Module 2, Lecture 12

EECS 16A

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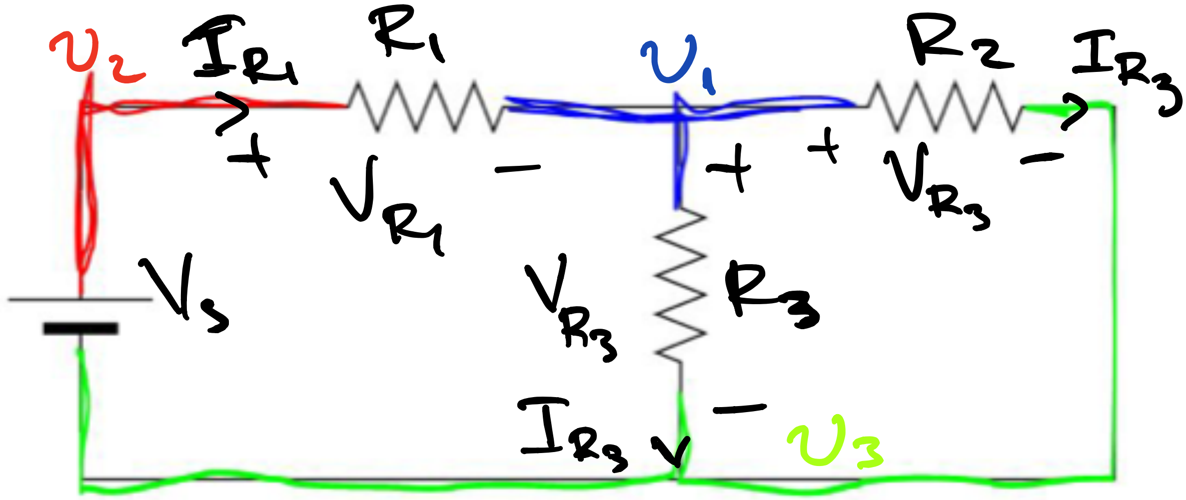
TOPIC REVIEW:

* Fundamentals
* NVA
* Power, I & V Measurements
* Resistor & Capacitor Physics
* Thevenin and Norton Equiv
  - Dep. Sources
  - // + series
* Op Amps
  - NFB
  - Comparators
* Charge Sharing
* Practice Problems

Fundamentals:

Voltages & Currents
Nodes & Branches
} Lec 2D
Note 11



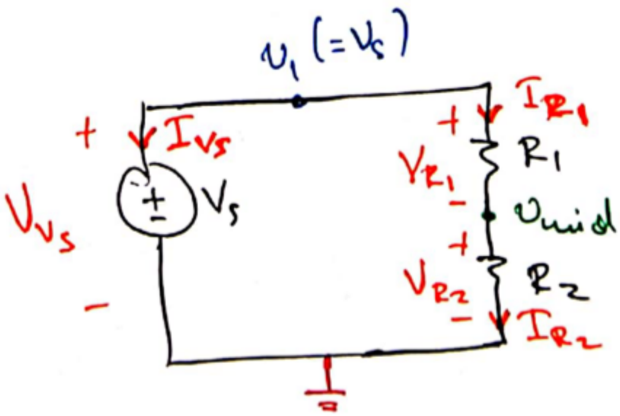
Node: Any point where 2 or more elements intersect.

e.g. $V_{R_1} = v_2 - v_1$

Node Voltage Analysis

Knowns: Source values, Resistor values
Unknowns: Node voltages

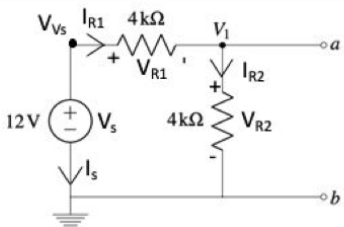
1. Select a reference node
2. Label nodes with known voltages
3. Label remaining nodes
4. Label element voltages & currents, following passive sign convention
5. Write KCL Equations at each unknown node voltage
6. Write Element Current Equations
7. Substitute Element Currents into KCL equations
8. Solve!



$$v_{mid} = \frac{R_2}{R_1 + R_2} \cdot V_s$$

Universal algorithm: never fails!

Labeling elements: What if we flip the current directions?



KCL:

$I_{R1} = I_{R2}$ $I_{R_1} - I_{R_2} = 0$

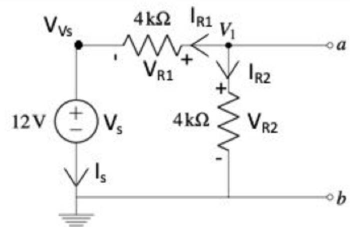
Element Equations:

$$I_{R1} = \frac{V_{R1}}{4k\Omega} = \frac{V_{Vs} - V_1}{4k\Omega}$$

$$I_{R2} = \frac{V_{R2}}{4k\Omega} = \frac{V_1 - 0}{4k\Omega}$$

Substituting element equations into KCL:

$$\frac{V_{Vs} - V_1}{4k\Omega} = \left(\frac{V_1 - 0}{4k\Omega}\right)$$



KCL:

$I_{R1} + I_{R2} = 0$

Element Equations:

$$I_{R1} = \frac{V_{R1}}{4k\Omega} = \frac{V_1 - V_{Vs}}{4k\Omega}$$

$$I_{R2} = \frac{V_{R2}}{4k\Omega} = \frac{V_1 - 0}{4k\Omega}$$

Substituting element equations into KCL:

$$0 = \left(\frac{V_1 - 0}{4k\Omega}\right) + \frac{V_1 - V_{Vs}}{4k\Omega}$$

Notice that if we rearrange the equations a bit, they should give the same solution for V_1.
I_{R1} will be equal in magnitude but negative, however, the "real" direction of the current is the same

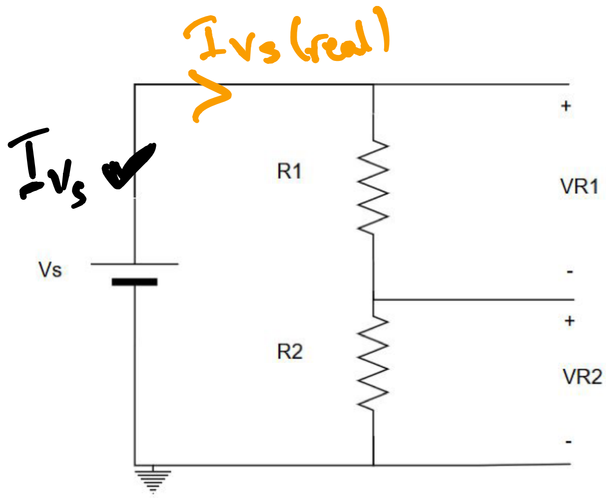
Common Practice:

Invoke already done analysis results on "common" ckts that show up often:

$\Downarrow$ Dividers (Voltage, Current)

Voltage Divider

Note, the resistors MUST be in SERIES to use this equation



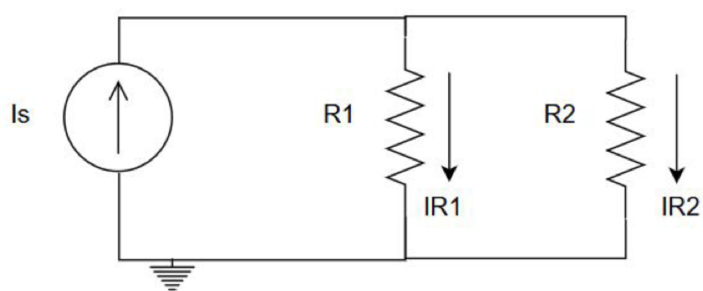
$$V_{R_1} = V_s \frac{R_1}{R_1 + R_2}$$

$$V_{R_2} = V_s \frac{R_2}{R_1 + R_2}$$

Current Divider

Note, the resistors MUST be in PARALLEL to use this equation

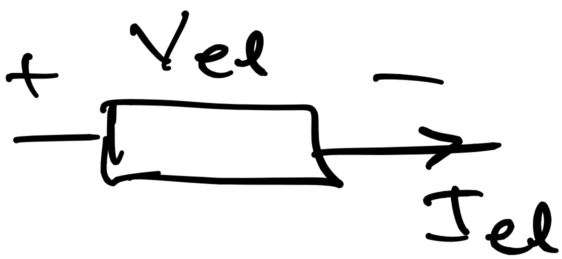
Also note that the resistor in the numerator is opposite of the one you'd expect. Intuitively, path of least resistance*.



$$I_{R_1} = I_s \frac{R_2}{R_1 + R_2}$$

$$I_{R_2} = I_s \frac{R_1}{R_1 + R_2}$$

Power:



$$P_{el} = V_{el} \cdot I_{el}$$

ALWAYS!

