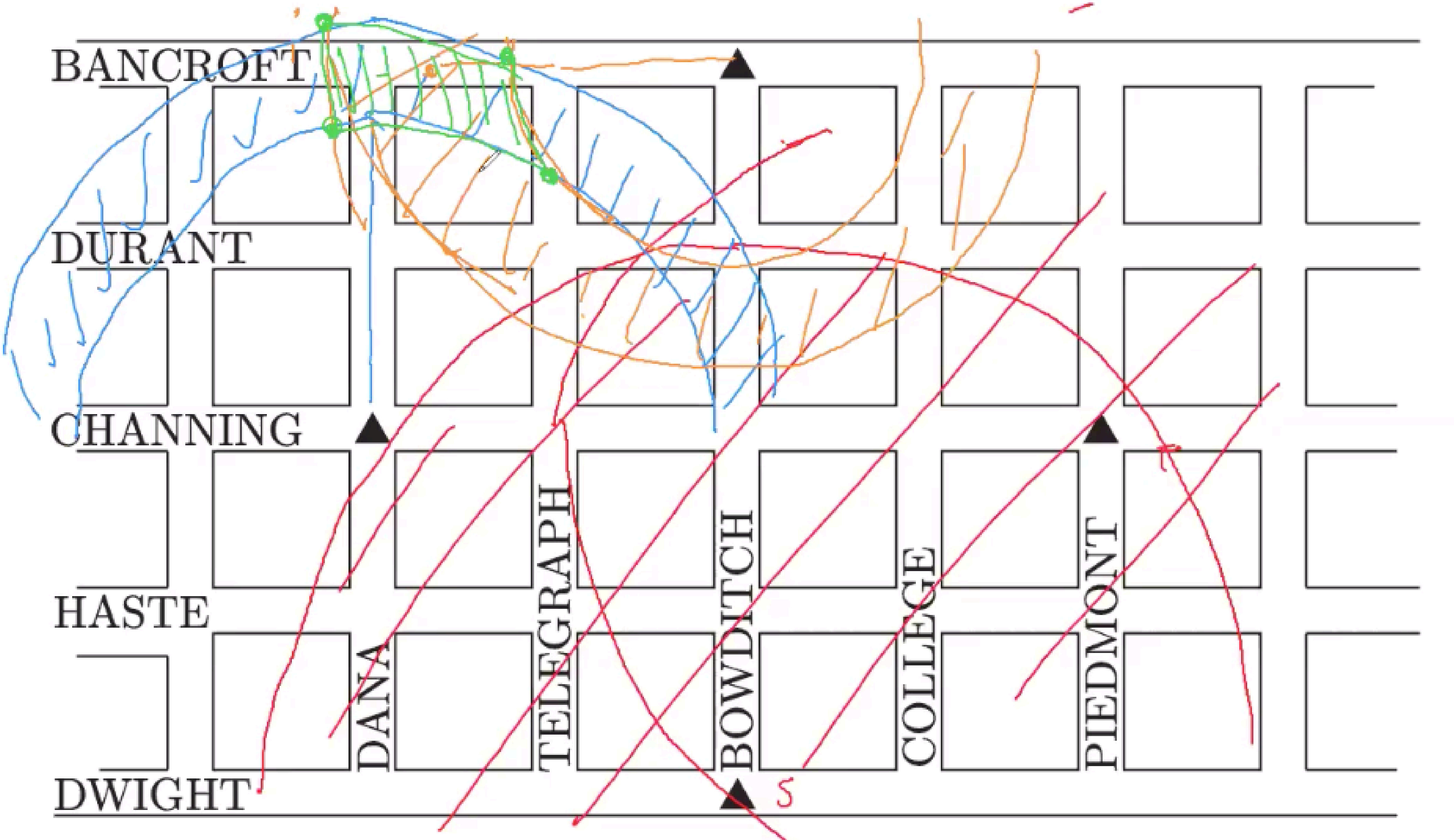
(d) Mr. Muffin is a very mischievous puppy, and while playing and running around he damaged his collar. The transmitter on his collar will still send a signal to the receiver towers, but the distance sensor has noise. You check the logs of the cell towers, and they have received the following messages:

Sensor	Distance
N	1.7 ± 0.5
W	2.1 ± 0.2
E	Out of Range
S	Out of Range

On the map provided, identify where Mr. Muffin is! Can you find exactly where he is?

