P1

Module 2, Lecture 5

EECS 16A
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Last time:
* An interesting circuit
* 2D Resistive Touchscreen
* Superposition - Intro

} Note 14

Today:
* Superposition (Cont.)
* Equivalence

} Ckt Jedi techniques - Note 15 A,B

Goal: Build Interesting Systems!

→ Need: tools to provide insight (analysis techniques)

Superposition (aka ckt analysis Jedi technique #1)

Reminder: Linear function $f(x+y) = f(x) + f(y)$

* Imagine a circuit with multiple sources (voltage or current)

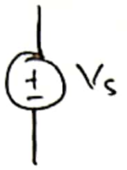
Superposition says that we can analyze the ckt by looking at the effect of each source independently and summing up all these at the end.

* Procedure:
  1) For each source k: zero-out all other sources and compute the output $V_{out,k}$ due to just source k.
  2) $V_{out} = \sum_k V_{out,k}$

(p2)

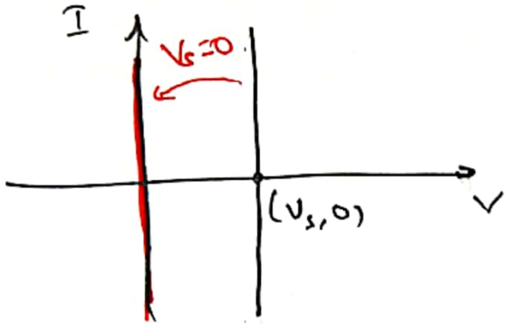
What does it mean to zero-out a source?