$P_{el} > 0 \Rightarrow$

1) Element dissipates power
2) It follows passive sign convention
3) It is "passive"

$V_{el} > 0, I_{el} > 0$
$V_{el} < 0, I_{el} < 0$

$P_{el} < 0 \Rightarrow$

1) Element generates power
2) It "violates" passive sign convention
3) It is "active"

(i.e current flows out of the "+" in reality)
e.g. in voltage source + resistors

$V_{el} > 0, I_{el} < 0$
$V_{el} < 0, I_{el} > 0$

* For Resistors :

$$P_R = I_R V_R = I_R^2 R = \frac{V_R^2}{R} \geq 0$$

$\Rightarrow$ A resistor always dissipates power!

* Power is always conserved!

$$\underline{\underline{P_{el,tot} = 0}}$$

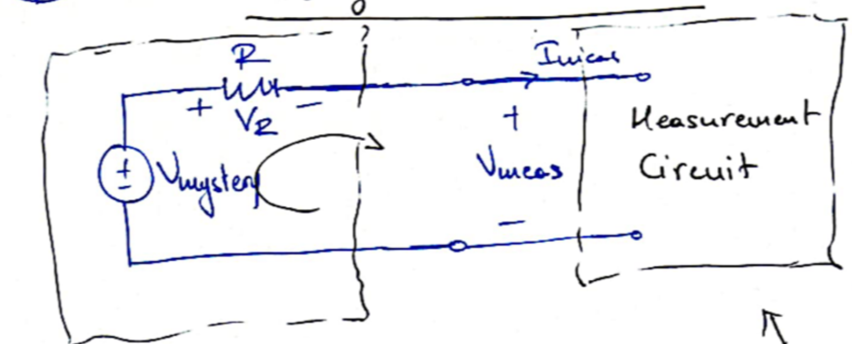
V & I Measurements:

Ideal Measurement Device should dissipate 0 power

Ref: Lec 3B - Note 13

p3

Voltage Measurement



Goal:

Measure $V_{mystery}$

i.e. $V_{meas} = V_{mystery}$

Problem:

R is "in the way"

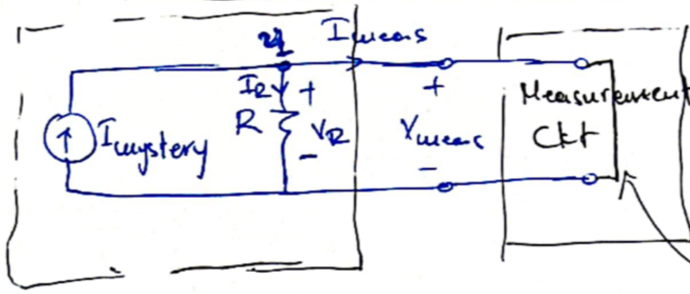
KVL: $V_{mystery} - V_R - V_{meas} = 0$

$\Rightarrow V_{mystery} = I_{meas} \cdot R + V_{meas}$

$\Rightarrow V_{mystery} = V_{meas} \Big|_{\text{if } I_{meas} = 0}$

measurement circuit needs to look like an open-circuit p5

Current Measurement



Goal:
Measure $I_{mystery}$
i.e. $I_{meas} = I_{mystery}$

Problem:
R is "in the way"

KCL on v_1: $I_{mystery} = I_R + I_{meas}$

$\Rightarrow I_{mystery} = \frac{V_{meas}}{R} + I_{meas}$

$\Rightarrow I_{mystery} = I_{meas} \Big|$ if $V_{meas} = 0$

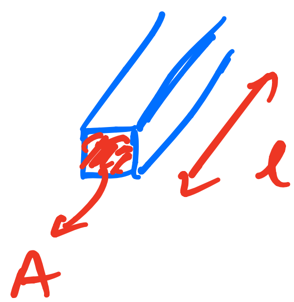
needs to look like a wire

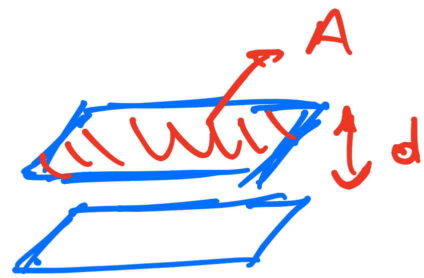
$P_{meas} = \cancel{V_{meas}}^{\,0} \cdot I_{meas} = 0$ ✓ (measurement ckt does not dissipate any power)

Disclaimer:

If approximations are required we will explicitly tell you so in an exam. (i.e $\frac{R_{VM} \cdot R_x}{R_{VM} + R_x} \to R_x$ if $R_{VM} \to \infty$)

Resistors and Capacitors





$$R = \rho \cdot \frac{\ell}{A}$$

ρ → material resistivity

$$C = \varepsilon \cdot \frac{A}{d}$$

ε → material (insulator) permittivity

Superposition

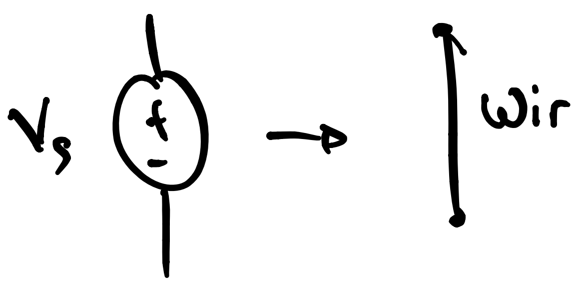
Def of linear: T(ax + by) = aT(x) + bT(y)

The passive components we use have linear I-V relations, so we can solve for the effect of each source individually and add the responses to get the same result.

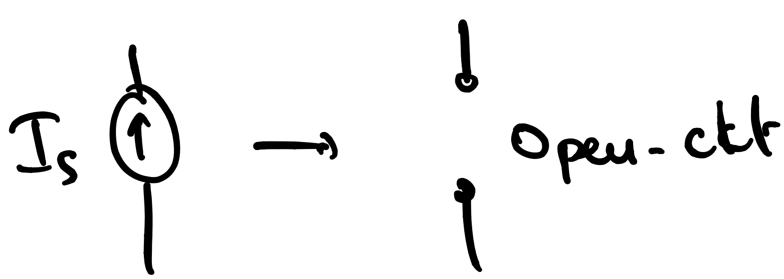
When to use? When you have **multiple sources** and c**onsidering all at the same time is confusing**

Note: you can solve any circuit in this class without superposition with just normal nodal analysis.

Zero a voltage source



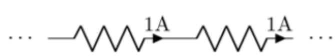
Zero a current source



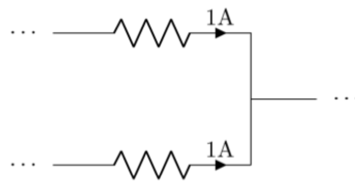
Series?

Two ways to tell if components are in series:

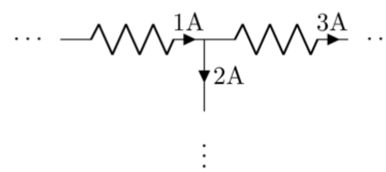
1. same EXACT current. Not same value of current, but same exact current through elements by KCL. If the current can split off then they aren't series!
2. **If two and only two elements are connected to a single node** (no other elements are connected to that node), the elements are in series.



Series



Same VALUE of current, but not same current => NOT series



Current does not go entirely through second resistor. NOT series

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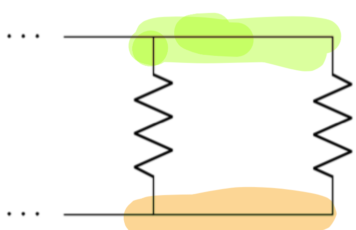
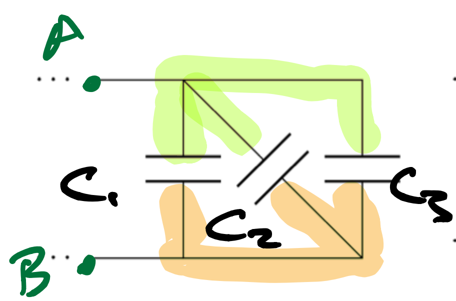
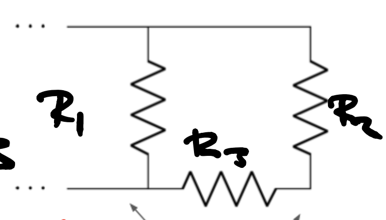
Parallel?

Parallel: they share the same NODES on either side. Not just the same voltage, but same NODES







Yes!

