(p1)

Module 2, Lecture 6

EECS 16A
Panos Zarkos

Last time:
* Superposition
* Equivalence

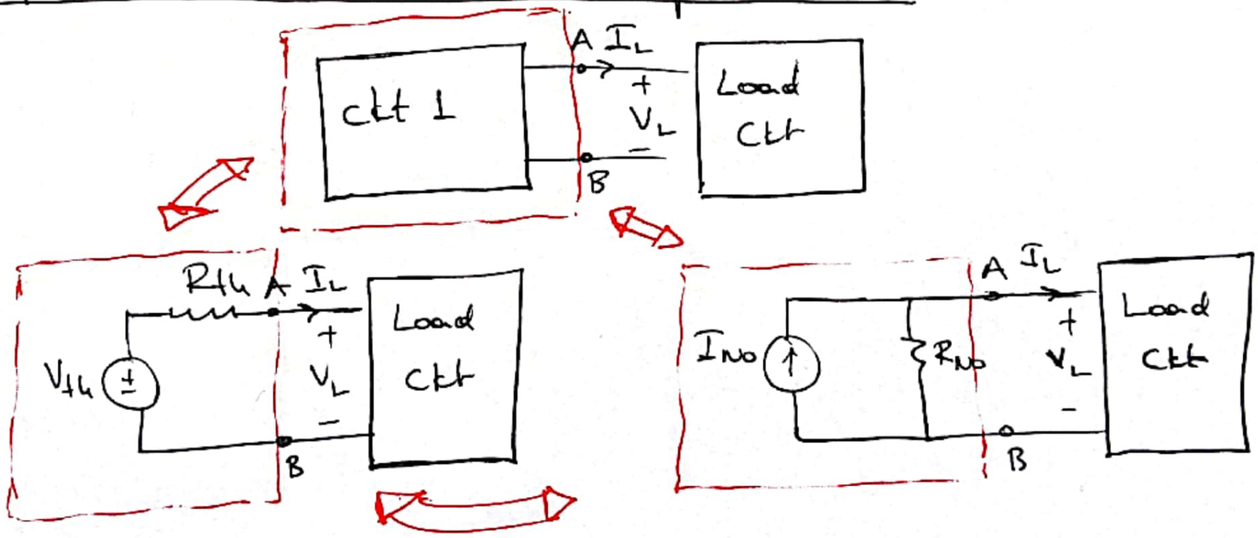
} Note 15 A, B

Today:
* Equivalence Recap
* Capacitive Touchscreen
* Capacitor Equivalence
* Capacitors as Batteries
* Capacitor Physics

} Note 16

Announcements: Panos extra OH, Mon-Thu, 2-3pm PST. Starting today, use oh.eecs16a.org

Recap: Thévenin and Norton Equivalents

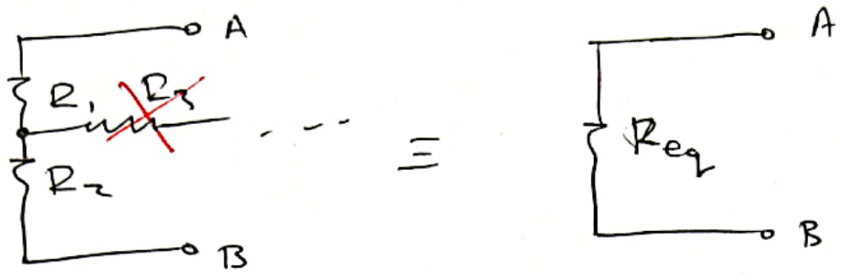


Thévenin to Norton: $I_{No} = \frac{V_{th}}{R_{th}}$, $R_{No} = R_{th}$

Remarks:
1) Thévenin and Norton equivalents are only equivalent with ckt 1 when looking into terminals A, B!
2) Equivalence refers only to "I-V" characteristics and not any other quantities (such as power)

Recap #2 : Series and parallel

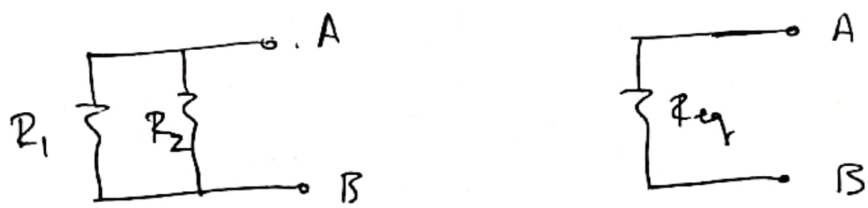
series



$$R_{eq} = R_1 + R_2$$

Resistors in series have the same <u>current</u> through them!

