EECS 16A 7/30/2020
Module 3
lecture 1

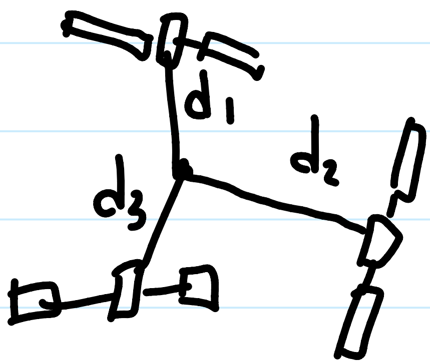
Announcements
1) HW 5B is up
- One mandatory problem
- Solution upload today
2) OH: Friday 1-2pm
Tues/Wed 3-4 pm

Module 1: system modeling
System analysis
Module 2: Designing & implementation
Sensing
Module 3: Building blocks of Machine Learning

Classification

Ex: Image recognition

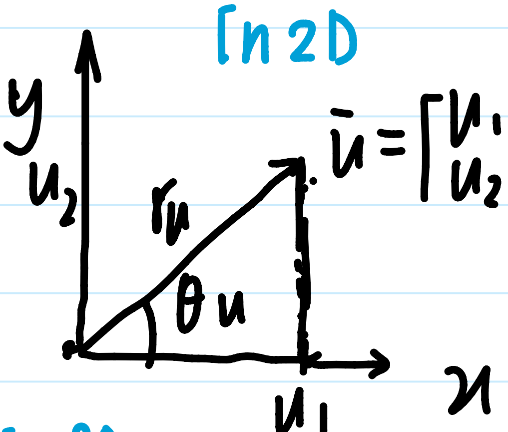
Ex: GPS problem



Which satellite is sending signals

Pattern recognition

In 2D



$$\bar{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$u_1 = r_u \cos\theta_u$$

$$u_2 = r_u \sin\theta_u$$

$$r_u = \sqrt{u_1^2 + u_2^2}$$

In 3D

$$\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad r_u = \sqrt{u_1^2 + u_2 + \dots + u_n^2}$$

$$\boxed{\begin{aligned} r_u &= \text{Magnitude of } \bar{u} \\ &= \text{Norm of } \bar{u} \\ &= \|u\| = \sqrt{u_1^2 + u_2 + \dots + u_n^2} \end{aligned}}$$

Norm

Transpose:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$\bar{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \qquad \bar{u}^T = \begin{bmatrix} u_1 & \cdots & u_n \end{bmatrix}$$

$$(\bar{a} + \bar{b})^T = \bar{a}^T + \bar{b}^T$$

$$\bar{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$(\bar{a} + \bar{b})^T = \begin{bmatrix} 5 & 7 & 9 \end{bmatrix}$$

$$\bar{a}^T + \bar{b}^T = \begin{bmatrix} 5 & 7 & 9 \end{bmatrix}$$

Same

Inner product:

$\bar{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_2 \end{bmatrix} \quad \bar{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$

Inner product of $\bar{u}$ & $\bar{v}$

$= \langle u, v \rangle = \underbrace{u_1 v_1 + u_2 v_2 + \dots + u_n v_n}_{\text{Scalar}}$

$\bar{u}^T = [u_1 \cdot \dots \cdot u_n]$

$\bar{u}^T \bar{v} = [u_1 \ \cdot \ \cdot \ \cdot \ u_n] \begin{bmatrix} v_1 \\ \\ v_n \end{bmatrix}$

$= u_1 v_1 + \dots u_n v_n$

$= \langle \bar{u}, v \rangle$

$\langle \bar{u}, \bar{v} \rangle = \bar{u}^T \bar{v} = u_1 v_1 + u_2 v_2 + \cdot \cdot + u_n v_n$