C_1, C_2, C_3 in parallel

X No

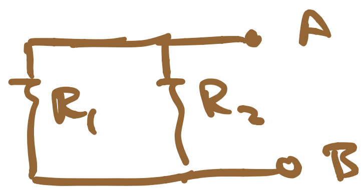
res. R_1, R_2, R_3 are in parallel

$$C_{AB} = C_1 + C_2 + C_3$$

Resistors

Capacitors

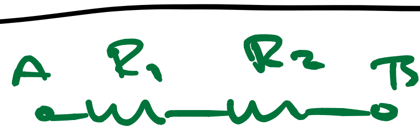
Parallel



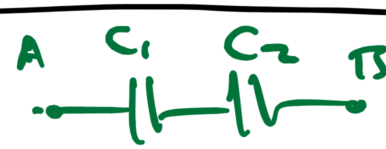
$$R_{AB} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$C_{AB} = C_1 + C_2$$

Series

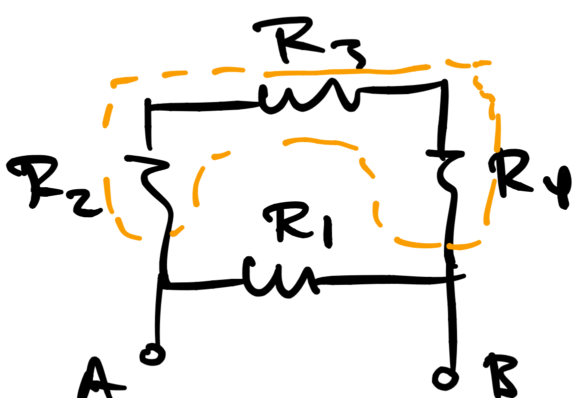


$$R_{AB} = R_1 + R_2$$



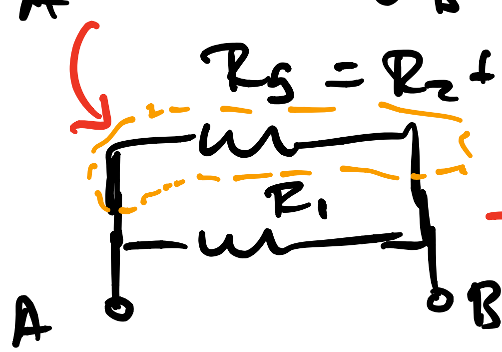
$$C_{AB} = C_1 // C_2$$

$$= \frac{C_1 C_2}{C_1 + C_2}$$



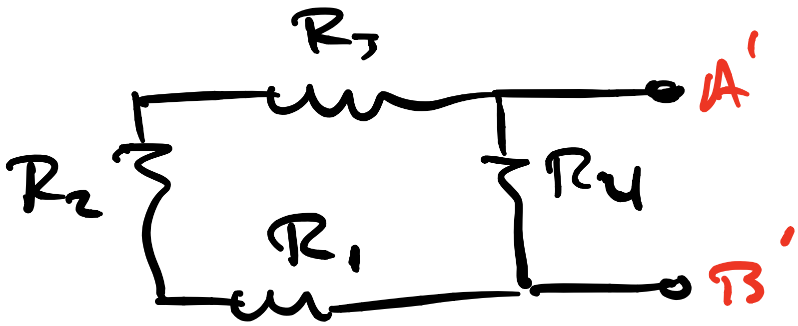
$$R_{AB} = ?$$

$$R_5 = R_2 + R_3 + R_4$$



$$R_{AB} = R_1 // R_5$$

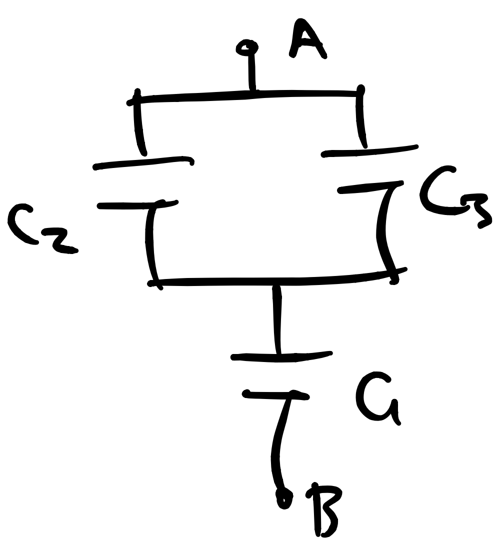
Note: R_{eq} depends on where I connect my terminals:



$R_{A'B'} = R_4 \parallel (R_1 + R_2 + R_3)$

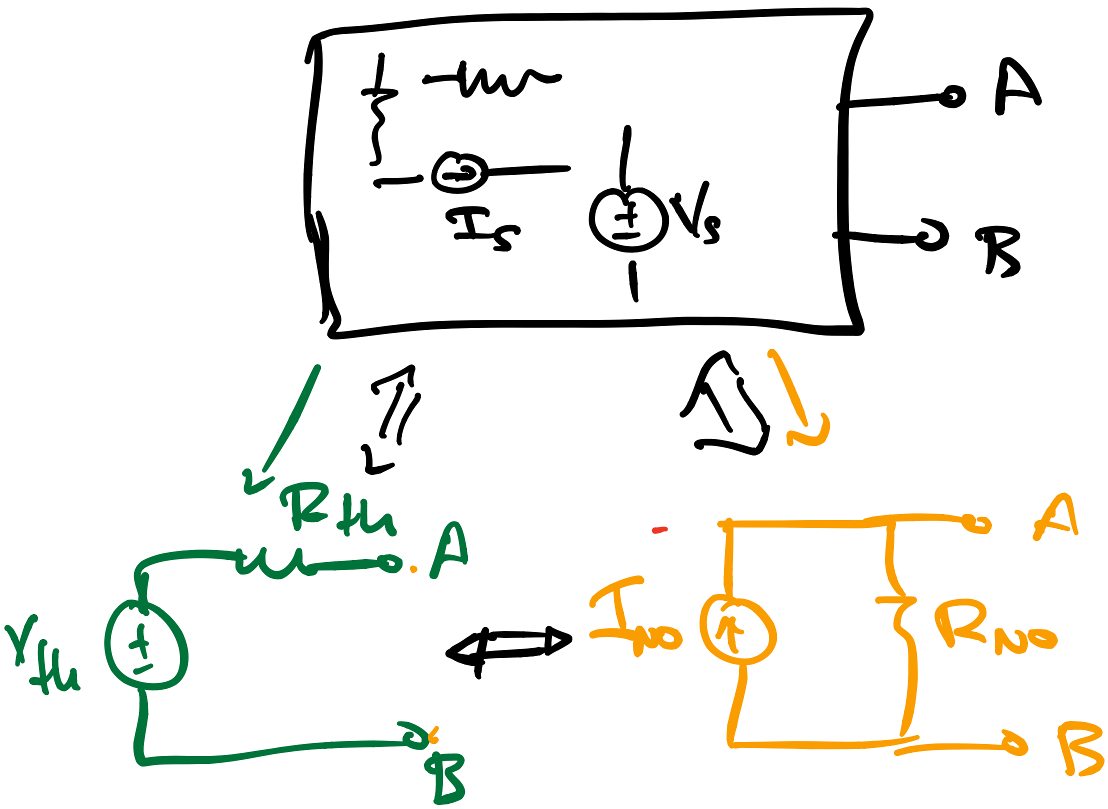
$\neq R_{AB}$ from above

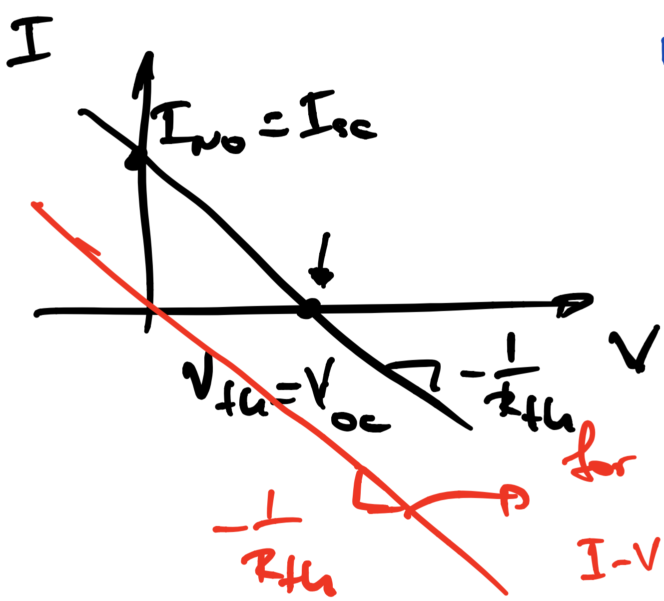
Capacitor Example



$C_{AB} = C_1 \parallel (C_2 + C_3)$

Thevenin and Norton Equiv.:





1) $V_{th} = I_{No} \cdot R_{No}$

2) $I_{No} = \dfrac{V_{th}}{R_{th}}$

3) $R_{th} = R_{No}$

for resistive only ckts
I-V goes through the origin

Finding Thevenin + Norton Equiv.:

1) Find V_{th} by open-ckting the terminals A-B and finding $V_{oc} = V_{th}$

How: NVA or Superposition

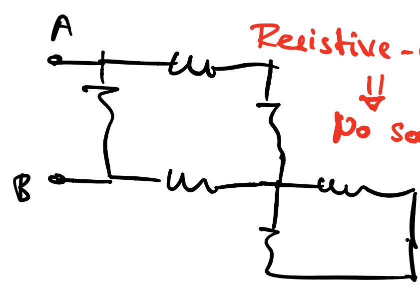
2) Find I_{NO} by short-ckting the terminals A-B (i.e. connect them with a wire) and find $I_{sc} = I_{NO}$

How: NVA or Superposition

3) Find R_{th}:

How: 3 options → pick the easiest!

a) $R_{th} = V_{th} / I_{NO}$ → fails with res. only ckts* where $V_{th} = I_{NO} = 0$

E.g. 

Resistive-only ckt
⇓
no source → cannot apply method a) since $V_{th} = I_{NO} = 0$

b) Use // and series combination with independent source zeroed-out → fastest

→ fails when dep. sources are not zeroed out (see ex. below)

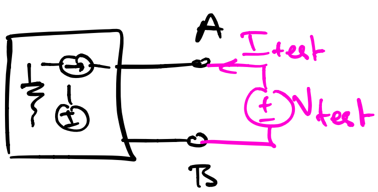
c) Use V_{test} - I_{test} method with independent sources zeroed → (ALWAYS WORKS)

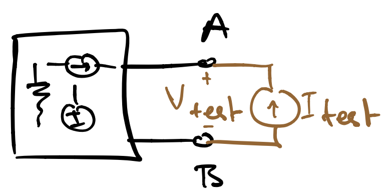
→ But takes the longest

Procedure:

With Indep. Sources zeroed-out

Apply V_{test} → measure I_{test} OR Apply I_{test} → measure V_{test}



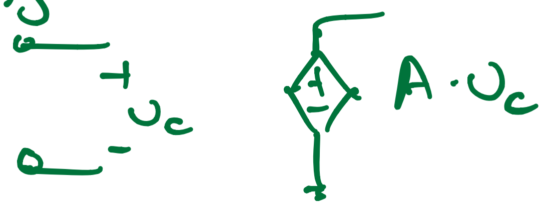


$$R_{th} = \frac{V_{test}}{I_{test}}$$

Dependent Sources:

Model some complicated circuit element in our "Black Box" ckt (e.g. transistors, opamps)

e.g.

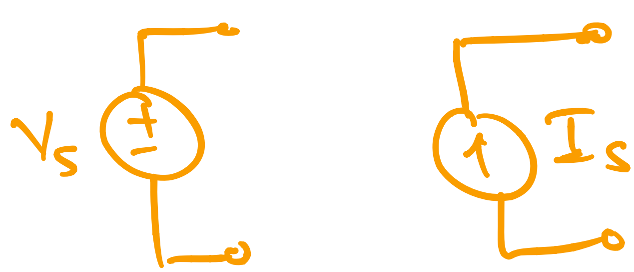


Should not be shut off when finding R_{th}.

Independent Sources:

Inputs to our ckt.

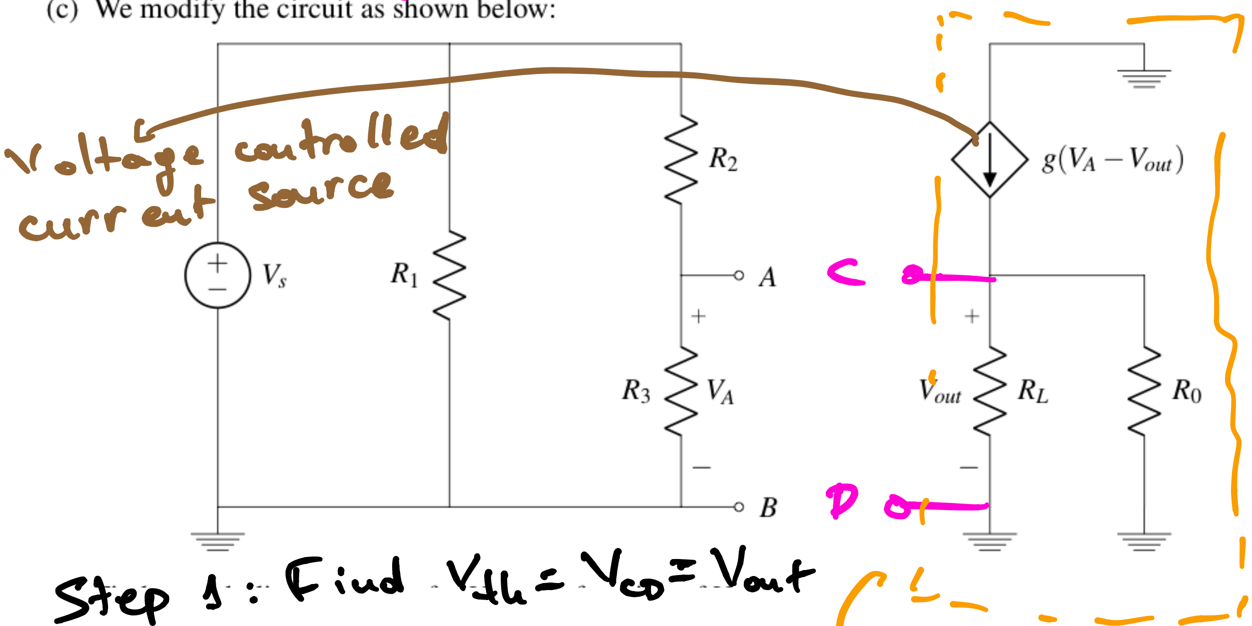
e.g.



Find V_{th}, R_{th} looking into terminals C,D:

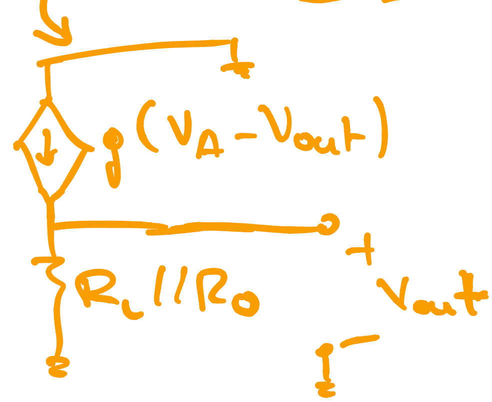
Disclaimer: g has units of $\frac{1}{\Omega}$

(c) We modify the circuit as shown below:



Step 1: Find $V_{th} = V_{CD} = V_{out}$

$$V_{AB} = V_A = \frac{R_3}{R_3 + R_2} \cdot V_s \quad (1)$$



$$V_{out} = g(V_A - V_{out}) R_L // R_0$$

$$\Rightarrow V_{out} = \frac{g \cdot R_L // R_0}{1 + g R_L // R_0} \cdot V_A$$

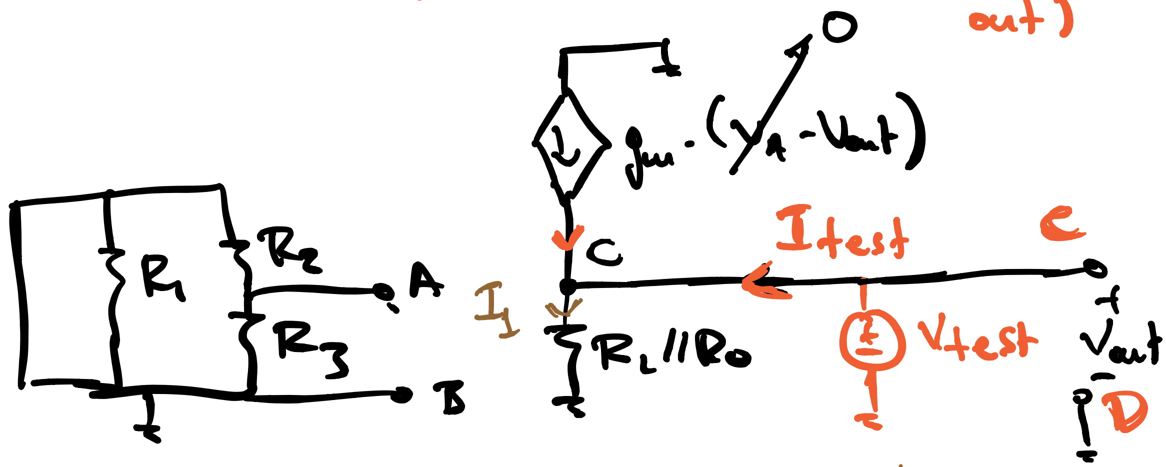
$$\stackrel{(1)}{=} \frac{g R_L // R_0}{1 + g R_L // R_0} \cdot \frac{R_3}{R_3 + R_2} \cdot V_s$$

$$V_{out} = V_{oc} = V_{th}.$$

Step 2 : Find R_{th}

a) zero out indep. sources

b) Apply V_{test} and measure I_{test} using NVA. (I need to do this here since the dep. source is not zeroed out)



