EECS 16A Designing Information Devices and Systems I
Summer 2022 Homework 6

This homework is due Friday, August 5, 2022, at 23:59.
Self-grades are due Monday, August 8, 2022, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw5.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.
  If you do not attach a PDF "printout" of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

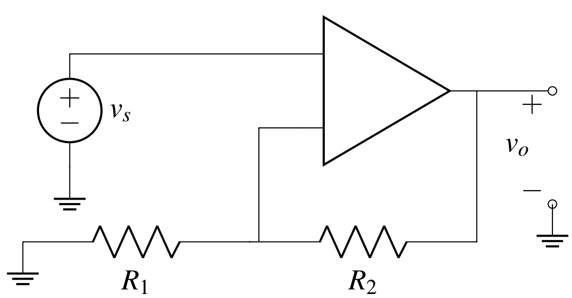
Submit the file to the appropriate assignment on Gradescope.

1. Reading Assignment

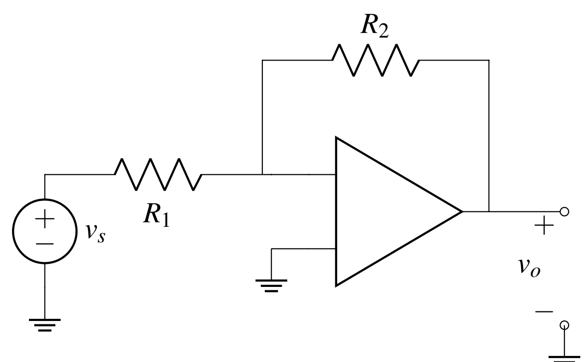
For this homework, please review and read Note 11A/B, which introduces the basics of circuit analysis and node voltage analysis. You are always welcome and encouraged to read beyond this as well.

2. Basic Amplifier Building Blocks

The following amplifier stages are used often in many circuits and are well known as (a) the non-inverting amplifier and (b) the inverting amplifier.



(a)



(b)

(a) Label the input terminals of the op-amp with $(+)$ and $(-)$ signs in Figure (a), so that it is in negative feedback. Then derive the voltage gain ($G = \frac{v_o}{v_s}$) of the non-inverting amplifier in Figure (a) using the Golden Rules. Why do you think this circuit is called a non-inverting amplifier?

(b) Label the input terminals of the op-amp with $(+)$ and $(-)$ signs in Figure (b), so that it is in negative feedback. Then derive the voltage gain ($G = \frac{v_o}{v_s}$) of the inverting amplifier using the Golden Rules. Can you explain why this circuit is called an inverting amplifier?

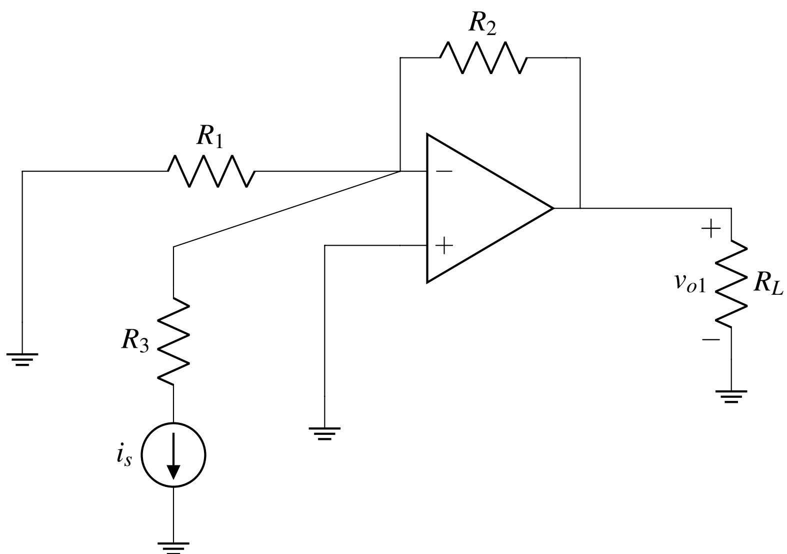
(c) Using your toolkit of circuit topologies, design blocks that implement the following equations. Feel free to reference Discussion 5C for circuit topologies:

i. $v_o = 2v_s$

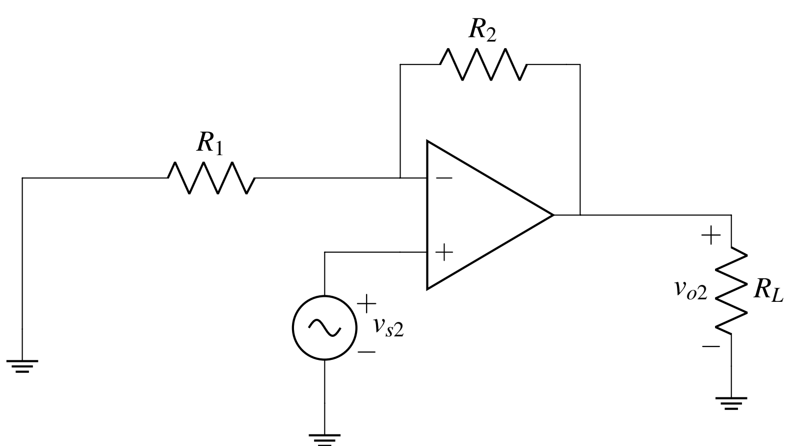
ii. $v_o = -3v_s + 8$

3. Amplifier with Multiple Inputs

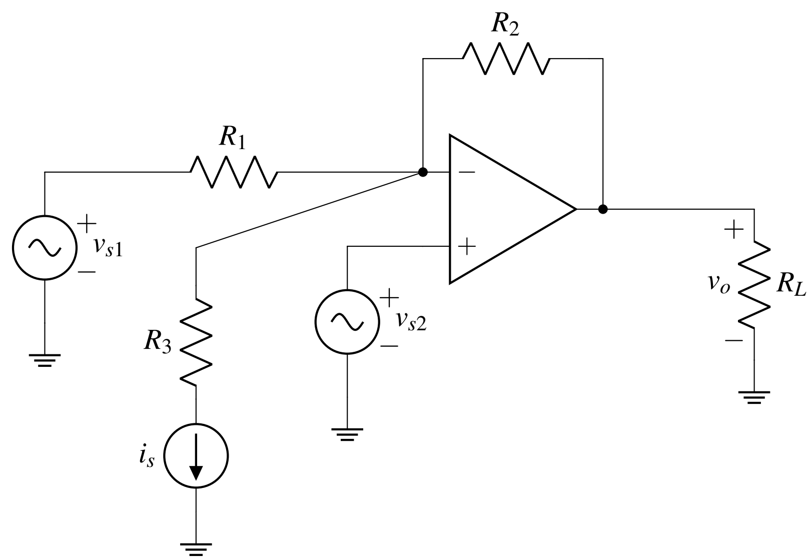
(a) Use the Golden Rules to find v_{o1} for the circuit below.



(b) Use the Golden Rules to find v_{o2} for the circuit below.