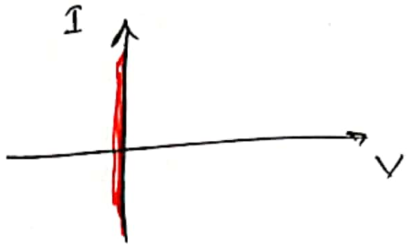
A) Voltage-Source



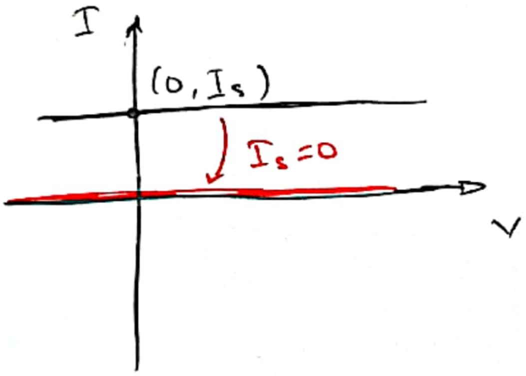
$\Rightarrow$

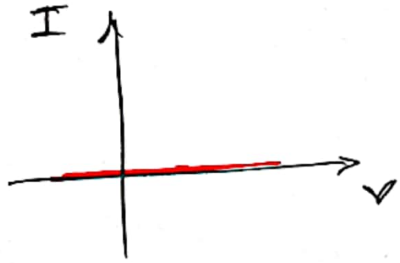
Wire

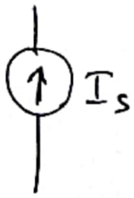




B) Current-Source $\Rightarrow$ open-circuit





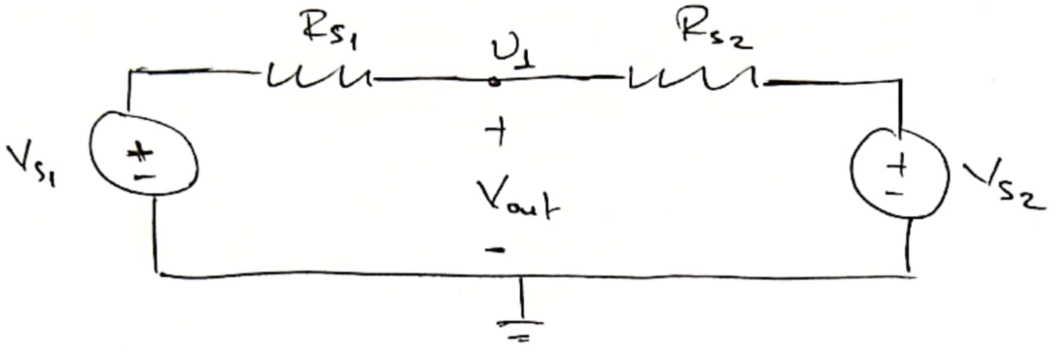


$\Rightarrow$

open-circuit

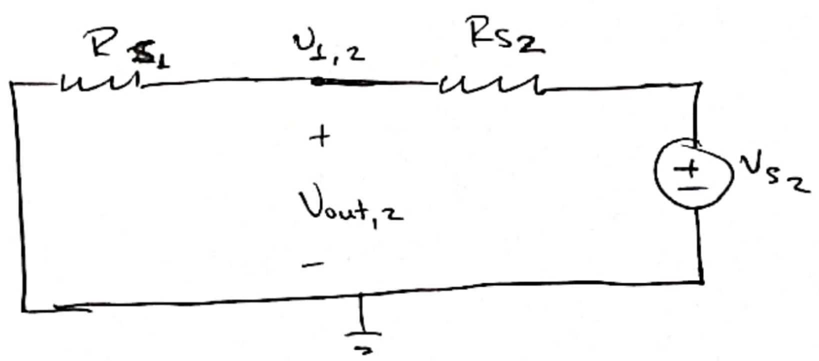
P3

Example: Voltage Summer



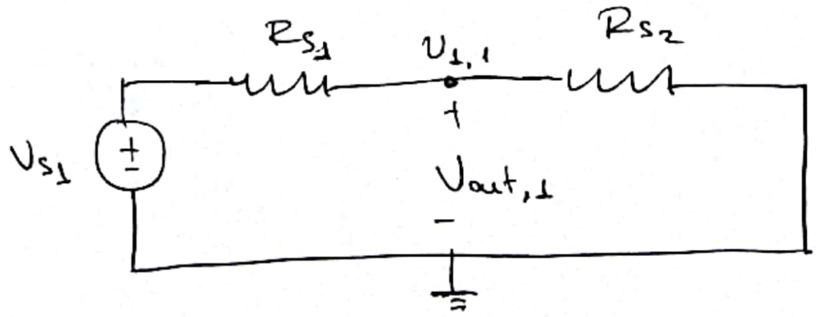
Step 1:

a) Compute the response due to V_{S_2}:



Volt. divider: $V_{out,2} = v_{1,2} - 0 = \dfrac{R_{S_1}}{R_{S_1} + R_{S_2}} \cdot V_{S_2}$

b) Compute the response due to V_{S_1}:



created a weighted sum of V_{S_1}, V_{S_2}!

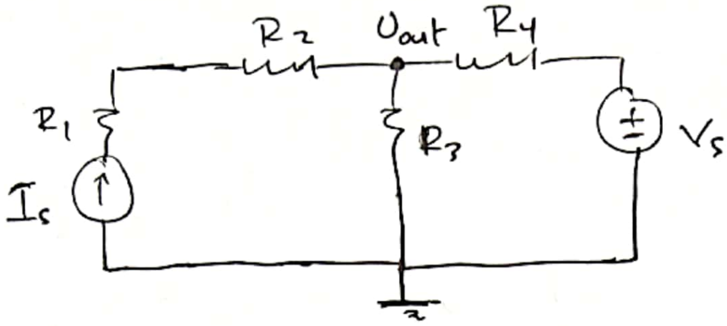
Volt. divider: $V_{out_1} = v_{1,1} - 0 = \dfrac{R_{S_2}}{R_{S_1} + R_{S_2}} V_{S_1}$

Step 2:

$$V_{out} = V_{out_1} + V_{out_2} = \underbrace{\frac{R_{S_2}}{R_{S_1} + R_{S_2}}}_{\alpha < 1} V_{S_1} + \underbrace{\frac{R_{S_1}}{R_{S_1} + R_{S_2}}}_{\beta < 1} V_{S_2}$$