KCL (on node C) : $\overset{\text{entering}}{I_{test} + g_m(-V_{out})} \overset{\text{exiting}}{- I_1} = 0$

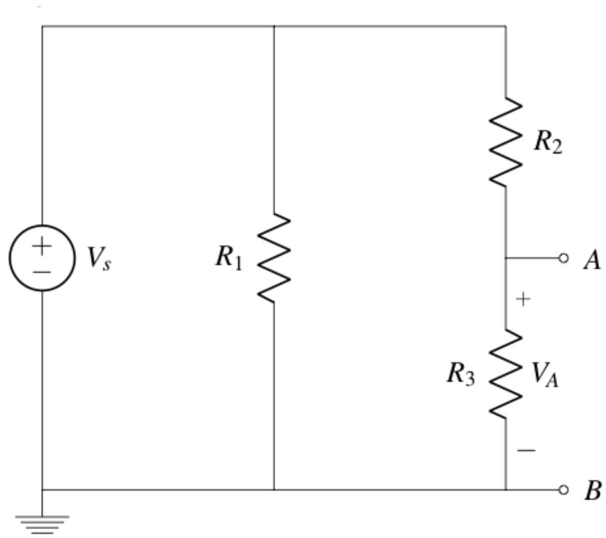
$$I_{test} + g_m(-V_{out}) - V_{out}/R_L // R_o = 0$$

$$\Rightarrow -\left(g_m + \frac{1}{R_L // R_o}\right) \cdot V_{test} = -I_{test}$$

$$\Rightarrow \boxed{\frac{V_{test}}{I_{test}} = \frac{1}{g_m + \frac{1}{R_L \parallel R_O}}}$$

$$R_{th} = \frac{1}{g_m + \frac{1}{R_L \parallel R_O}}$$

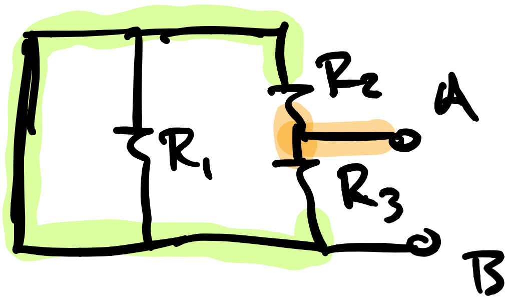
Looking only at this:



Find Vth, Rth looking at A-B.

$$V_{th} = V_{AB} = \frac{R_3}{R_2+R_3} V_s$$

<u>Rth</u> : (zero V_s)



Now I can use series and // comb!

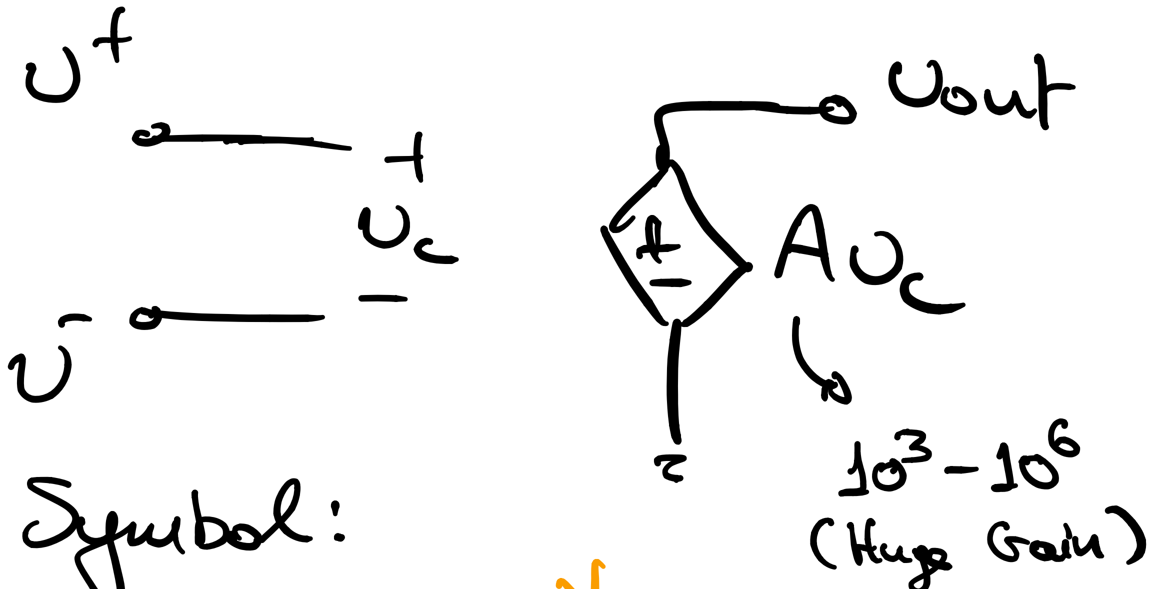
$$R_{AB} = R_2 // R_3$$

R_2, R_3 are conn. in parallel since $V_s \to$ wire

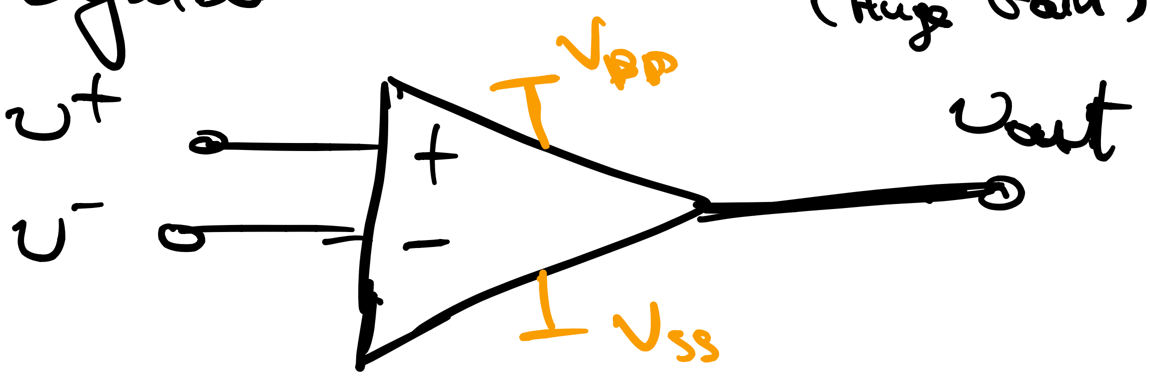
(R_1 is shorted so neglect - was done wrong in lec.)

