Lecture 7A

Agenda:

→ Energy stored in capacitors

→ NVA Hack

→ Steady State analysis for capacitors

→ Thevenin/Norton Equivalence

Energy Stored in a Capacitor

$$\boxed{P = IV = \frac{dE}{dt}}$$

$$\frac{dE}{dt} = I \cdot V \Rightarrow dE = V\underline{I\,dt}$$

$$\boxed{I = C\frac{dV}{dt}}$$

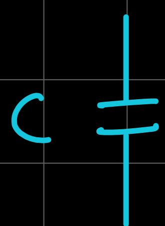
$$I\,dt = C\,dV$$

$$dE = V \cdot C\,dV = \underline{C\,V\,dV}$$

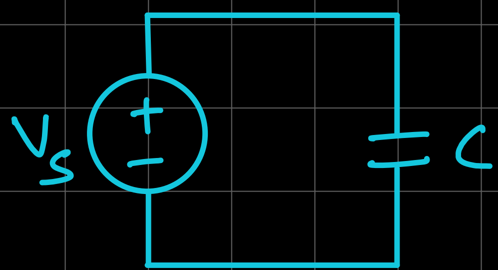
$$\int dE = \int C\,V\,dV \Rightarrow E = C\left(\frac{V^2}{2}\right)$$

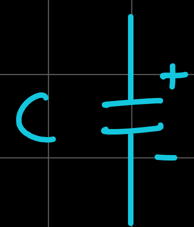
$$(Q = CV)$$

$$E = \frac{1}{2}CV^2 = \frac{C^2V^2}{2C} = \frac{Q^2}{2C}$$



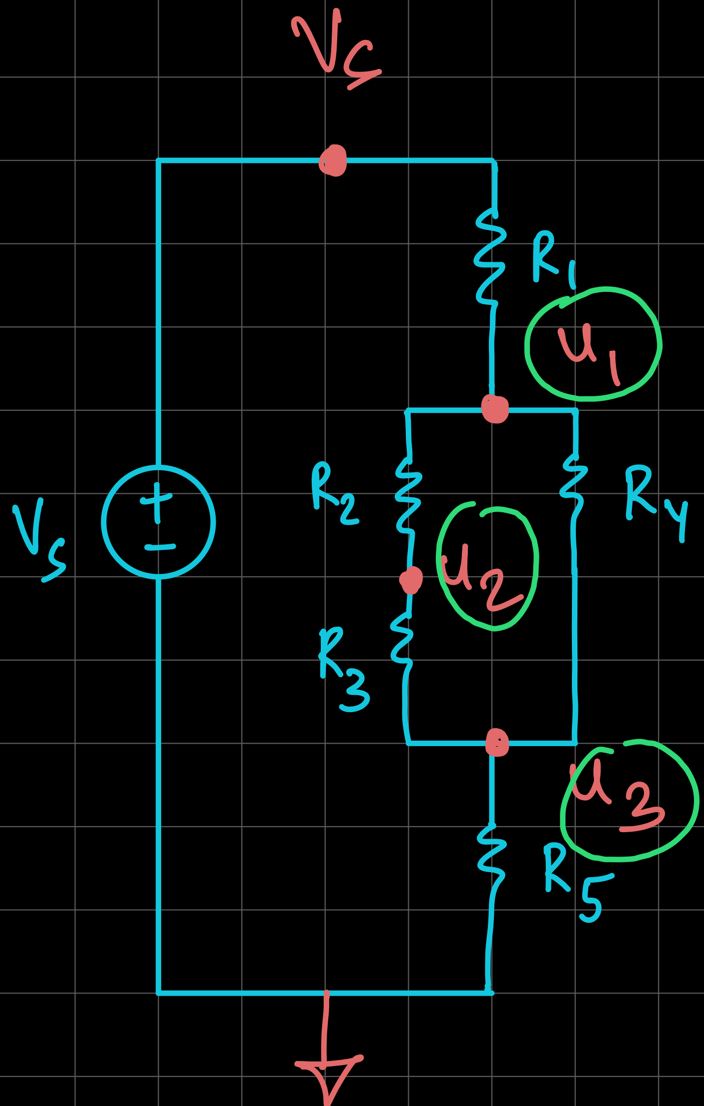
$Q = 0$





$$\frac{1}{2}C\,V_s^2$$

NVA Hack



$$\underbrace{\frac{u_1}{R_1}+\frac{u_1}{R_2}+\frac{u_1}{R_4}} - \frac{u_2}{R_2} - \frac{u_3}{R_4} = \boxed{\frac{V_s}{R_1}}$$