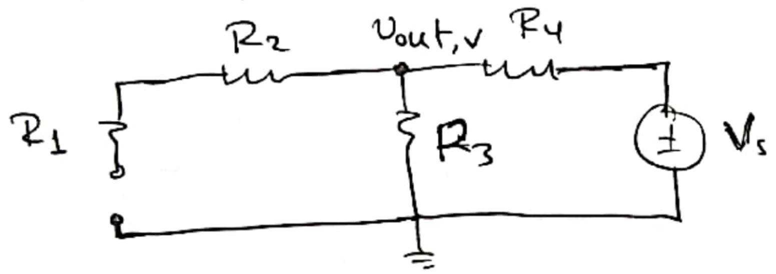
$$V_{out} = \alpha V_{S_1} + \beta V_{S_2}$$

(p4) Example #2 : Some "scary-looking" ckt.

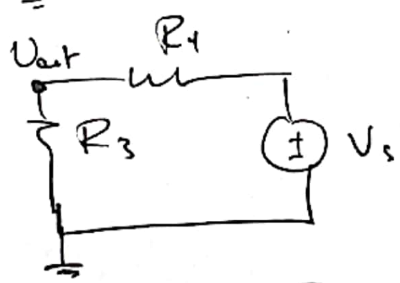


Step 1:

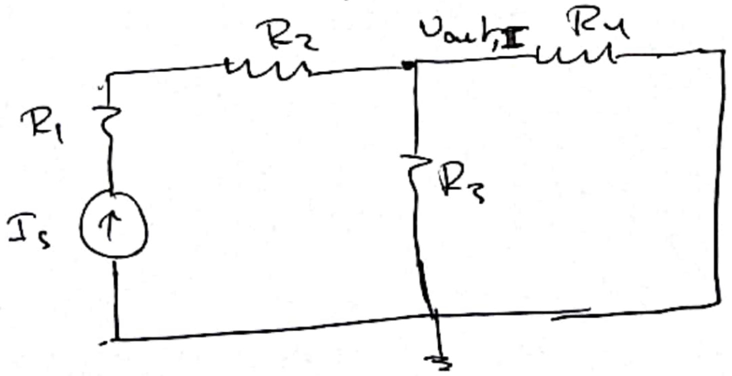
a) Response due to V_s



R_1, R_2 are open! no voltage drop across them!

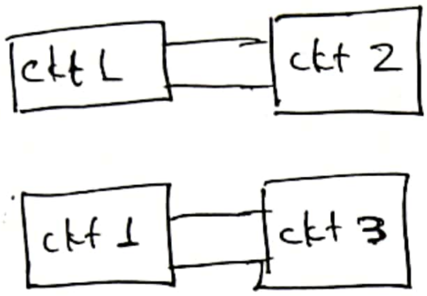


b)

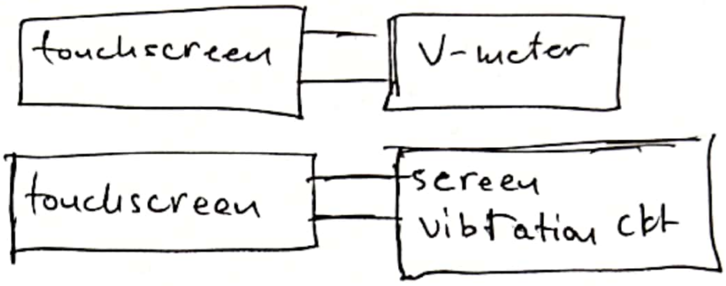


Step 2: $V_{out} = V_{out,V} + V_{out,I}$

Equivalence (aka ckt Jedi technique #2)