Op Amps

Model:



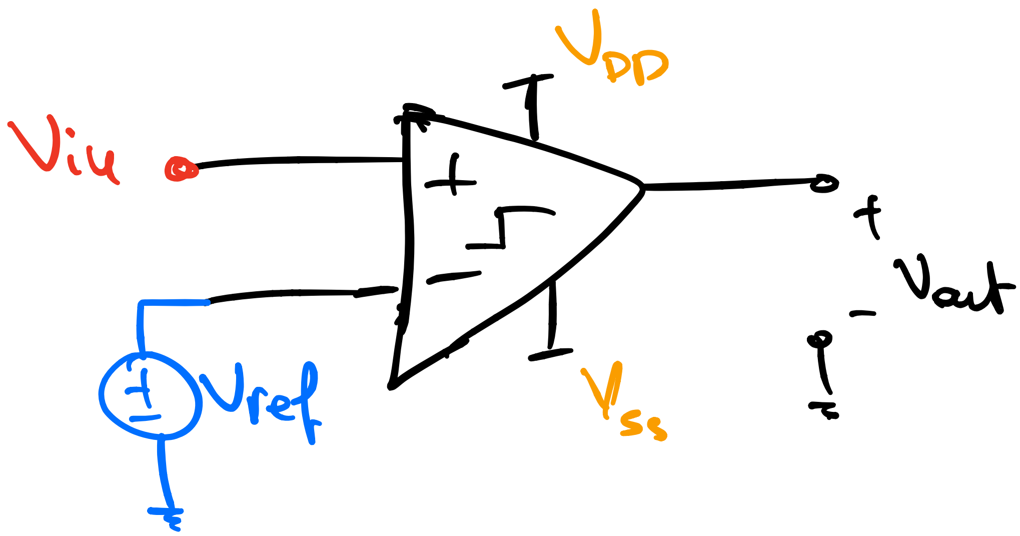
Symbol:

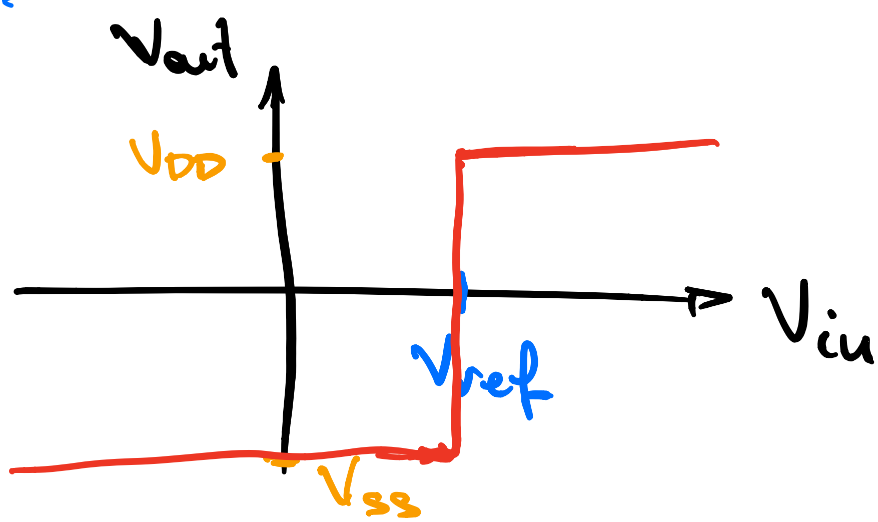


Caveat: V_{DD} and V_{SS} supply power to the op amp (often not drawn for convenience)

Uses :

a) As a comparator



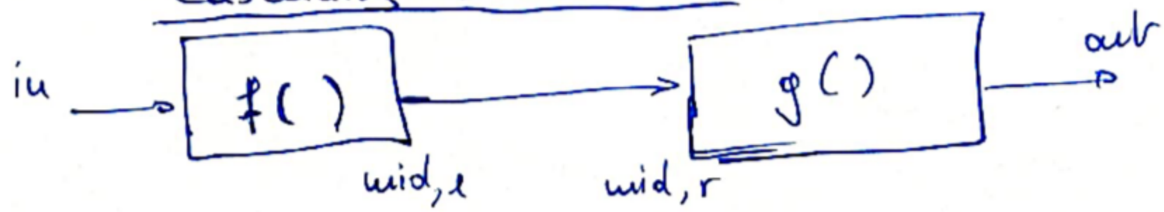


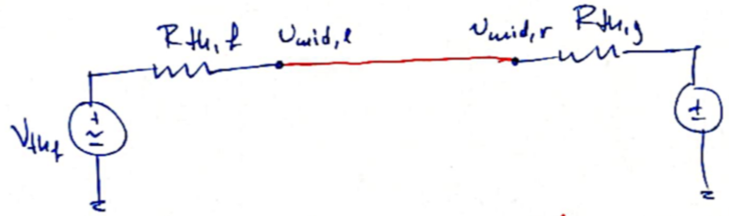
Is this a linear function?

NO!

b) As an amplifier and a means of cascading blocks → Lec. 4D, 5A!

Cascading Ckt Blocks





(most likely zero since we are looking at the in of block g( ))

Before connection: $\quad$ After connection:

$$v_{mid,l} = V_{th,f} \underset{\text{in general}}{\neq} v_{mid,l} = v_{mid,r} = \frac{R_{th,g}}{R_{th,g}+R_{th,f}} V_{th,f} + \frac{R_{th,f}}{R_{th,g}+R_{th,f}} V_{th,g}$$

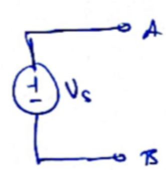
:(except when: $R_{thf} = 0$ (wire)

or $R_{thg} = \infty$ (open-circuit)

Ideal Isolation

From the perspective of block f: see an open-circuit $R_{th,g} = \infty$

— " — " — g: see a voltage source $R_{th,f} =$



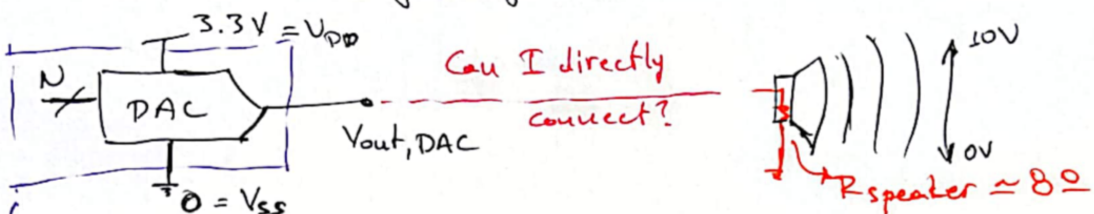
$R_{th,\text{volt source}}$
$= 0$

Motivation:

(p4)

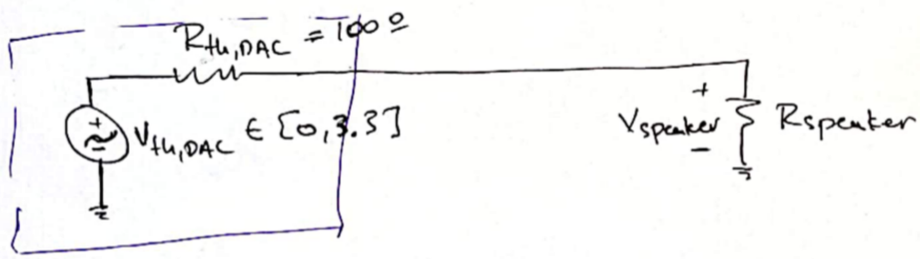
Audio System – "16A Boombox"

Digital to Analog Converter (DA)
Converts a digital-binary value to an analog voltage



$V_{out,DAC} \in [0, 3.3V]$

Thevenin Equivalent



Voltage Divider

$$V_{speaker} = \frac{R_{speaker}}{R_{speaker} + R_{th,DAC}} \cdot V_{th,DAC}$$

$$= \frac{8\Omega}{8\Omega + 100\Omega} \cdot V_{th,DAC} \simeq 0.1\, V_{th,DAC}$$