At u_1: $$\boxed{\frac{u_1 - V_s}{R_1} + \frac{u_1 - u_2}{R_2} + \frac{u_1 - u_3}{R_4} = 0}$$

u_2: $$\frac{u_2 - u_1}{R_2} + \frac{u_2 - u_3}{R_3} = 0$$

u_3: $$\frac{u_3 - u_2}{R_3} + \frac{u_3 - 0}{R_5} + \frac{u_3 - u_1}{R_4} = 0$$

$$\underbrace{\frac{u_1}{R_1}+\frac{u_1}{R_2}}+\frac{u_1}{R_4}-\underbrace{\frac{u_2}{R_2}}-\underbrace{\frac{u_3}{R_4}}=\boxed{\frac{V_s}{R_1}}$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)u_1-\frac{1}{R_2}u_2-\frac{1}{R_4}u_3=\frac{V_s}{R_1}$$

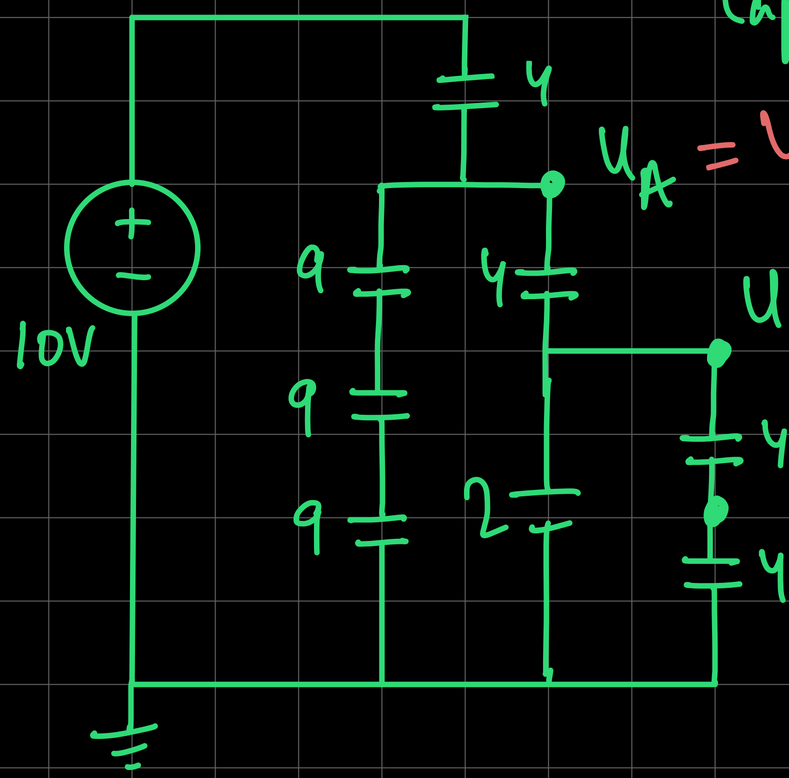
$$\left[\begin{array}{ccc|c} \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right) & -\frac{1}{R_2} & -\frac{1}{R_4} & \frac{V_s}{R_1} \\ \cdots & \cdots & \cdots & \cdots \end{array}\right]$$