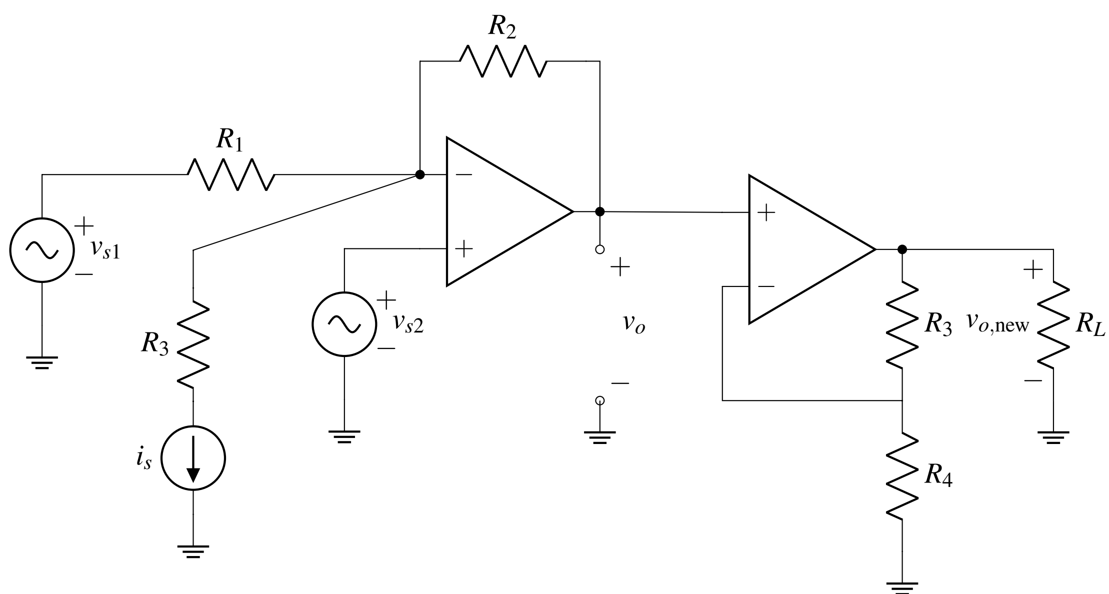


(c) Use the Golden Rules to find the output voltage v_o for the circuit shown below.



(d) Use superposition and the answers to the first few parts of this problem to verify your answer to part c. *Hint: See if you can generate some combination of the circuits in a & b that is equivalent to the one in c.*

(e) Now add a second stage as shown below. What is $v_{o,\text{new}}$? Does v_o change between part (c) and this part? Does the voltage $v_{o,\text{new}}$ depend on R_L?



4. Cool For The Summer

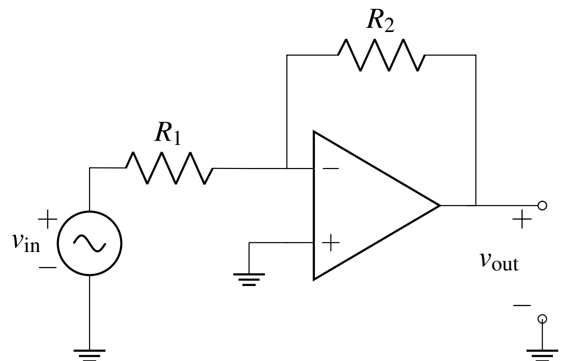
You and a friend want to make a box that helps control an air conditioning unit based on both your inputs. You both have individual dials which you can use to control the voltage. An input of 0 V means that you want to leave the temperature as is. A **negative voltage input** means that you want to **reduce** the temperature. (It's hot out, so we will assume that you never want to increase the temperature – so no, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds only to **positive voltages**. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off. You also need a system that **sums up** both you and your friend's control inputs.

Therefore, you need a box that acts as an **an inverting summer** – *it outputs a weighted sum of two voltages where the weights are both negative*. The sum is weighted because one room is bigger, so you need to compensate for this.

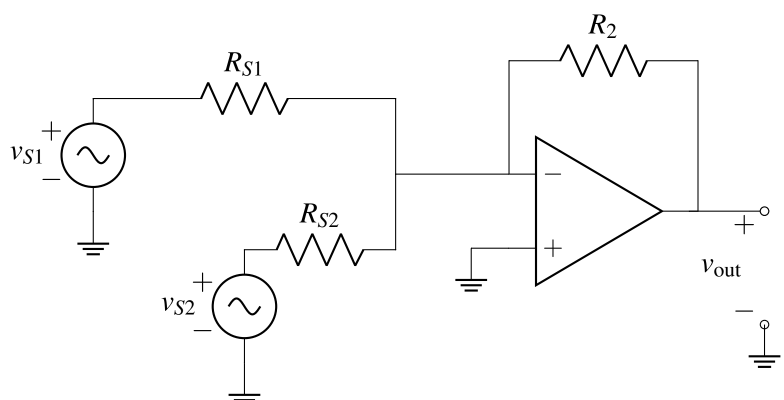
(a) As a first step, derive v_{out} in terms of R_2, R_1, and v_{in}.