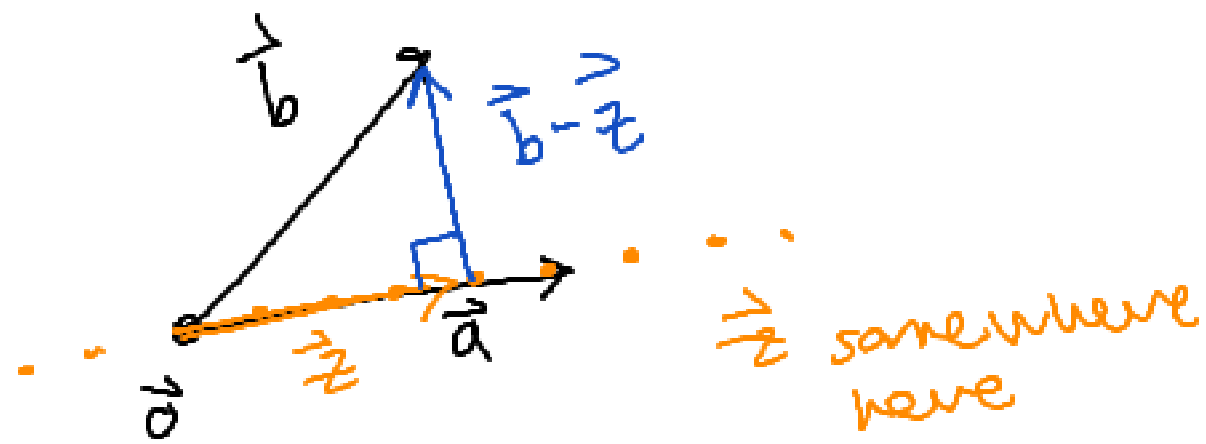
CAL CAMPUS





BANCROFT
DURANT
CHANNING
HASTE
DWIGHT
DANA
TELEGRAPH
BOWDITCH
COLLEGE
PIEDMONT

Thm: The point along $\vec{a}$ (in the span of $\vec{a}$) that's closest to $\vec{b}$ is $\vec{z}$ where $\vec{b}-\vec{z} \perp \vec{a}$. $\vec{z} = \dfrac{\langle \vec{a}, \vec{b} \rangle}{\|\vec{a}\|^2} \vec{a}$

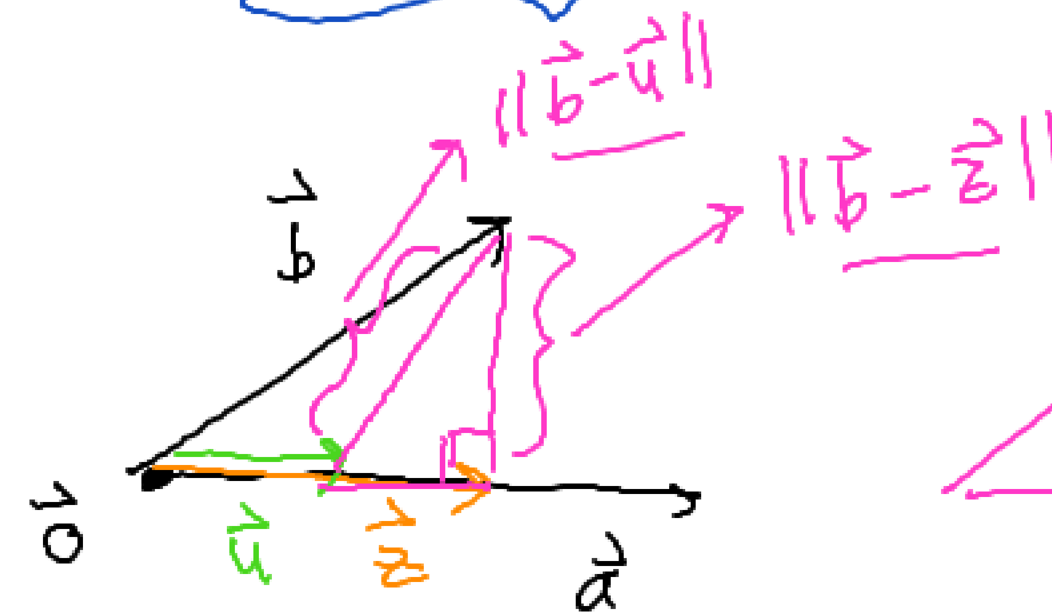


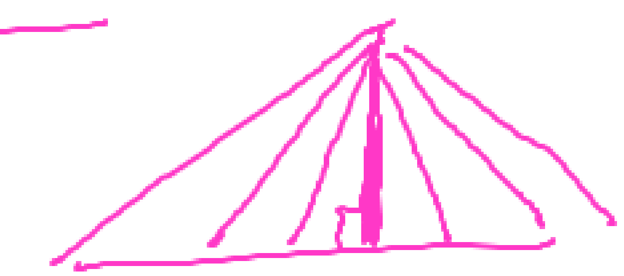
What distance is the smallest distance?