Capacitors (Steady State)

$$\boxed{V = \frac{Q}{C}}$$

u_A, u_B, u_C

Capacitors were initially uncharged

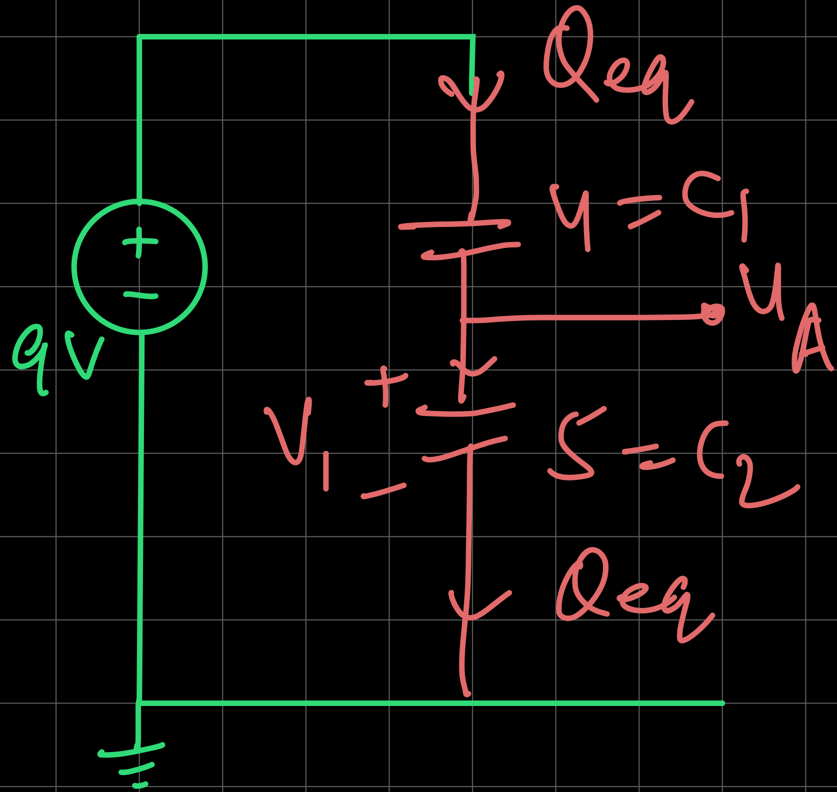


$u_A = 4$

$u_B = 2V$

$u_C = 1V$

$Q = 8C$



$$Q_{eq} = CV = \frac{20}{9} \times 9 = \boxed{20}$$

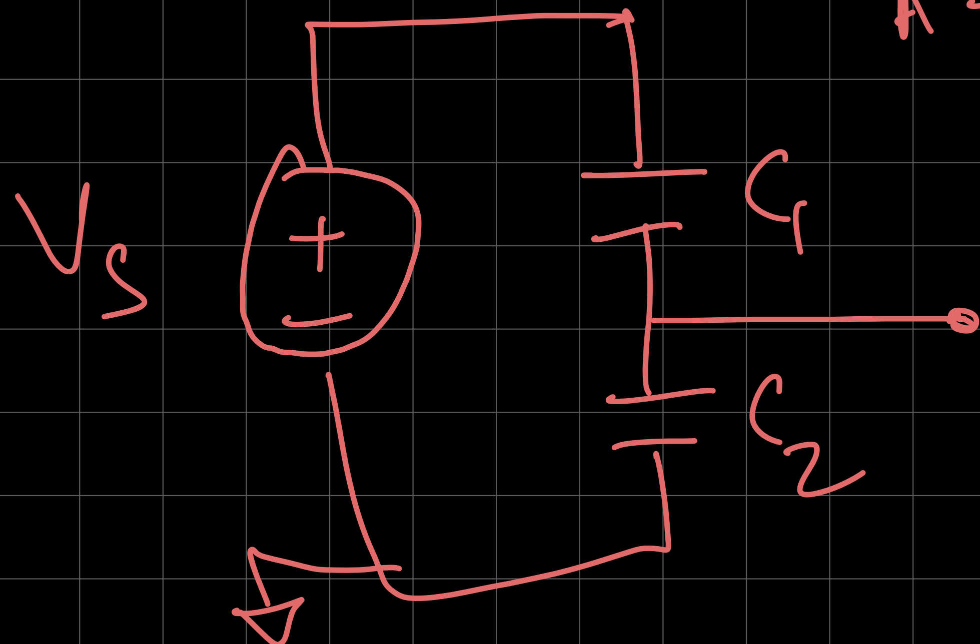
$$Q_{eq} = 20 = 5V_1$$

$$V_1 = 4$$

$$u_A - 0 = 4$$

$$\boxed{u_A = 4}$$

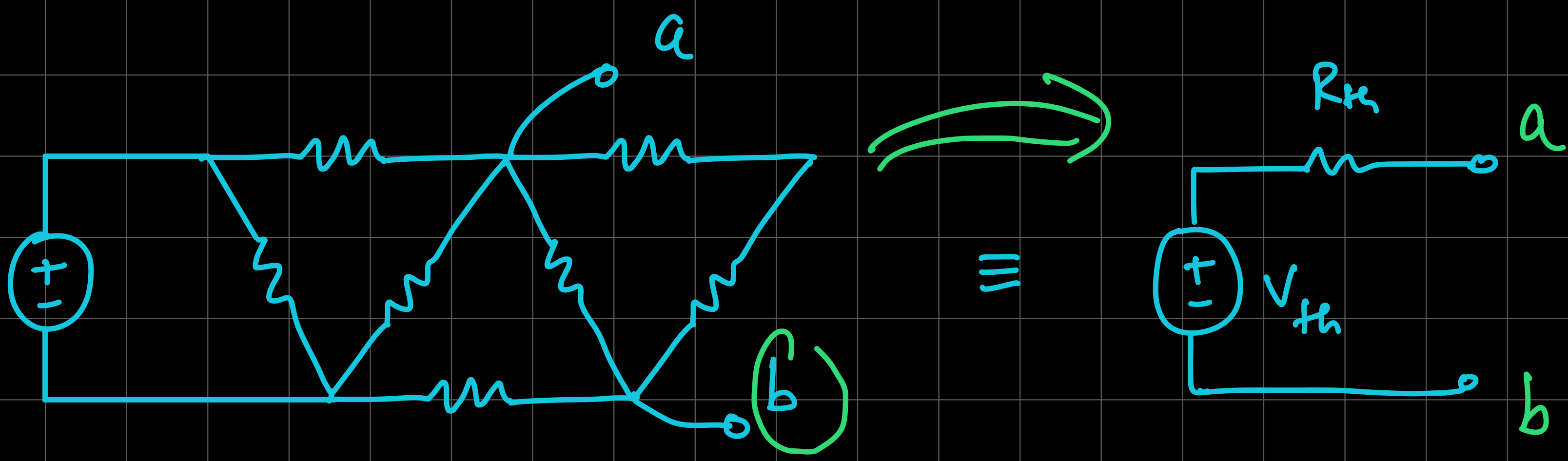