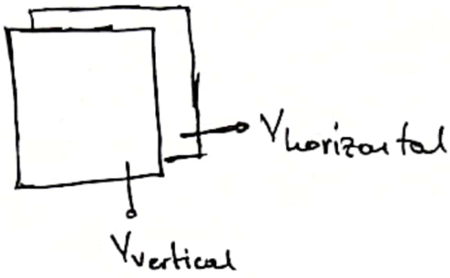
parallel



$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} = R_1 // R_2$$

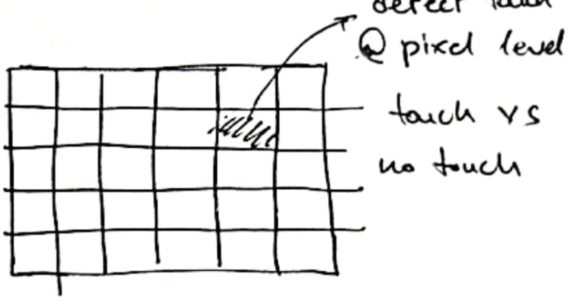
Resistors in parallel have the same <u>voltage</u> across them!

An improved touchscreen:



Resistive Touch

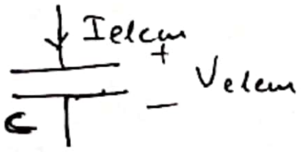
Arbitrary # $\Rightarrow$ of touch points



Capacitive Touch

Capacitor Circuit Model

Symbol



"I-V" characteristic

$$\boxed{Q_{elem} = C \cdot V_{elem}}$$

$[V]$

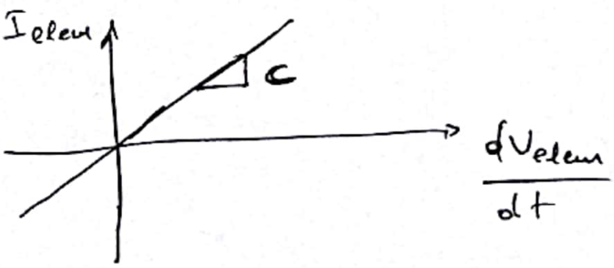
$[C]$ $\quad [F] = \dfrac{[C]}{[V]}$

I law: $\quad I_{elem} = \dfrac{dQ_{elem}}{dt}$

$$\frac{dQ_{elem}}{dt} = C \cdot \frac{dV_{elem}}{dt} \Bigg|_{C=\text{const.}}$$

$$\Rightarrow \boxed{I_{elem} = C \cdot \frac{dV_{elem}}{dt}}$$

"Ohm's law" for a capacitor



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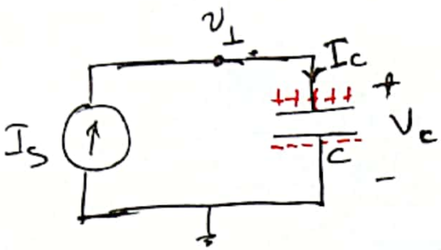
Example ckt #1:



$I_s = I_c$ (KCL on v_1)

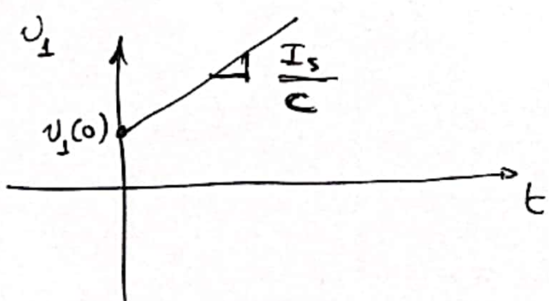
$I_c = C \cdot \frac{dV_c}{dt}$

$V_c = v_1 - 0$

$$I_s = C \cdot \frac{dv_1}{dt} \Rightarrow \int_{v_1(0)}^{v_1(t)} dv_1 = \int_0^t \frac{I_s}{C} \cdot dt$$