Hint: Have you solved for this particular amplifier configuration before? You can use your answer from the time you did this earlier.



(b) Now we will add a second input to this circuit as shown below. Find v_{out} in terms of v_{S1}, v_{S2}, R_{S1}, R_{S2} and R_2.

Hint: You can solve this problem using either superposition or our tried-and-true KCL analysis.



(c) Let's suppose that you want $v_{\text{out}} = -\left(\frac{1}{4}v_{S1} + 2v_{S2}\right)$ where again v_{S1} and v_{S2} represent the input voltages from you and your friend's control knobs. Select resistor values such that the circuit from part (b) implements this desired relationship.

(d) Suppose that you have a new AC unit that you want to use with your original control inputs v_{S1} and v_{S2}. **This unit, however, responds only to negative voltages – the opposite of your previous air conditioning unit, which only responded to positive input voltages.** The higher the magnitude of the negative voltage, the stronger the AC runs.

You want to modify your prior circuit for the new AC unit. Your circuit takes in two control voltages and outputs a weighted sum, but the sum should now become more negative as you increase your input voltages.

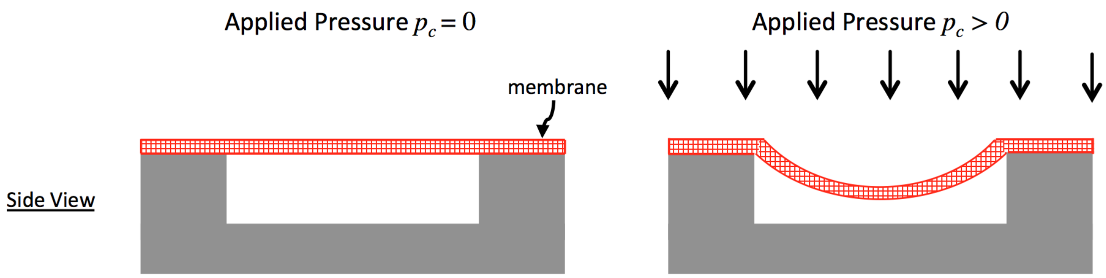
Hint: Consider adding another op-amp circuit to the output of your circuit from part (b), such that you invert the output of the op-amp circuit of part (b) ***without*** *adding additional gain.*

5. Putting on the Pressure: Build Your Own InstantPot

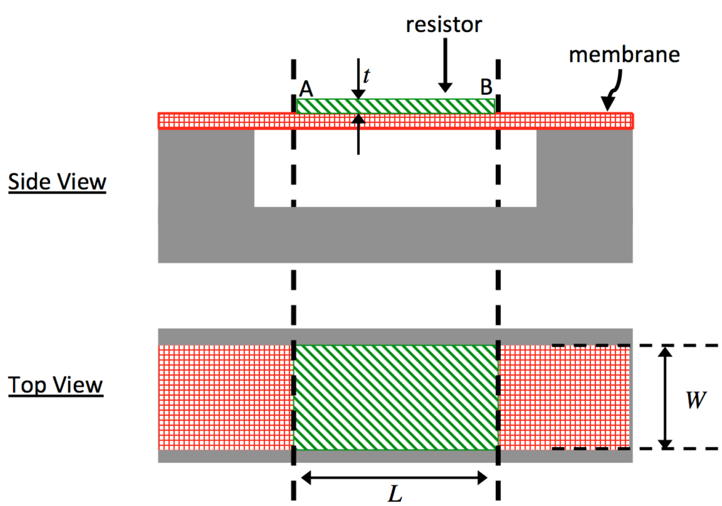
Prof. Arias had a great experience with her automatic pressure cooker, so she was inspired to try and build her own. She's enlisting your help! The design of the pressure cooker uses a pressure sensor and a heating element. Whenever the pressure is below a set target value, an electronic circuit turns on the heating element.