$$u_A = \frac{4}{4+5} \times 9 = 4V$$



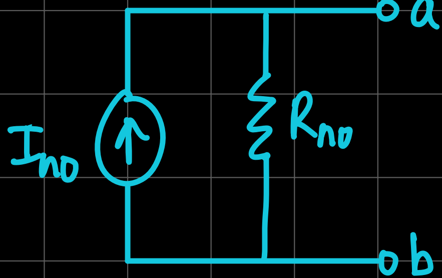
$$\boxed{u_{mid} = \frac{C_1}{C_1 + C_2} V_s}$$

Thevenin/Norton Equivalence

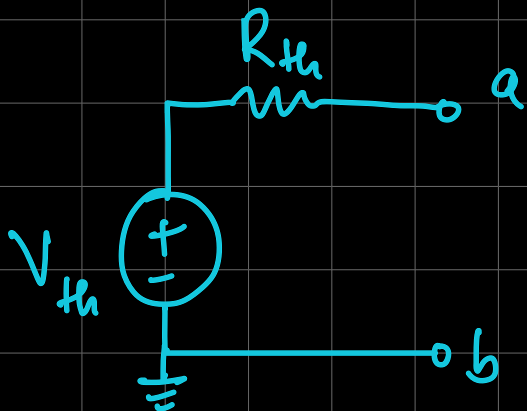
Any linear electrical network containing only Voltage sources, current sources, and resistors can be replaced at terminals A-B by a single voltage source (V_{th}) and resistance (R_{th}).

