EECS 16A
Module 3
Lecture 3 (6B)
8/4/2020

Topics
Trilateration
Noisy Measurements

- Extra OH: Today/tomorrow 3-4pm
- Regular OH: Friday 1-2pm
- MEME contest !!!

GPS Problem:

What the GPS receiver knows
1) Database signature sequence: $S_1, S_2 \dots$
2) Which satellite sends which signal
3) Position of satellites
4) When signals are transmitted

What the GPS receiver doesn't know
1) What signature signals are present in received signal $\vec{r}$? → Correlation
2) What are the delays → Correlation
3) Its own position - trilateration

GPS satellites: Baseline of 24 satellites

$$\left.\begin{array}{l} \bar{s}_1 = \underbrace{[1\ 1\ -1 \cdots \qquad]}_{1023}^T \\ \vdots \\ \bar{s}_{24} = [1\ -1 \ldots \qquad]^T \end{array}\right\} \text{Gold codes}$$

Properties:

1) $\|\bar{s}_1\| = \|\bar{s}_2\| = \cdots \|\bar{s}_{24}\| = \sqrt{1023}$

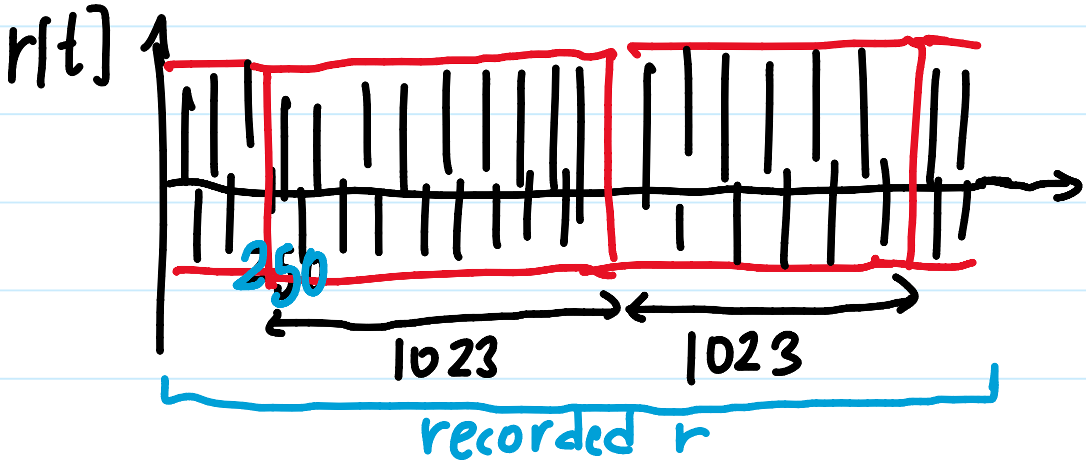
   $\langle \bar{s}_i, \bar{s}_i \rangle \simeq 1023$

2) $\langle \bar{s}_i, \bar{s}_j \rangle \simeq 0 \ll 1023$, if $i \neq j$ by design

   $\mathrm{corr}_{\bar{s}_i}(\bar{s}_j)[k] \simeq 0$ by design

3) $\bar{s}_1, \bar{s}_2, \ldots$ are transmitted periodically

Ex: $r[t] = s_1[t-250]$



Find:

$corr_r(S_1)[k] \rightarrow$ Is the peak close to 1023?

$corr_r(S_2)[k] \rightarrow$ "

⋮

$corr_r(S_{24})[k] \rightarrow$ "

* Ipython demo of GPS prob

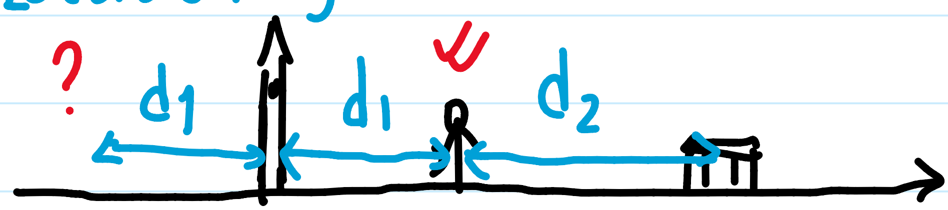
Positioning:

Assume $\bar{s}_1, \bar{s}_2, \bar{s}_3$ are present in $\bar{r}$

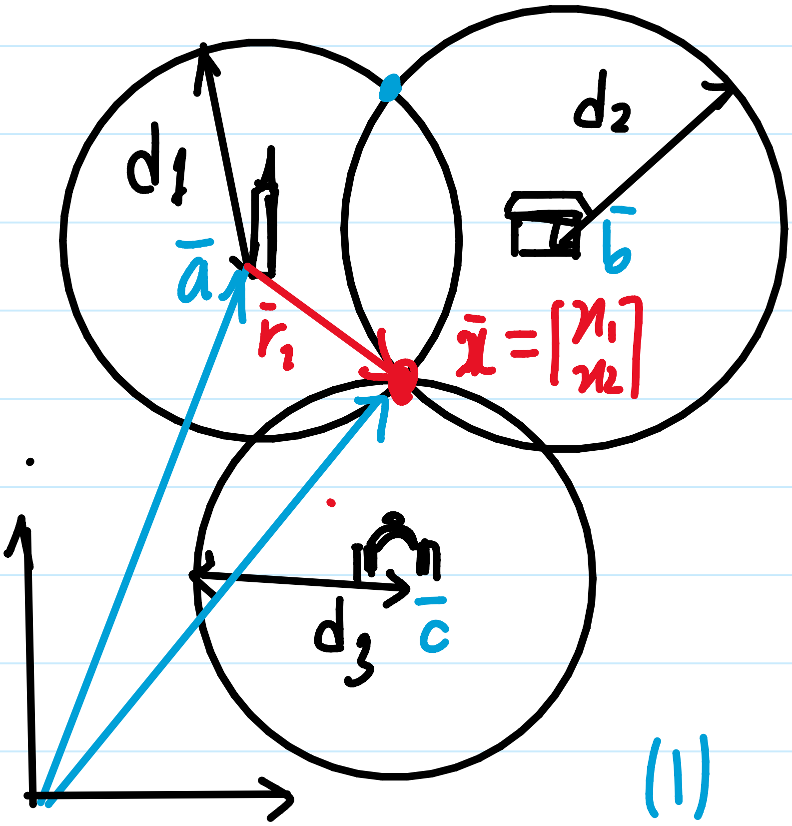
t_{d1}, t_{d2}, t_{d3}

Satellite	Delay	Distance
1	t_{d1}	$d_1 = v t_{d1}$
2	t_{d2}	$d_2 = v t_{d2}$
3	t_{d3}	$d_3 = v t_{d3}$

1D Locationing:



2D Locationing :



Triangle law

$$\bar{a} + \bar{r}_1 = \bar{x}$$

$$\Rightarrow \bar{x} - \bar{a} = \bar{r}_1$$

$$\Rightarrow \|\bar{x} - \bar{a}\| = \|\bar{r}_1\|$$
$$= d_1$$

(1) $\|\bar{x} - \bar{a}\|^2 = d_1^2$

(2) $\|\bar{x} - \bar{b}\|^2 = d_2^2$

(3) $\|\bar{x} - \bar{c}\|^2 = d_3^2$

(1): $\|\bar{x} - \bar{a}\|^2 = d_1^2$

$$\Rightarrow \langle (\bar{x} - \bar{a}), (\bar{x} - \bar{a}) \rangle = d_1^2$$

$$\Rightarrow (\bar{x} - \bar{a})^T (\bar{x} - \bar{a}) = d_1^2$$

$(a+b)^T$
$= a^T + b^T$

$$\Rightarrow (\bar{x}^T - \bar{a}^T)(\bar{x} - \bar{a}) = d_1^2$$

$$\Rightarrow \bar{x}^T\bar{x} + \bar{a}^T\bar{a} - \bar{x}^T\bar{a} - \bar{a}^T\bar{x} = d_1^2$$

$$\Rightarrow \|\bar{x}\|^2 + \|\bar{a}\|^2 - \langle \bar{x}, \bar{a} \rangle - \langle \bar{a}, \bar{x} \rangle = d_1^2$$

$$\Rightarrow \|\bar{x}\|^2 + \|\bar{a}\|^2 - 2\langle \bar{a}, \bar{x} \rangle = d_1^2$$

$$\Rightarrow \underline{\|x\|}^2 + \|a\|^2 - 2\bar{a}^T\bar{x} = d_1^2 \quad (4)$$

$$\underline{\|x\|}^2 + \|b\|^2 - 2\bar{b}^T\bar{x} = d_2^2 \quad (5)$$

$$\underline{\|x\|}^2 + \|c\|^2 - 2c^T\bar{x} = d_3^2 \quad (6)$$

(4)−(5) $\|\bar{a}\|^2 - \|\bar{b}\|^2 - 2\bar{a}^T\bar{x} + 2\bar{b}^T\bar{x} = d_1^2 - d_2^2$

$$(2\bar{a}^T - 2\bar{b}^T)\cdot\bar{x} = \|\bar{a}\|^2 - \|\bar{b}\|^2 - d_1^2 + d_2^2 \quad (A)$$

(4)−(6)

$$(2\bar{a}^T - 2\bar{c}^T)\bar{x} = \|\bar{a}\|^2 - \|\bar{c}\|^2 - d_1^2 + d_3^2 \quad (B)$$

Is $\|a\|^2$ a linear term?