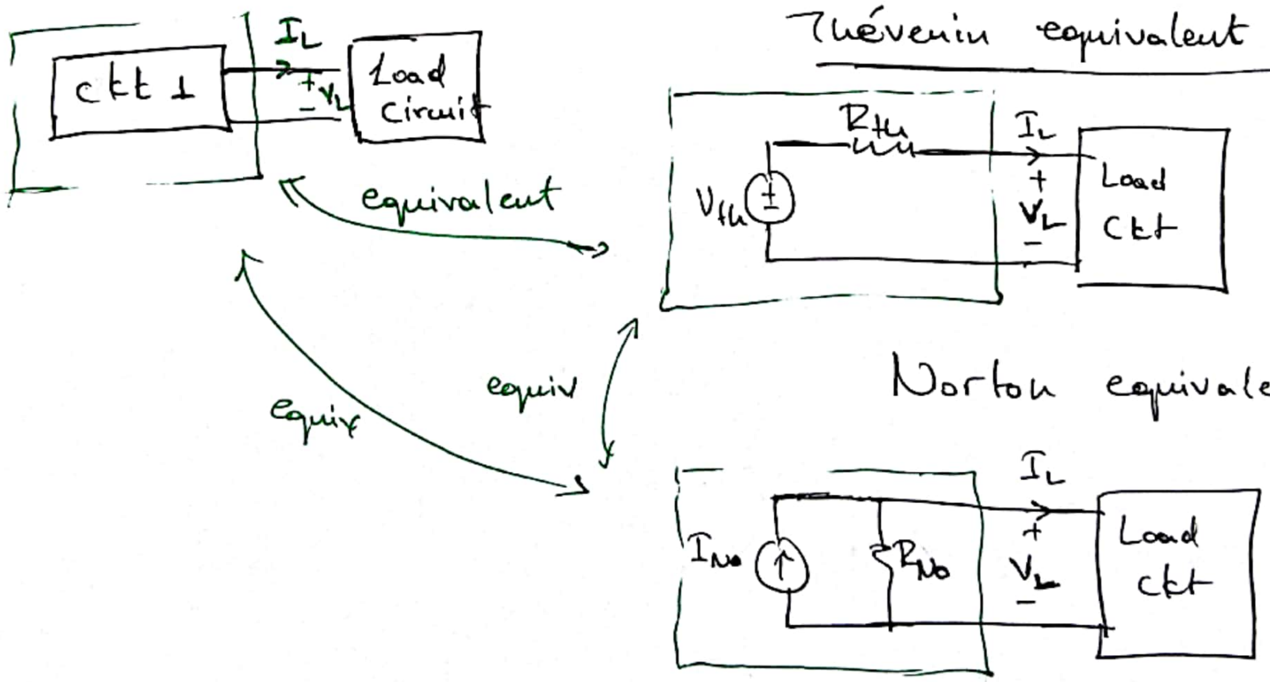


example :

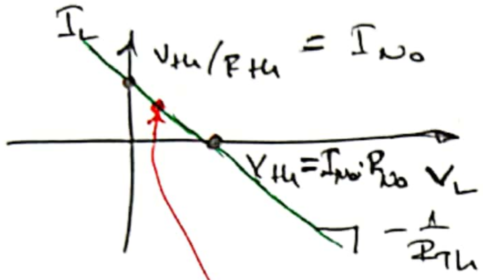


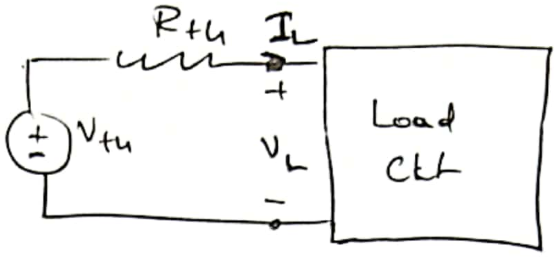
Definition : Two circuit are equivalent if they have the same I-V characteristics.

Theorem : Any linear circuit (no matter how complicated) can be replaced with an equivalent simpler linear circuit with only 2 elements.

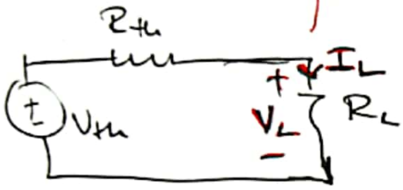


p6 Let's find $I_L - V_L$ for the Thévenin Equivalent:



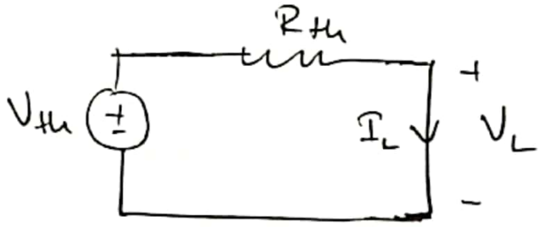


What if Load-Ckt is a resistor?