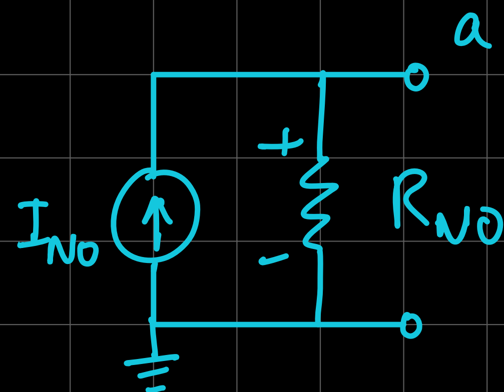


Norton Equivalent



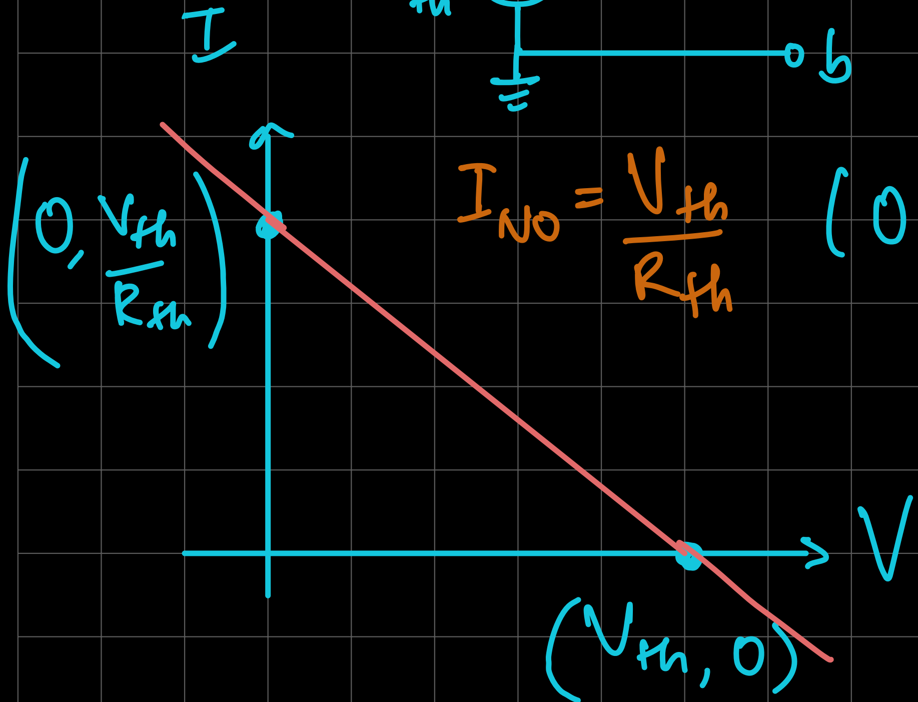
Let's see how these are equivalent →



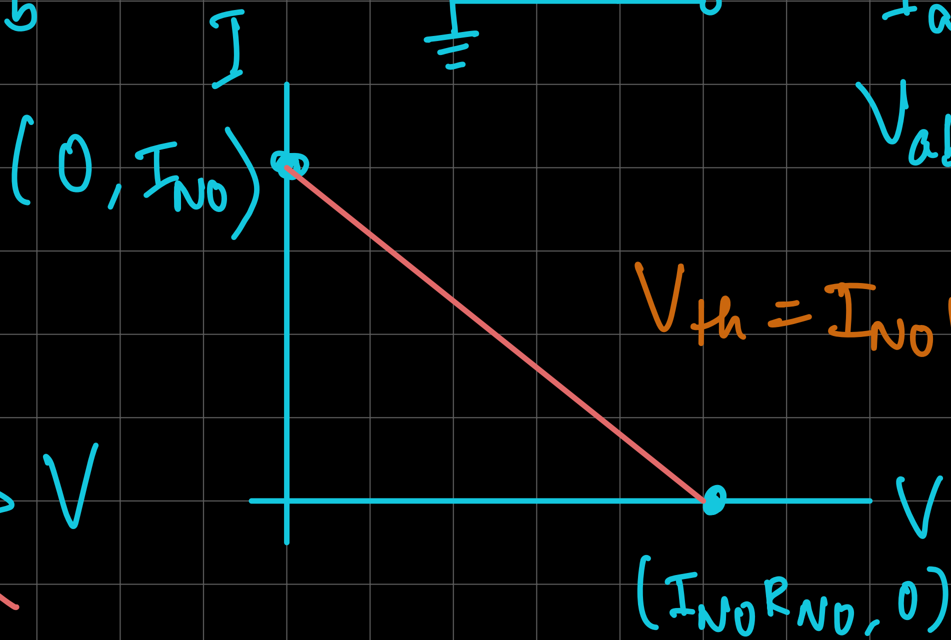