$$\Rightarrow v_1(t) - v_1(0) = \frac{I_s}{C}(t - 0)$$

$$\Rightarrow v_1(t) = \frac{I_s}{C} \cdot t + v_1(0)$$



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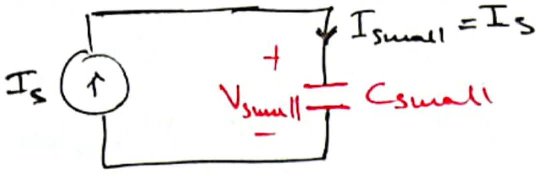
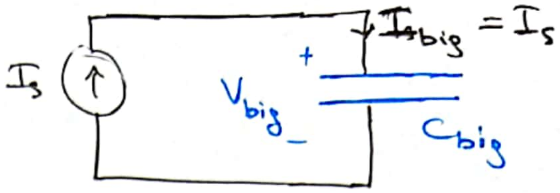
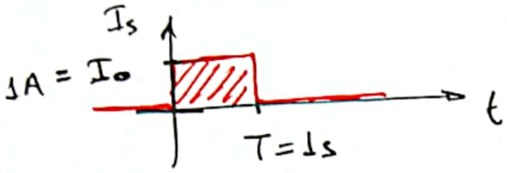
Intuition on ckt #1:





Let's Assume $C_{big} > C_{small}$

And I_s is the same for both:



Then $Q_{small} = Q_{big} = 1 \cdot A \cdot 1s = 1C$

But because: $Q_{small} = Q_{big}$

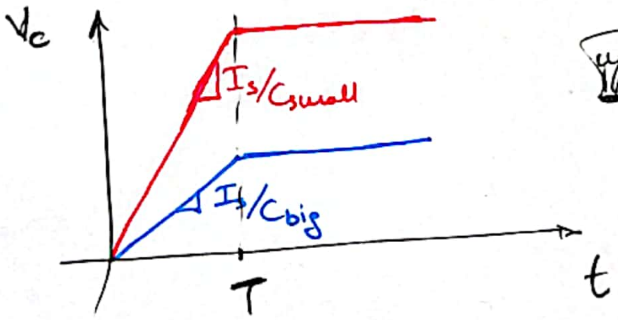
and $Q_{small} = C_{small} \cdot V_{small}$

$Q_{big} = C_{big} \cdot V_{big}$

$V_{big} < V_{small}$

Intuitive takeaway:

A larger capacitor requires a smaller voltage to store the same amount of charge.

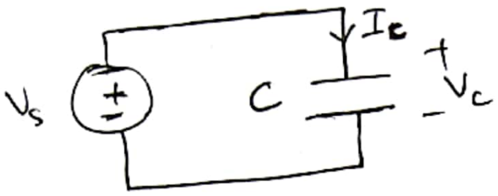


Smaller capacitor means larger voltage slope!

(Remember: $\frac{dV_c}{dt} = \frac{I_s}{C}$)

Scanned with CamScanner

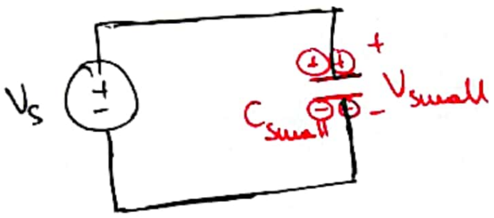
Example ckt #2

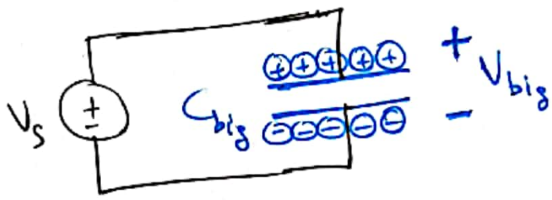


$V_c = V_s = \text{const.}$

$$I_s = C \cdot \frac{dV_c}{dt} = C \cdot \frac{\cancel{dV_s}^{\,0}}{dt} = 0$$

Intuition on ckt #2:





$$\left.\begin{array}{l} V_{small} = V_{big} = V_s \quad \text{(KVL)} \\ C_{small} < C_{big} \\ Q_{small} = C_{small} V_{small} \\ Q_{big} = C_{big} V_{big} \end{array}\right\}$$

$\Rightarrow \quad Q_{big} > Q_{small}$

Intuitive Takeaway:

A larger capacitor has the capacity to store more charge for a given (fixed) voltage across it.