Pressure Sensor Resistance

The first step is designing a pressure sensor. The figure below shows your design. As pressure p_c is applied, the flexible membrane stretches.



(a) You attach a resistor layer R_p with resistivity $\rho = 0.1\Omega$m, width W, length L, and thickness t to the pressure sensor membrane, as illustrated in the figure below. When the pressure $p_c = 0\,\text{Pa}$ (i.e. there is no applied pressure), $W = 1\,\text{mm}$, $L = L_0 = 1\,\text{cm}$, $t = 100\,\mu\text{m} = 100 \times 10^{-6}$ m.
R_{p0} is the value of R_p when there is no applied pressure. Calculate R_{p0}. Note that direction of current flow in the resistor is from A to B as marked in the diagram.



(b) When pressure is applied, the length of the resistor L changes from L_0 and is a function of applied pressure p_c, and is given by

$$L = L_0 + \beta p_c,$$

where L_0 is the nominal length of the resistor with no pressure applied, and β is a constant with units m/Pa. As a result of the length change, the value of resistance R_p also changes from its nominal value R_{p0} (the value of R_p with no pressure applied).

Derive an expression for R_p as a function of resistivity ρ, width W, thickness t, nominal length L_0, constant β, and applied pressure p_c, when pressure is applied.

Note: The width and thickness of the resistor will also change with applied pressure. However, we ignore this to keep the math simple.

(c) **Pressure Sensor Circuit Design**

For this sub-part and the following sub-parts, we will use a new model for pressure-sensitive resistance R_p. Assume that the resistance R_p is a function of applied pressure p_c according to the relationship $R_p = R_o \times \frac{p_c}{p_{\text{ref}}}$, where $R_o = 1\text{k}\Omega$, and $p_{\text{ref}} = 100\text{kPa}$.

To complete our sensor circuit, we would like to generate a voltage V_p that is a function of the pressure p_c.

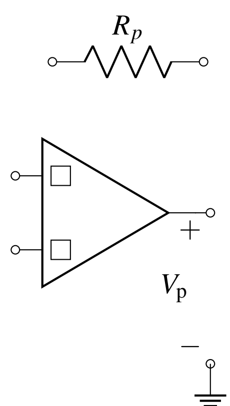
Complete the circuit below so that the output voltage V_p depends on the pressure p_c as:

$$V_p = -V_o \times \frac{p_c}{p_{\text{ref}}}, \text{ where } V_o = 1\,\text{V}.$$

Restrictions on your pressure sensor circuit design are as follows:

- **You may add at most one ideal voltage source and one additional resistor besides R_p to the circuit, but you must calculate their values and mark them in the diagram.**
- **Mark the positive and negative inputs of the operational amplifier with "+" and "-" symbols, respectively, in the boxes provided.**
- **Assume op-amp supply voltages V_{DD} and $V_{SS} = -V_{DD}$ are already provided.**

You may assume that the operational amplifier is ideal.



(d) **Resistive Heating Element**

To heat the pressure cooker, you use a heating element with resistance R_{heat}. Calculate the value of R_{heat} such that the power dissipated is $P_{\text{heat}} = 1000\,\text{W}$ with $V_{\text{heat}} = 100\,\text{V}$ applied across the heating element.