$I_{ab} = 0$

$V_{ab} = I_{NO} R_{NO}$



$I_{NO} = \frac{V_{th}}{R_{th}}$



$V_{th} = I_{NO} R_{NO}$

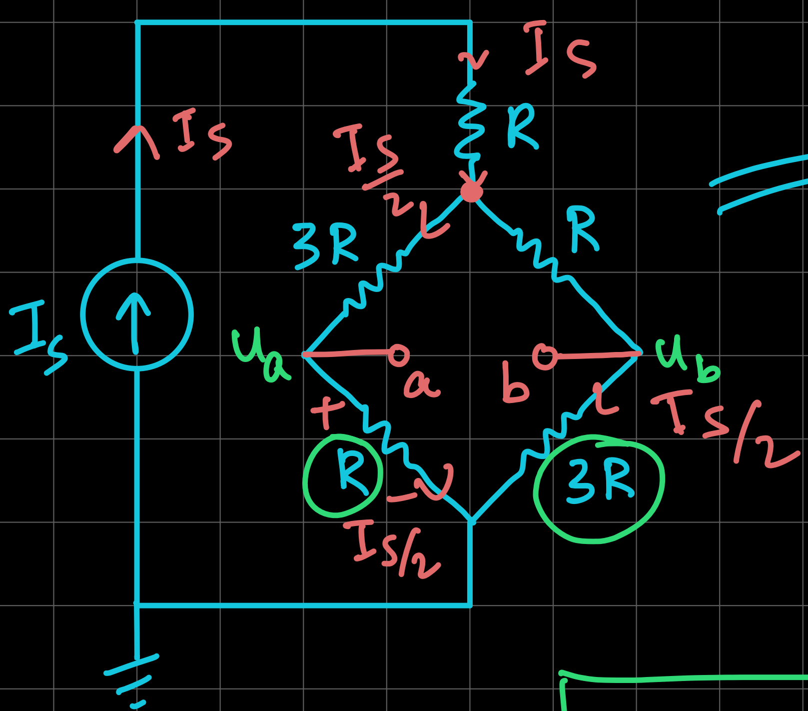
$$\boxed{I_{NO} = \frac{V_{th}}{R_{th}}}$$