Equivalent circuits with capacitors

→ capacitor-only circuits

* Step 1: ~~Find V_{Th} or I_{No}~~ No source, no problem

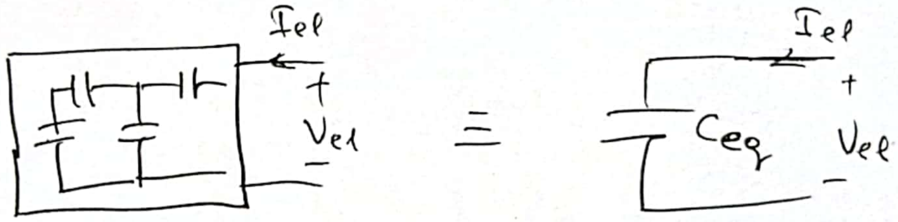
* Step 2: $C_{eq} = \dfrac{I_{elem}}{\frac{dV_{elem}}{dt}}$

a) Apply I_{test} and measure $\frac{dV_{test}}{dt}$

b) Apply $\frac{dV_{test}}{dt}$ and measure I_{test}

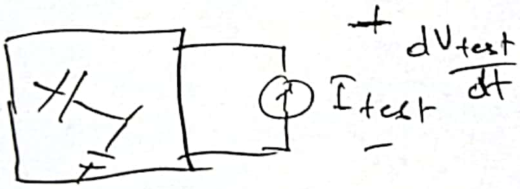
$\Bigg\}$ $C_{eq} = \dfrac{I_{test}}{\frac{dV_{test}}{dt}}$

Goal:

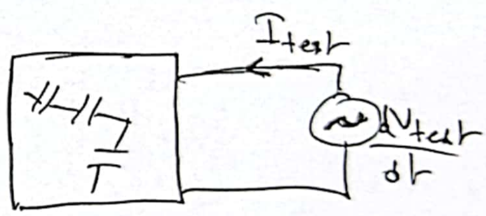


(match $\frac{dV_{el}}{dt}$)

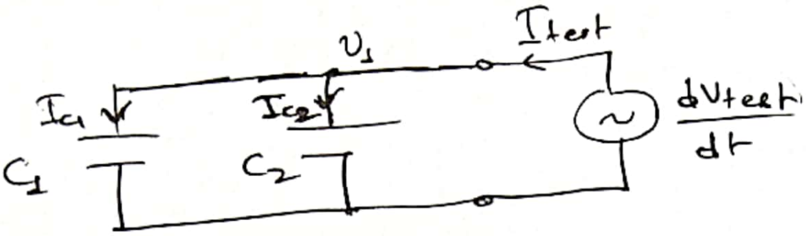
a)



b)



Capacitors in parallel:



some time-varying voltage source (only there so that $\frac{dV_{test}}{dt} \neq 0$)

$$V_{C_1} = V_{C_2} = V_{test}$$

$$\frac{dV_{C_1}}{dt} = \frac{dV_{C_2}}{dt} = \frac{dV_{test}}{dt}$$

KCL on v_1:

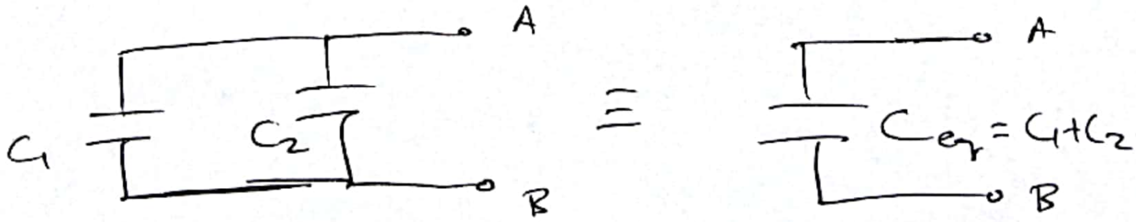
$$I_{test} = I_{C_1} + I_{C_2}$$

$$= C_1 \frac{dV_{C_1}}{dt} + C_2 \frac{dV_{C_2}}{dt}$$

$$\Rightarrow I_{test} = (C_1 + C_2) \cdot \frac{dV_{test}}{dt}$$

$$\Rightarrow \frac{I_{test}}{\frac{dV_{test}}{dt}} = C_1 + C_2$$

$$\Rightarrow C_{eq} = C_1 + C_2$$



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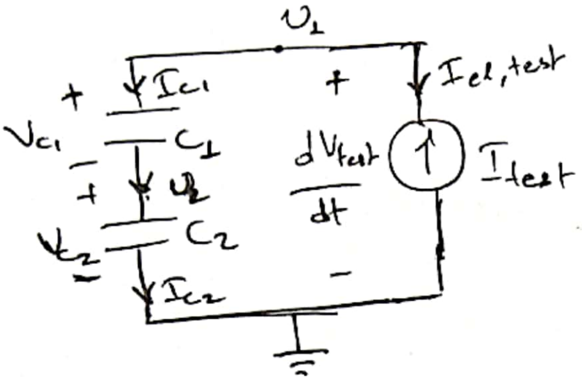
Capacitors in series:



$$I_{test} = I_{c_1} = I_{c_2}$$

$$V_{test} = v_1 - 0$$

$$V_{c_2} = v_2 - 0$$

$$V_{c_1} = v_1 - v_2$$

Element equations

$$I_{c_1} = C \cdot \frac{dV_{c_1}}{dt} = I_{test}$$

$$I_{c_2} = C \cdot \frac{dV_{c_2}}{dt} = I_{test}$$

$$V_{test} = V_{c_1} + V_{c_2}$$

$$\frac{dV_{test}}{dt} = \frac{dV_{c_1}}{dt} + \frac{dV_{c_2}}{dt}$$

$$\Rightarrow \frac{dV_{test}}{dt} = \frac{I_{c_1}}{C_1} + \frac{I_{c_2}}{C_2} = I_{test}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Rightarrow \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = C_1 \| C_2$$

parallel operator

$$\boxed{C_{eq} = C_1 \| C_2}$$

for capacitors in series

parallel mathematical operator:

$$x \| y = \frac{x \cdot y}{x + y}$$

Do not confuse w/ topology!

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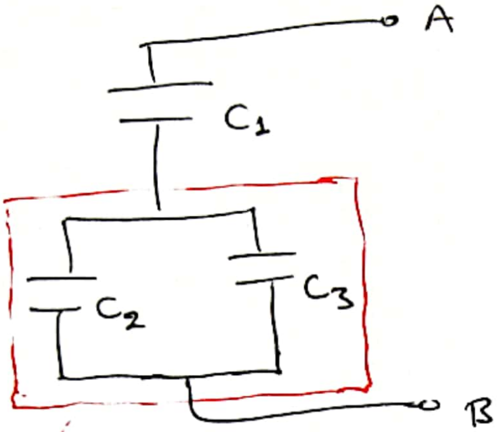
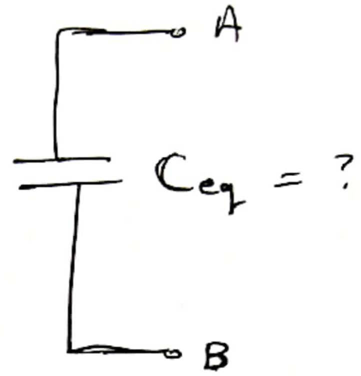
Example 3:

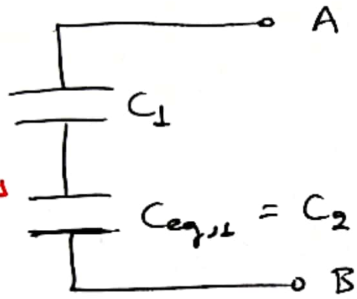


$\equiv$



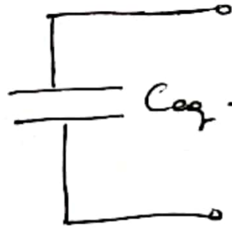
$C_{eq} = ?$

$\equiv$



$C_{eq,12} = C_2 + C_3$

$\equiv$

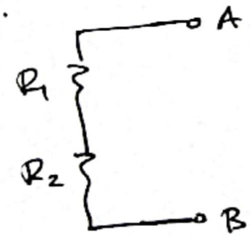


$C_{eq} = C_1 \parallel C_{eq,12}$

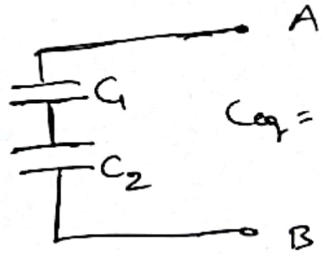
$$C_{eq} = \frac{C_1 (C_2 + C_3)}{C_1 + (C_2 + C_3)}$$

Summing it up:

series:

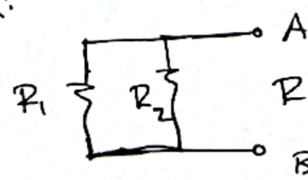


$R_{eq} = R_1 + R_2$

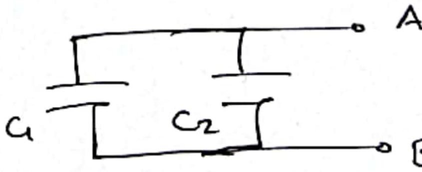


$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = C_1 \parallel C_2$

parallel:



$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = R_1 \parallel R_2$



$C_{eq} = C_1 + C_2$

Resistors **Capacitors**

Resistor and capacitor equivalents are "swapped".

Reason: $V = R \cdot I$ (represents resistance) $I = C \cdot \frac{dV}{dt}$ (represents conductance) | C and R are on the opposite side of the "I–V" equation