(e) **Pressure Regulation**

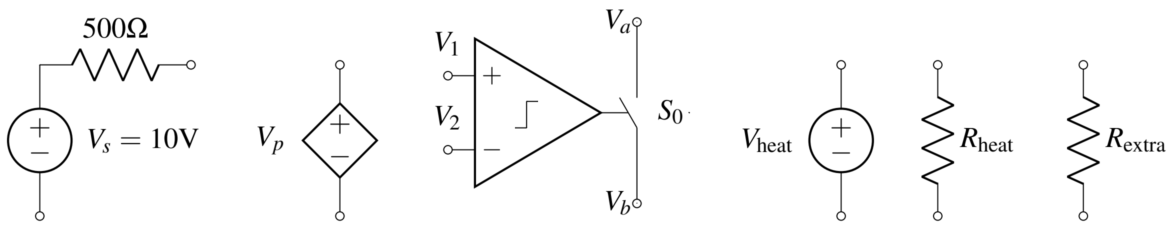
You are finally ready to complete the design of your pressure cooker.

Using all of the circuit elements below, make a circuit that will turn the heater on (i.e. will cause a current to flow through R_{heat}) when the pressure is less than 500 kPa, and off (i.e. will cause no current to flow through R_{heat}) when the pressure is greater than 500 kPa.

The elements are:

- A voltage source $V_s = 10\text{V}$ in series with a resistance of 500Ω.

- A dependent voltage source $V_p = V_o \times \frac{p_c}{p_{\text{ref}}}$, with V_o = 1V and $p_{\text{ref}} = 100\text{kPa}$. (This is a voltage source whose voltage is a function of pressure p_c, unrelated to any previous parts of the question.)
- A comparator that controls switch S_0. The switch is normally opened (i.e. an open circuit between nodes V_a and V_b), and is closed only when $V_1 > V_2$ (i.e. a short circuit between nodes V_a and V_b).
- The heater supply ($V_{\text{heat}} = 100\text{V}$).
- The heater resistor R_{heat}.
- One additional resistor R_{extra} that can have **any value**.
- Assume comparator supply voltages V_{DD} and $V_{SS} = -V_{DD}$ are already provided.

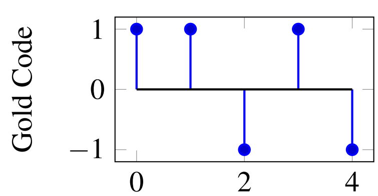


6. ***Gold*en Positioning System**

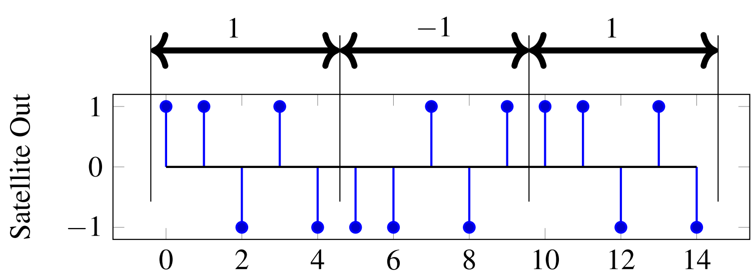
Learning Goal: *This problem is meant to help practice computing cross-correlation. It also covers the concept of trilateration.*

In this problem we will explore how real GPS systems work, and touch on a few aspects of implementing GPS receivers.

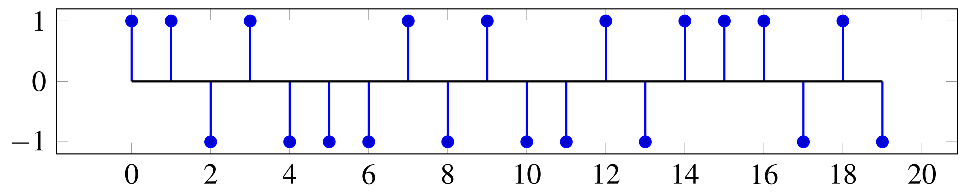
A Gold code is a sequence of 1's and -1's that has a high autocorrelation at a shift of 0, and small autocorrelations otherwise. Every GPS satellite has a unique Gold code assigned to it, and users are aware of the Gold code used by each satellite. The plot below shows a Gold code of length 5.