$\bar{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \rightarrow$ known

$\|a\|^2 = a_1^2 + a_2^2 \rightarrow$ numerical value

$\rightarrow$ scalar

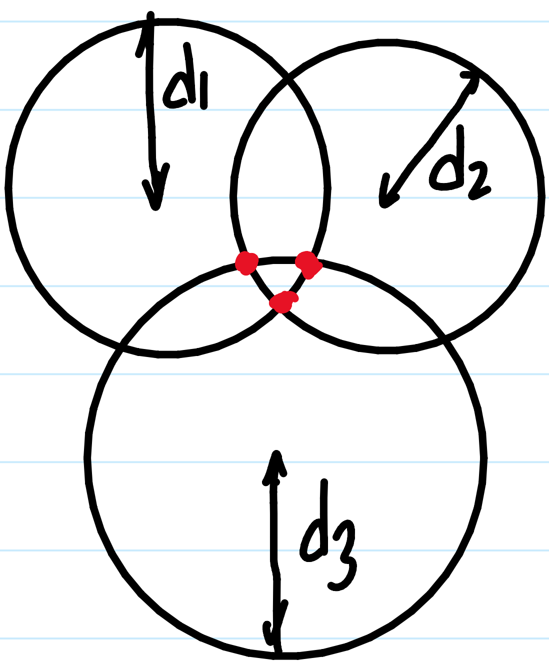
From equations A & B:

$$\begin{bmatrix} 2\bar{a}^T - 2\bar{b}^T \\ 2\bar{a}^T - 2\bar{c}^T \end{bmatrix} \bar{x} = \begin{bmatrix} \|\bar{a}\|^2 - \|b\|^2 - d_1^2 + d_2^2 \\ \|\bar{a}\|^2 - \|c\|^2 - d_1^2 + d_3^2 \end{bmatrix}$$

2 variables, 2 lin indep equations

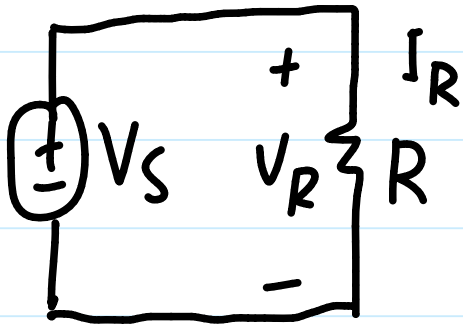
Dis 6C: Mr Muffin
Numerical Example

Noisy measurement: Measurements containing errors



No exact solution
→ Approximate the solution
→ Least squares (Linear regression analysis)

Example:



$V_R = V_S$

$I_R = \frac{V_R}{R}$

$\Rightarrow R = \frac{V_R}{I_R}$

Find R

V_R	I_R
4V	2mA
2V	0.9mA

$\rightarrow R = \frac{4}{2m} = 2k\Omega$

$\rightarrow R = \frac{2}{0.9} = 2.2k\Omega$

$\bar{i}_R R = \bar{v}_R$, where $\bar{v}_R = \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \bar{b}$ $\quad \bar{i}_R = \begin{bmatrix} 2m \\ 0.9m \end{bmatrix} = A$

$A\bar{x} = \bar{b}$

Gaussian elimination:

$$\begin{bmatrix} 2m \\ 0.9m \end{bmatrix} R = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} R = \begin{bmatrix} 2k \\ 1 \end{bmatrix}$$

$\rightarrow$ Inconsistent

No solution

Inconsistent system : $A\bar{x} = \bar{b}$

$$A\bar{x} = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$= x_1\bar{a}_1 + \bar{x}_2\bar{a}_2 \ \cdots$

$=$ linear combination of $\bar{a}_1, \bar{a}_2 \cdots \bar{a}_n$

$\in$ columnspace(A)

$A\bar{x} \in C(A)$

If $\bar{b} \notin C(A) \rightarrow$ No solution

For an inconsistent system : $A\bar{x} \neq \bar{b}$

$\Rightarrow A\bar{x} + \bar{e} = \bar{b}$

$\hookrightarrow$ error vector

$\star$ $A\bar{x} = \bar{b}_0$, where $\bar{b}_0 \in C(A)$

Find $\bar{b}_0$ so that $A\bar{x} = \bar{b}_0$ is consistent.