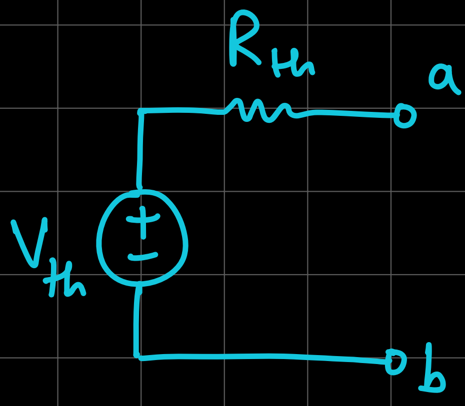
$$V_{th} = I_{NO} R_{NO}$$

$$\cancel{V_{th}} = \frac{\cancel{V_{th}}}{R_{th}} R_{NO}$$

$$\boxed{R_{NO} = R_{th}}$$

Example

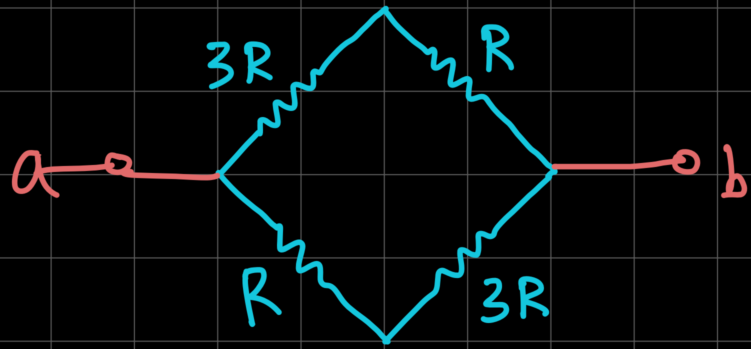




$$V_R = u_A - 0 = \frac{I_s R}{2} = \frac{1}{2} I_s R$$

$$V_R = u_b - 0 = \frac{I_s}{2}(3R) = \frac{3}{2} I_s R$$

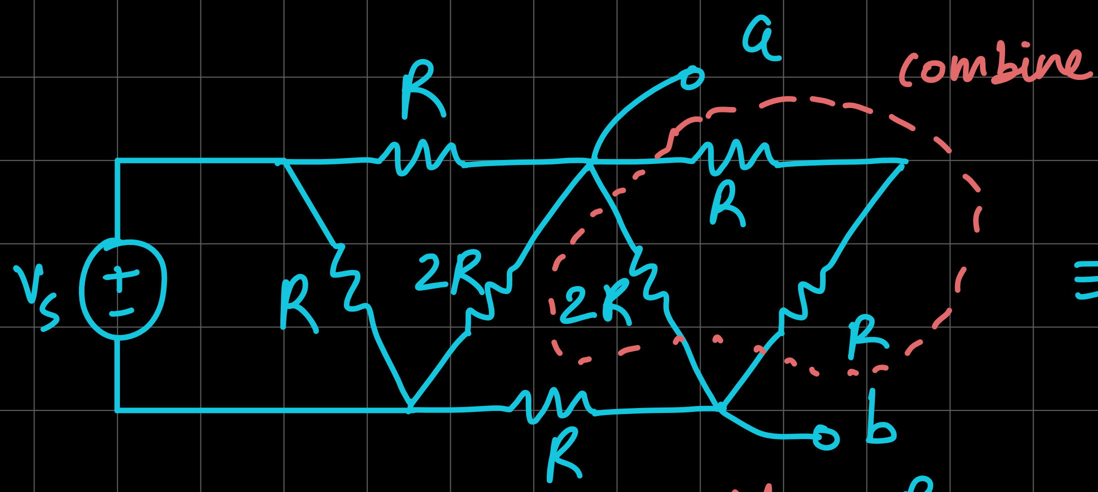
$$\boxed{u_A - u_b = (-I_s R) = V_{th}} \Rightarrow \text{open circuit voltage}$$