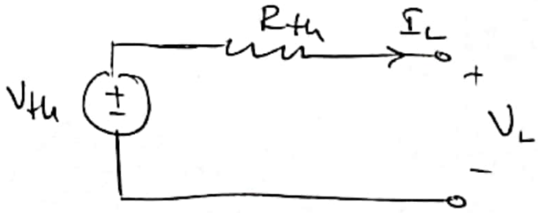
Note: Each load corresponds to a different point in the $I_L - V_L$ line.

Load Ckt is a wire:



$V_L = 0$, $I_L = \frac{V_{th}}{R_{th}}$ (y-intercept)

Load ckt is an open-ckt:

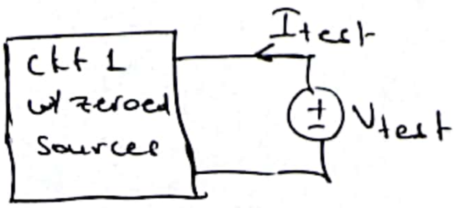


$I_L = 0$ $\quad V_L = V_{th}$ (x-intercept)

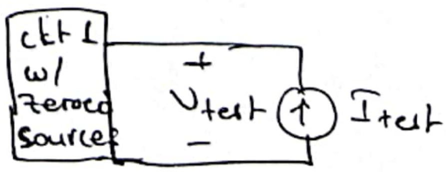
Algorithm to find V_{th}, R_{th} (i.e. the Thevenin equiv.)

1) To find V_{th} : "Connect" an open-circuit across the two output terminals and measure/compute $V_{open\text{-}circuit} = V_{th}$

2) To find R_{th} : Zero-out all independent sources and apply a test voltage or current. (find the slope of the I-V)

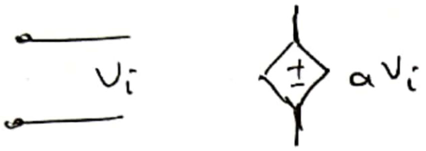


$$\boxed{R_{th} = \frac{V_{test}}{I_{test}}}$$



Dependent sources

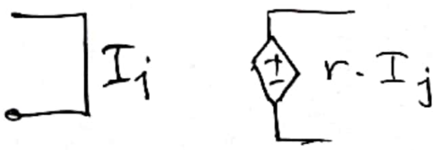
(used to model some ckt block or element; they don't get zeroed out when calculating Rth!)



Voltage - Controlled Voltage Source
(VCVS)
α : unitless

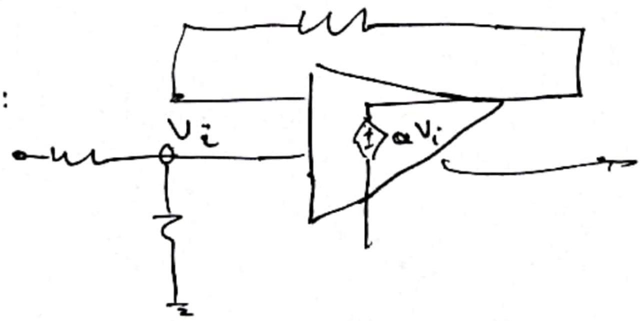
Voltage - Controlled Current Source
(VCCS)
g : $\frac{1}{[\Omega]}$

Current - Controlled Current Source
(CCCS)
α : unitless

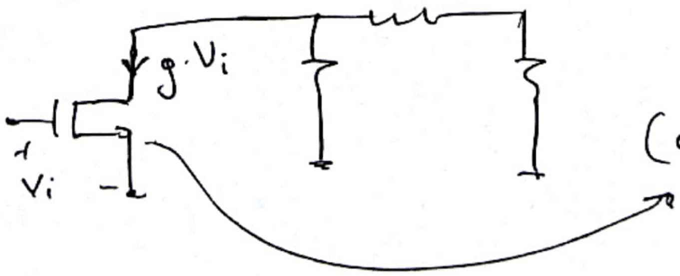


Current - Controlled Voltage Source
(CCVS)
r : $[\Omega]$

examples :



op-amp (VCVS)

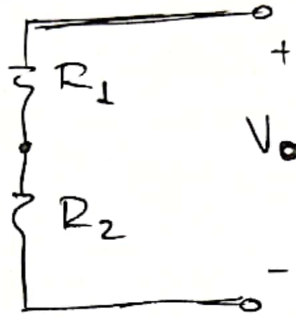


transistor : (VCCS)
(out-of-scope!)