$\boxed{\text{Loading Effect!}}$

Want Linear Amplification

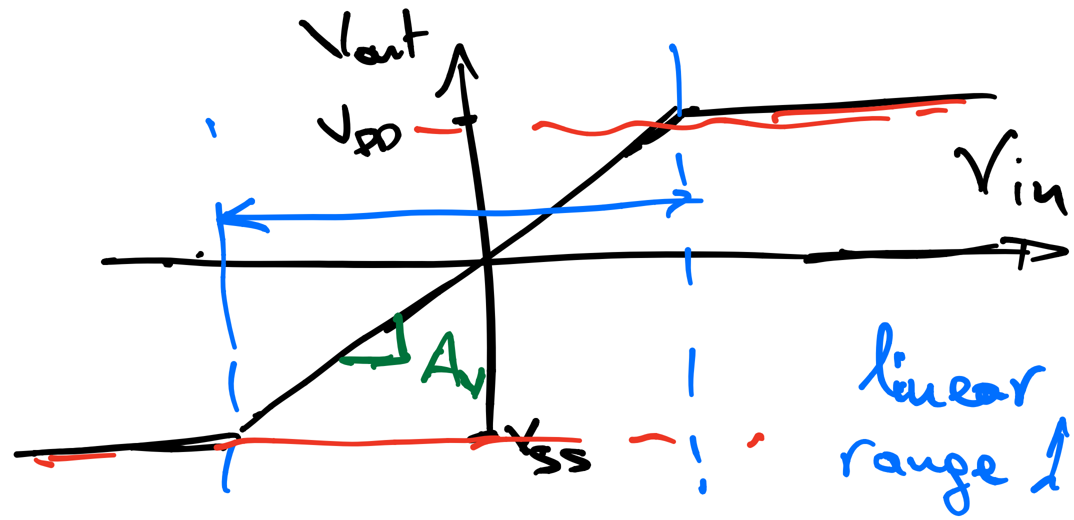
$V_{out} = A_v \cdot V_{in}$

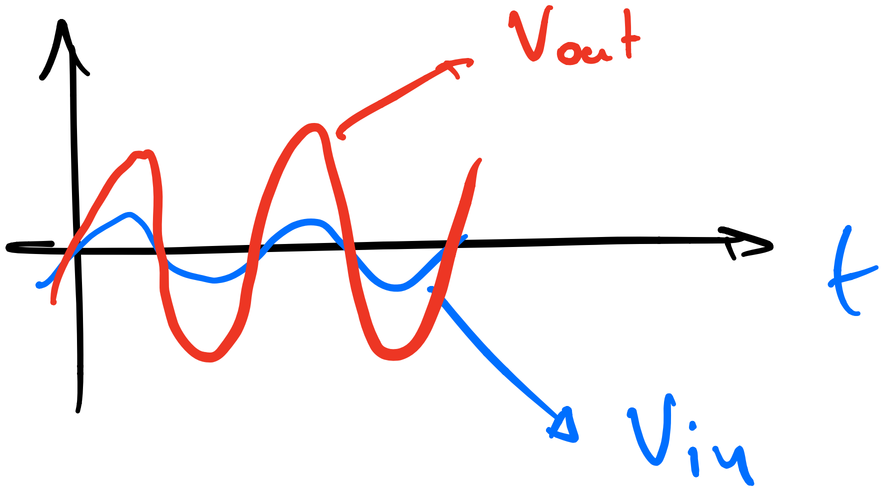
A_v → linear gain factor!

→ Goal: A_v needs to be reasonable and well contr

→ Problem: Huge uncertain gain → non-linear $V_{in} - V_{out}$

→ Solution: Use NFB!





$$V_{out} = A_v \cdot V_{in}$$

different than A_v (total ckt gain in NFB)

opamp gain

$$A_v = \frac{A}{1+Af} \xrightarrow{A \to \infty} \frac{1}{f}$$

f → feedback

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{f}$$

→ not that easy to calculate.

That's why we invoked the ...

GOLDEN RULES!

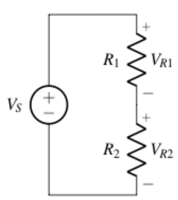
#1 $i^+ = i^- = 0$ (ALWAYS)

#2 $v^+ = v^-$ when $A \to \infty$ and op-amp is in NFB.

Using Golden Rules (+ NVA) we derived the following:

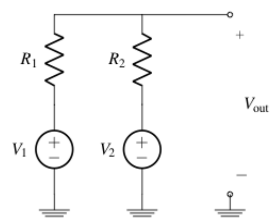
For Reference: Example Circuits

Voltage Divider



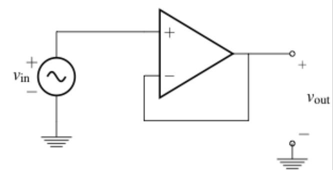
$$V_{R2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$$

Voltage Summer



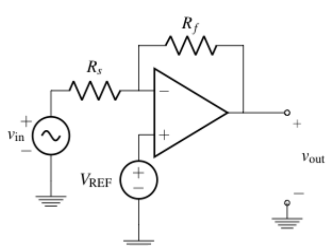
$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



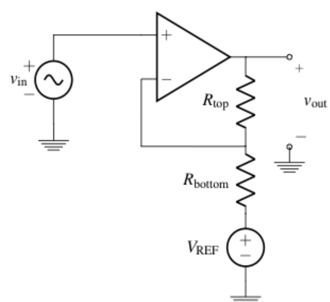
$$\frac{v_{out}}{v_{in}} = 1$$

Inverting Amplifier



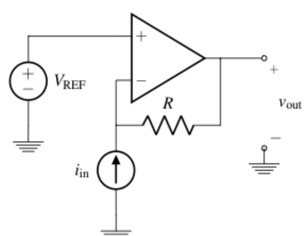
$$v_{out} = v_{in}\left(-\frac{R_f}{R_s}\right) + V_{REF}\left(\frac{R_f}{R_s}+1\right)$$

Non-inverting Amplifier



$$v_{out} = v_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right) - V_{REF}\left(\frac{R_{top}}{R_{bottom}}\right)$$

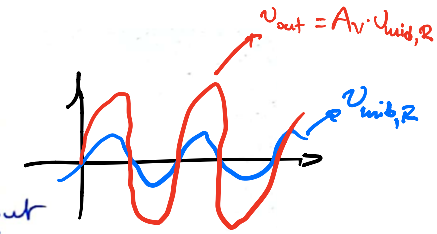
Transresistance Amplifier

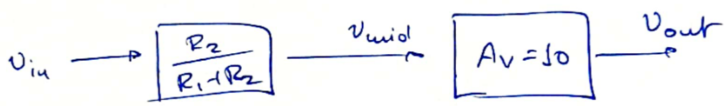


$$v_{out} = i_{in}(-R) + V_{REF}$$

Keep them handy - Learn to pattern match and recognize them p3

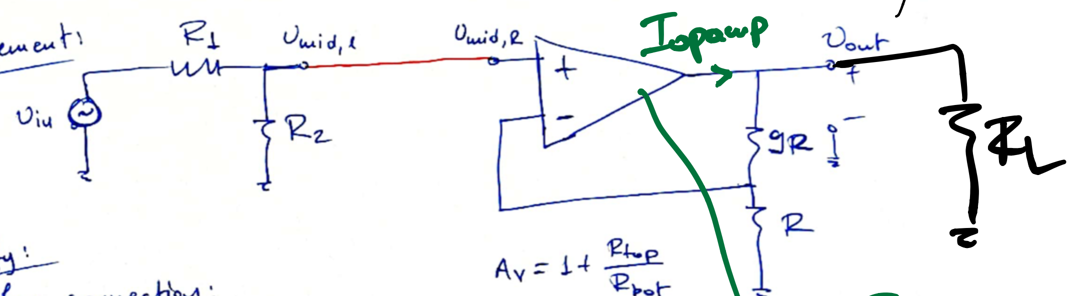
Example #1 : Want this:



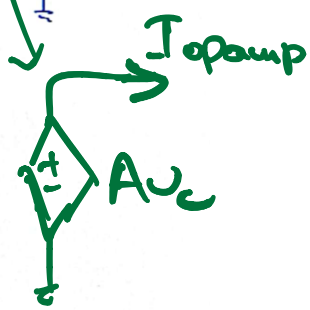


$\rightarrow$ $\underline{v_{out} = 10 \cdot v_{mid}}$

Implement:



$$A_V = 1 + \frac{R_{top}}{R_{bot}}$$



Verify:

Before connection:

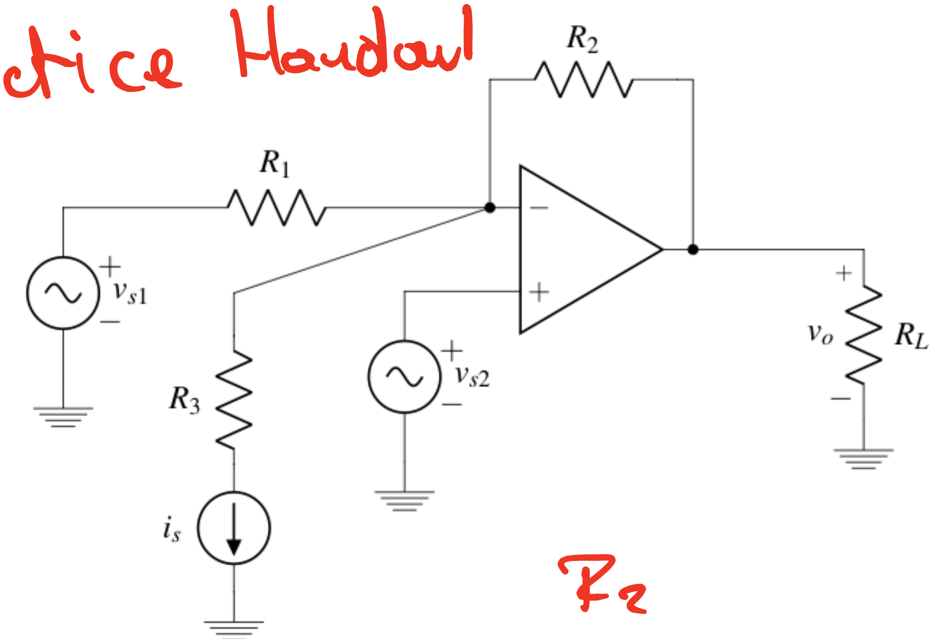
$$v_{mid,L} = \frac{R_2}{R_1+R_2} \cdot v_{in}$$