\# $\langle \bar{v}, \bar{u} \rangle = v_1 u_1 + \ \cdot \ \cdot \ + v_n u_n$

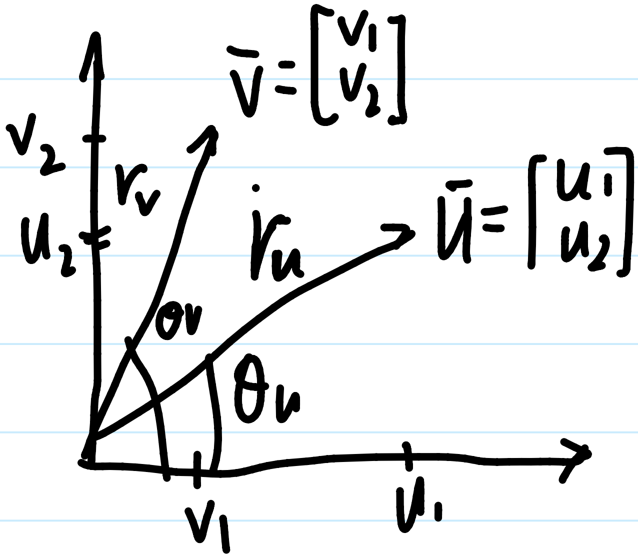
$= u_1 v_1 + \dots + u_n v_n$

$= \langle \bar{u}, \bar{v} \rangle$

$\bar{u}^T \bar{v} = \bar{v}^T \bar{u}$

\# $\langle \bar{u}, \bar{u} \rangle = u_1^2 + u_2^2 + \dots + u_n^2 = \|u\|^2$

Geometrically looking at $\langle \bar{u}, \bar{v} \rangle$



$$u_1 = r_u \cos\theta_u$$
$$u_2 = r_u \sin\theta_u$$
$$v_1 = r_v \cos\theta_v$$
$$v_2 = r_v \sin\theta_v$$
$$r_u = \sqrt{u_1^2 + u_2^2} = \|u\|$$
$$r_v = \sqrt{v_1^2 + v_2^2} = \|v\|$$

$$\begin{aligned} \langle \bar{u}, \bar{v} \rangle &= u_1 v_1 + u_2 v_2 \\ &= r_u r_v \cos\theta_u \cos\theta_v \\ &\quad + r_u r_v \sin\theta_u \sin\theta_v \\ &= r_u r_v [\cos\theta_u \cos\theta_v + \sin\theta_u \sin\theta_v] \\ &= r_u r_v \cos(\theta_u - \theta_v) \\ &= \|u\| \, \|v\| \cos(\theta_u - \theta_v) \end{aligned}$$

$\langle \bar{u}, \bar{v} \rangle$ is maximized when

$$\theta_u = \theta_v$$

$$\Rightarrow \cos(\theta_u - \theta_v) = 1$$

If, $\theta_u = \theta_v$, $\langle u, v \rangle = \|u\| \|v\|$

if $\theta_u - \theta_v = 90°$, $\langle u, v \rangle = 0$

$\downarrow$

vectors $\bar{u}$ & $\bar{v}$ are orthogonal

$$\cos(\theta_u - \theta_v) \leq 1$$

$$\Rightarrow \|u\| \|v\| \cos(\theta_u - \theta_v) \leq \|u\| \|v\|$$

$$\boxed{\Rightarrow \langle u, V \rangle \leq \|u\| \|v\|}$$

★ Cauchy Schwarz Inequality

Example of classification

Image database:

$$\bar{s}_A = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \qquad \bar{s}_B = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \qquad \bar{s}_C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

4 pixel image

Received image:

$$\bar{r} = \begin{bmatrix} 0.9 \\ 0.9 \\ -1.1 \\ -1.1 \end{bmatrix}$$

$$\bar{e}_A = \bar{r} - \bar{s}_A = \begin{bmatrix} -0.1 \\ -0.1 \\ 0.1 \\ 0.1 \end{bmatrix} \qquad \|\bar{e}_A\|$$

$$\bar{e}_B = \bar{r} - \bar{s}_B = \begin{bmatrix} \\ \end{bmatrix} \qquad \|\bar{e}_B\|$$

$$\bar{e}_C = \bar{r} - \bar{s}_C = \begin{bmatrix} \\ \end{bmatrix} \qquad \|\bar{e}_C\|$$

$\|\bar{e}_A\|$ is minimum

$$\bar{r} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \rightarrow \text{Noiseless}$$

If $\bar{r} = \bar{s}_A$, $\langle \bar{r}, \bar{s}_A \rangle = \langle \bar{s}_A, \bar{s}_A \rangle$

$$= \|\bar{s}_A\|^2$$
$$= 1^2 + 1^2 + (-1)^2 + (-1)^2$$
$$= 4 \rightarrow \text{highest}$$

$$\langle \bar{r}, \bar{s}_B \rangle = (1)(1) + (1)(-1) + (-1)(1) + (-1)(-1)$$
$$= 1 + (-1) + (-1) + 1$$
$$= 0 < 4$$

$$\langle \bar{r}, \bar{s}_C \rangle = 1 + 1 - 1 + 1$$
$$= 2 < 4$$

inner product can be a good metric of similarity

Relation between inner product & error:

Ipython example : Image search

$$\bar{r} = \begin{bmatrix} -1 \\ 1 \\ \vdots \\ 1 \\ -1 \end{bmatrix} \rightarrow \text{40 000 pixels}$$

Vector length = 40000

$$\|\bar{z}_A\| = \left\| \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix} \right\| = \sqrt{1^2 + (-1)^2 + \ldots}$$

$$= \sqrt{40000} = 200$$

$$\|\bar{z}_B\| = 200$$

$$\|\bar{z}_C\| = 200$$

For the best match, inner product of $\bar{r}$ & database image is going to be highest. (close to 40000)