Examples:

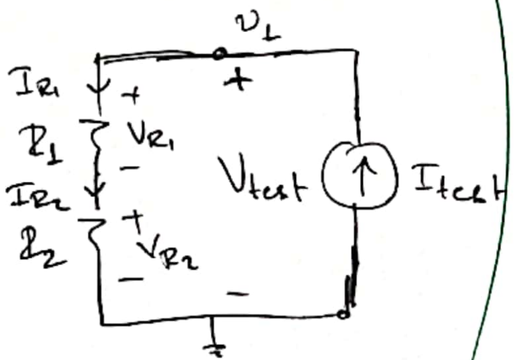
Ex. 1) "Series Resistors"

Step 1



$V_{oc} = 0 = V_{th}$

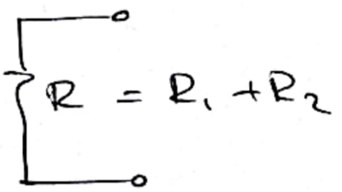
Step 2



$$V_{test} = V_{R_1} + V_{R_2}$$
$$= I_{R_1} \cdot R_1 + I_{R_2} \cdot R_2$$
$$= I_{test} R_1 + I_{test} R_2$$
$$V_{test} = I_{test}(R_1 + R_2)$$
$$\Rightarrow R_{th} = \frac{V_{test}}{I_{test}} = R_1 + R_2$$

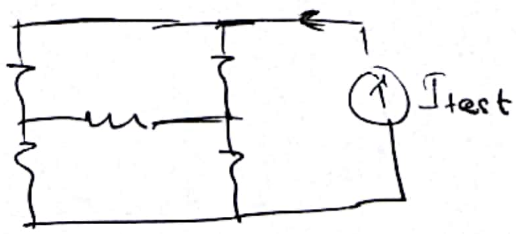
Thevenin Equivalent:

equiv.



Two resistors are connected "in series" when the same current flows through them.

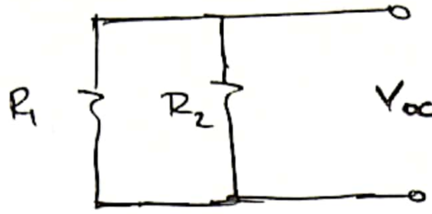
e.g.



Nothing is in series here!

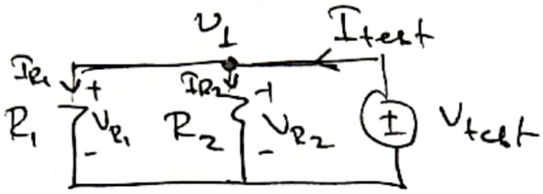
Ex. 2) "Parallel Resistors"

Step 1



$V_{oc} = 0 = V_{th}$

Step 2



$$V_{R_1} = V_{test} = I_{R_1} \cdot R_1$$

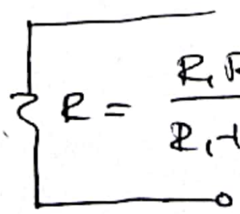
$$V_{R_2} = V_{test} = I_{R_2} \cdot R_2$$

$$I_{test} = I_{R_1} + I_{R_2} \quad \text{(KCL on } v_1\text{)}$$

$$= \frac{V_{R_1}}{R_1} + \frac{V_{R_2}}{R_2}$$

$$= \frac{V_{test}}{R_1} + \frac{V_{test}}{R_2}$$

$$\Rightarrow \frac{V_{test}}{I_{test}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$



$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Two resistors are connected in "parallel" when the voltage drop across them is the same.