$\Rightarrow \|\vec{b}-\vec{z}\|$ (for the $\vec{z}$ that makes $\vec{b}-\vec{z} \perp \vec{a}$)

① Smallest distance requires $\vec{b}-\vec{z} \perp \vec{a}$

② Formula





Pythagorean thm

sides: $\|\vec{z}-\vec{u}\|$, $\|\vec{b}-\vec{z}\|$, $\|\vec{b}-\vec{u}\|$

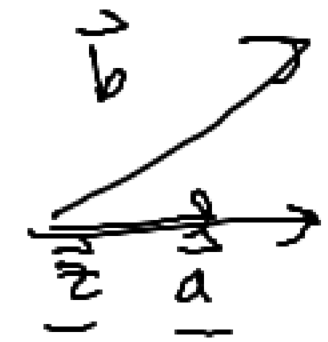
bottom leg, right leg, hypotenuse

$\|\vec{z}-\vec{u}\|^2 + \|\vec{b}-\vec{z}\|^2 = \|\vec{b}-\vec{u}\|^2$

($\|\vec{z}-\vec{u}\|^2 \geq 0$)

$\Rightarrow \|\vec{b}-\vec{u}\|^2 \geq \|\vec{b}-\vec{z}\|^2$

We know that shortest distance comes from $\vec{b}-\vec{z} \perp \vec{a}$



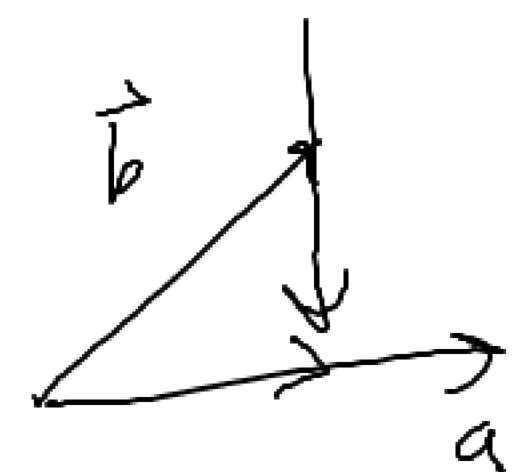
$\vec{z} = \alpha \vec{a}$

Ref. of orthogonal $\langle \vec{b}-\vec{z}, \vec{a} \rangle = 0$

$\langle \vec{b} - \alpha\vec{a}, \vec{a} \rangle = 0$

$\langle \vec{b}, \vec{a} \rangle - \alpha \underbrace{\langle \vec{a}, \vec{a} \rangle} = 0$

$\langle \vec{b}, \vec{a} \rangle - \alpha \|\vec{a}\|^2 = 0$



$\alpha = \dfrac{\langle \vec{b}, \vec{a} \rangle}{\|\vec{a}\|^2}$

$\vec{z} = \underbrace{\dfrac{\langle \vec{b}, \vec{a} \rangle}{\|\vec{a}\|^2}}_{\text{projection formula}} \vec{a} = \text{proj}_{\vec{a}}(\vec{b})$

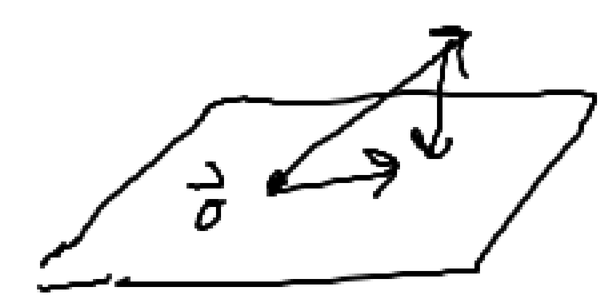
← vector we're squishing ($\vec{b}$)

← vector we flatten/squish onto ($\vec{a}$)

projection formula

projection of a vector onto another vector

Later:



Least squares