Each GPS satellite has a message that it transmits by modulating the Gold code. When the satellite is transmitting a 1, it sends just the Gold code sequence. When the satellite is transmitting a -1, it sends -1 times the Gold code. For example, if a satellite were transmitting the message [1, -1, 1], it would transmit the following:



(a) Suppose you receive the following from a GPS satellite that has the same Gold code as above. **What message is the satellite transmitting?**



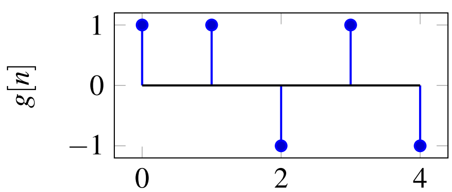
(b) In order to find the message being sent by the satellite, the receiver will find the linear cross-correlation of the received signal with a replica of the satellite Gold code.

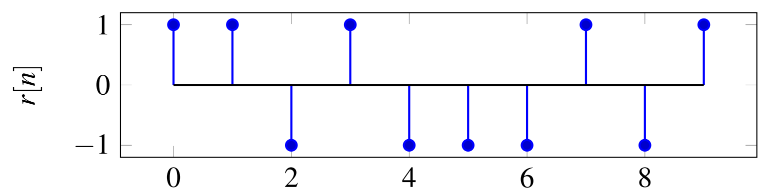
We need to find the **linear cross-correlation** of the signals shown below given by

$$\text{corr}_{\vec{r}}(\vec{g})[k] = \sum_{i=-\infty}^{\infty} r[i]g[i-k]$$

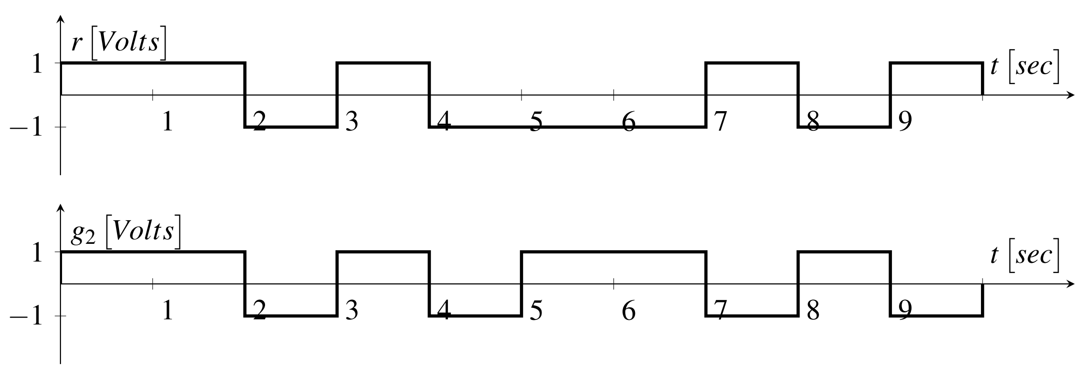
where $r[n]$ is the received signal and $g[n]$ is the Gold code sequence. *Note that neither of these signals is periodic in this part.*

Plot the values of corr$_{\vec{r}}(\vec{g})[k]$ for $-1 \leq k \leq 7$. What is the significance of the peaks in the linear cross-correlation?





(c) Real GPS receivers have specialized hardware to perform cross-correlation using circuits. However, since these transmissions are continuous signals instead of discrete values, we will model the received signal $r(t)$ and the Gold code signal $g_2(t)$ as square waves, as shown in the plot below. Notice that $g_2(t)$ shows two periods of the Gold code.



An essential hardware block to implementing a GPS correlator is *Multiply and Integrate*. The Multiply and Integrate block takes in two inputs, then integrates the product of the two inputs over time. For example, the output of the *Multiply and Integrate* block given the above two inputs would be:

$$y(t) = \int_0^t r(\tau)g_2(\tau)d\tau$$

where $y(t)$ is the circuit output at time t. **Draw $y(t)$ as a function of time, for $t = 0$ to $t = 10$ sec.**

(d) Receivers also need to use the received data to calculate the position of the satellite. Each receiver will receive data from k satellites. Each satellite transmits the time, S_i, at which it started sending the message, where i is the index of the satellite, and $1 \leq i \leq k$. The receiver knows the time, T_i, at which each message arrives. You may assume the receiver and transmitter clocks are synchronized perfectly. Let c represent the speed of the signal.