After connection:

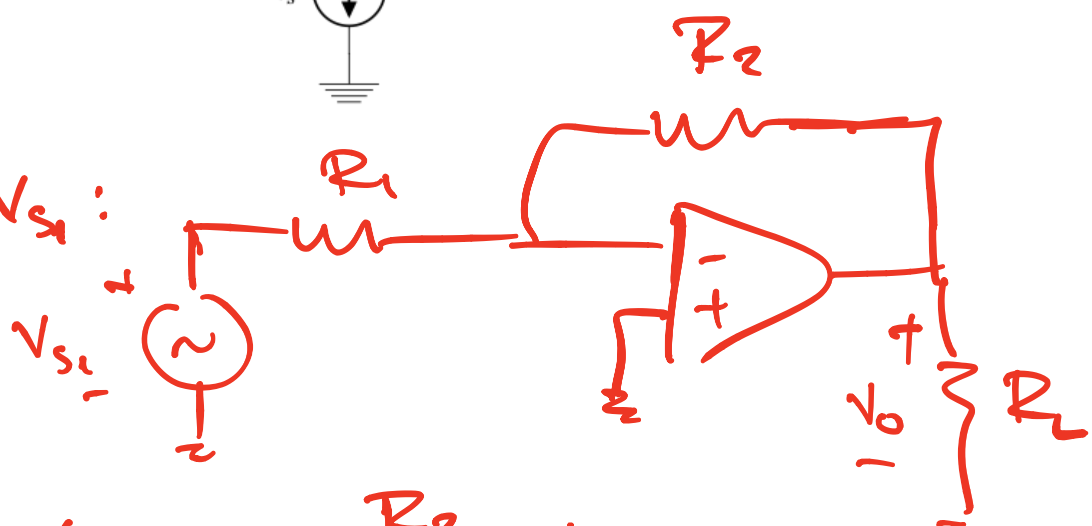
$$v_{mid,L} = v_{mid,R} = \frac{R_2}{R_1+R_2} \cdot v_{in}$$

$$\Rightarrow v_{out} = \frac{R_2}{R_1+R_2} \cdot \underbrace{\left(1 + \frac{R_{top}}{R_{bot}}\right)}_{10} \cdot v_{in}$$

Practice Handout

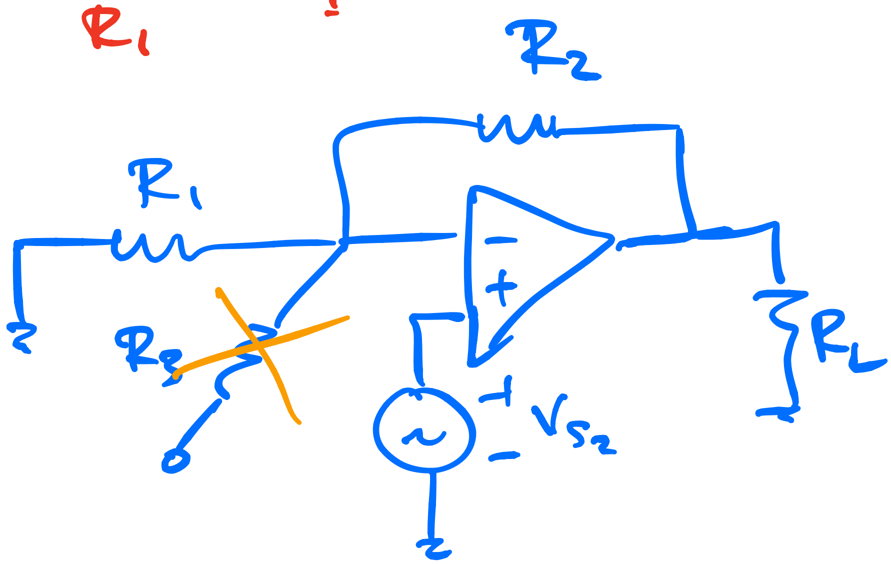


V_{s1} :

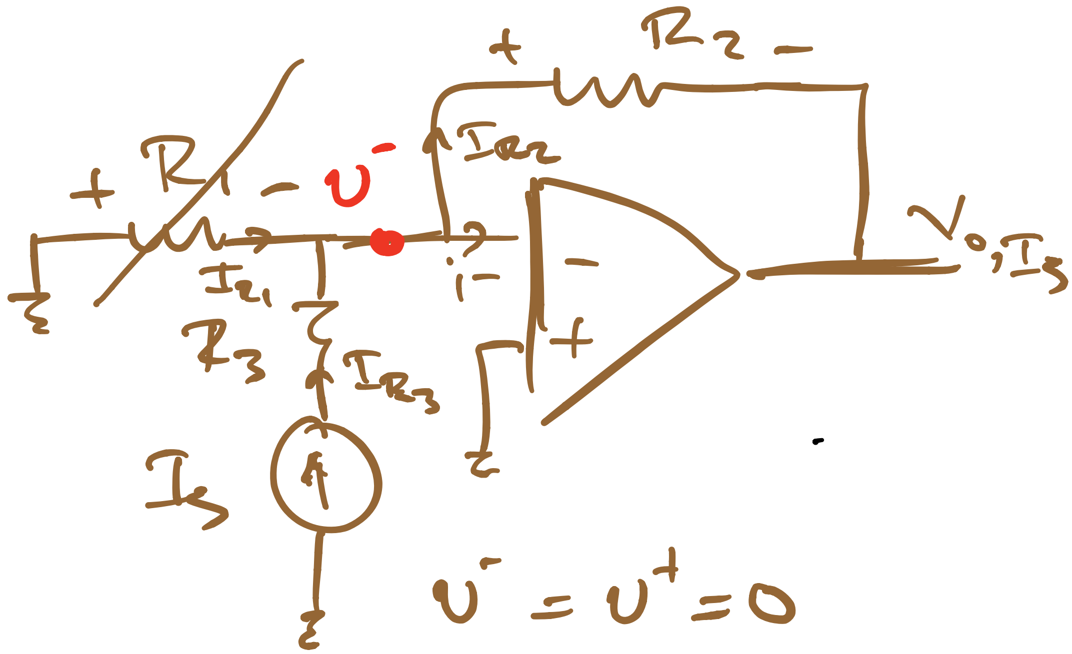


$$V_{o,s_1} = -\frac{R_2}{R_1} \cdot V_{s_1}$$

V_{s2} :



$$V_{o,s} = \left(1 + \frac{R_2}{R_1}\right) \cdot V_{s_2}$$



$$v^- = v^+ = 0$$

$$\Rightarrow V_{R_1} = ?$$

$$= 0 - v^-$$
$$= 0$$

KCL on v^- :

$$I_{R_3} = I_{R_2} = I_s$$

Ohm's law : $V_{R_2} = I_s \cdot R_2$

$$\Rightarrow 0 - V_{o_{I_s}} = I_s \cdot R_2$$

$$\Rightarrow V_{oI_s} = -I_s' R_2$$

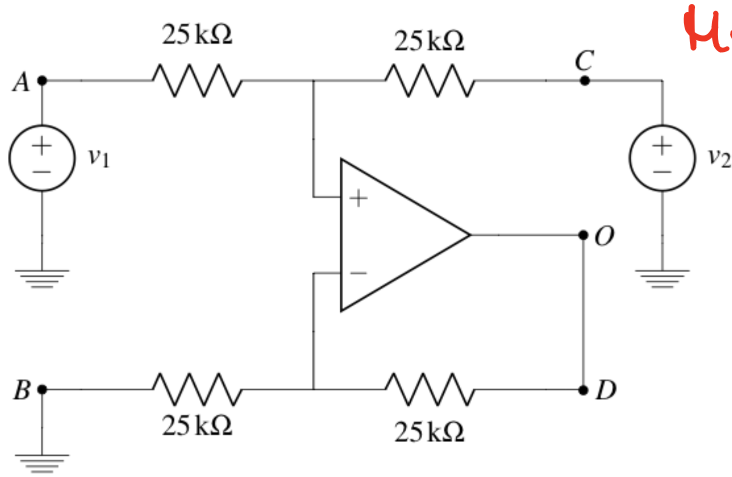
$$\boxed{\begin{aligned} V_o &= V_{os_1} + V_{os_2} + V_{oI_s} \\ &= -\frac{R_2}{R_1} \cdot V_{s_1} + \left(1 + \frac{R_2}{R_1}\right) V_{s_2} - I_s R_2 \end{aligned}}$$

(sol also on Class Website)

Sp 2020

Mt 2

(a) Determine the voltage at O for the following configuration.



Practice: show that

$v_o = v_1 + v_2$

(sols on Piazza)