Find an expression for d_i, the distance between the receiver and the i^{th} satellite, in terms of S_i, T_i, and other relevant parameters.

(e) Each satellite's position in 3D space is (u_i, v_i, w_i), where $1 \leq i \leq k$. The receiver position is given by (x, y, z). We need a linear system of equations the receiver can use to solve for its position, $\vec{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

Due to limitations of the hardware, the receiver can only handle **linear** systems of equations. **How many satellites must the receiver get data from to solve for the receivers position?**

7. Audio File Matching

Learning Goal: *This problem motivates the application of correlation for pattern matching applications such as Shazam.*

Many audio processing applications rely on representing audio files as vectors, referred to as audio *signals*. Every component of the vector determines the sound we hear at a given time. We can use inner products to determine if a particular audio clip is part of a longer song, similar to an application like *Shazam*.

Let us consider a very simplified model for an audio signal, $\vec{x}$. At each timestep k, the audio signal can be either $x[k] = -1$ or $x[k] = 1$.

(a) Say we want to compare two audio files of the same length N to decide how similar they are. First, consider two vectors that are exactly identical, namely $\vec{x}_1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ and $\vec{x}_2 = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$. What is the inner product of these two vectors? What if $\vec{x}_1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ but $\vec{x}_2$ oscillates between 1 and -1? Assume that N, the length of the two vectors, is an even number.

Use this to suggest a method for comparing the similarity between a generic pair of length-N vectors.

(b) Next, suppose we want to find a short audio clip in a longer one. We might want to do this for an application like *Shazam*, which is able to identify a song from a short clip. Consider the vector of length 8, $\vec{x} = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \end{bmatrix}^T$.

We want to find the short segment $\vec{y} := \begin{bmatrix} y[0] & y[1] & y[2] \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}^T$ in the longer vector. To do this, perform the linear cross correlation between these two finite length sequences and identify at what shift(s) the linear cross correlation is maximized. Apply the same technique to identify what shift(s) gives the best match for $\vec{y} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$.

(If you wish, you may use iPython to do this part of the question, but you do not have to.)

(c) Now suppose our audio vector is represented using integers beyond simply just 1 and -1. Find the short audio clip $\vec{y} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$ in the song given by $\vec{x} = \begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 2 & 3 & 10 \end{bmatrix}^T$. Where do you expect to see the peak in the correlation of the two signals? Is the peak where you want it to be, i.e. does it pull out the clip of the song that you intended? Why?

(If you wish, you may use iPython to do this part of the question, but you do not have to.)

(d) Let us think about how to get around the issue in the previous part. We applied cross-correlation to compare segments of $\vec{x}$ of length 3 (which is the length of $\vec{y}$) with $\vec{y}$. Instead of directly taking the cross correlation, we want to normalize each inner product computed at each shift by the magnitudes of both segments, i.e. we want to consider the inner product $< \frac{\vec{x}_k}{\|\vec{x}_k\|}, \frac{\vec{y}}{\|\vec{y}\|} >$, where $\vec{x}_k$ is the length 3 segment starting from the k-th index of $\vec{x}$. This is referred to as normalized cross correlation. Using this procedure, now which segment matches the short audio clip best?

(e) We can use this on a more 'realistic' audio signal – refer to the IPython notebook, where we use normalized cross-correlation on a real song. Run the cells to listen to the song we are searching through, and add a simple comparison function `vector_compare` to find where in the song the clip comes from. Running this may take a couple minutes on your machine, but note that this computation can be highly optimized and run super fast in the real world! Also note that this is not exactly how Shazam works, but it draws heavily on some of these basic ideas.

8. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.