$$R_{ab} = (3R+R) \parallel (3R+R)$$

$$= 4R \parallel 4R = \underline{\underline{2R}}.$$

Additional Questions during Lecture



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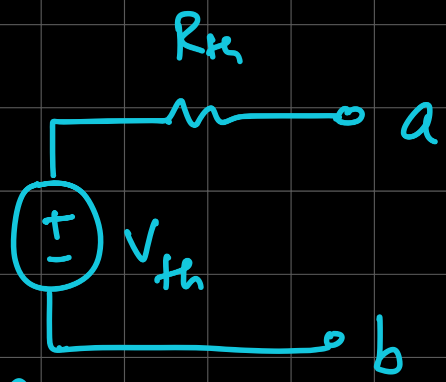
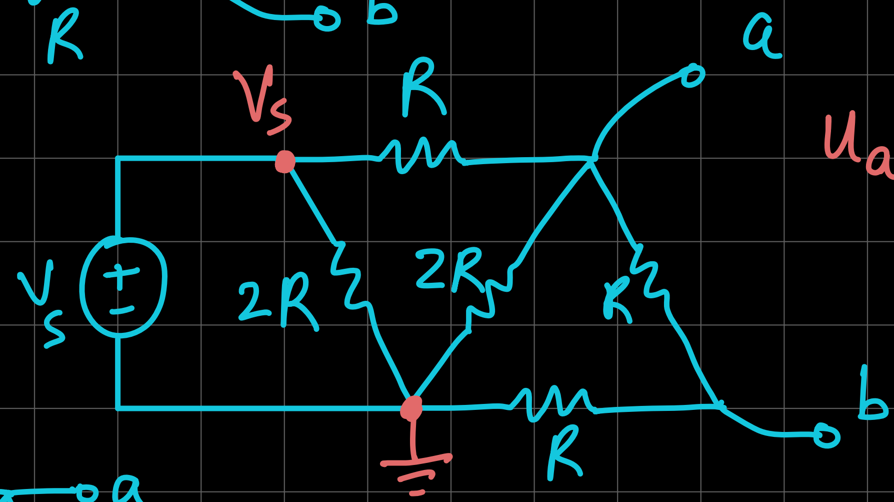
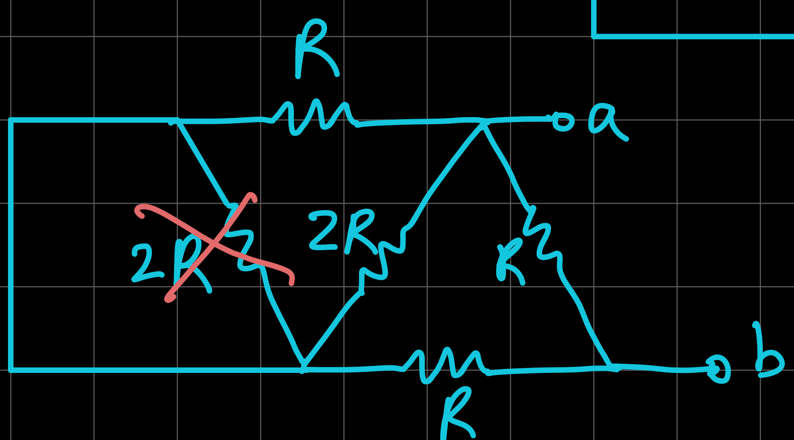


Finding V_{th} →

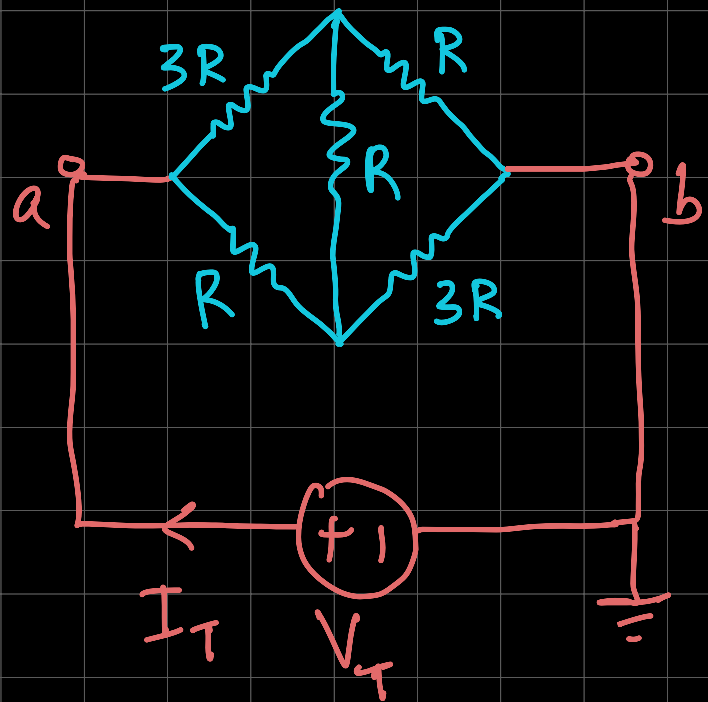


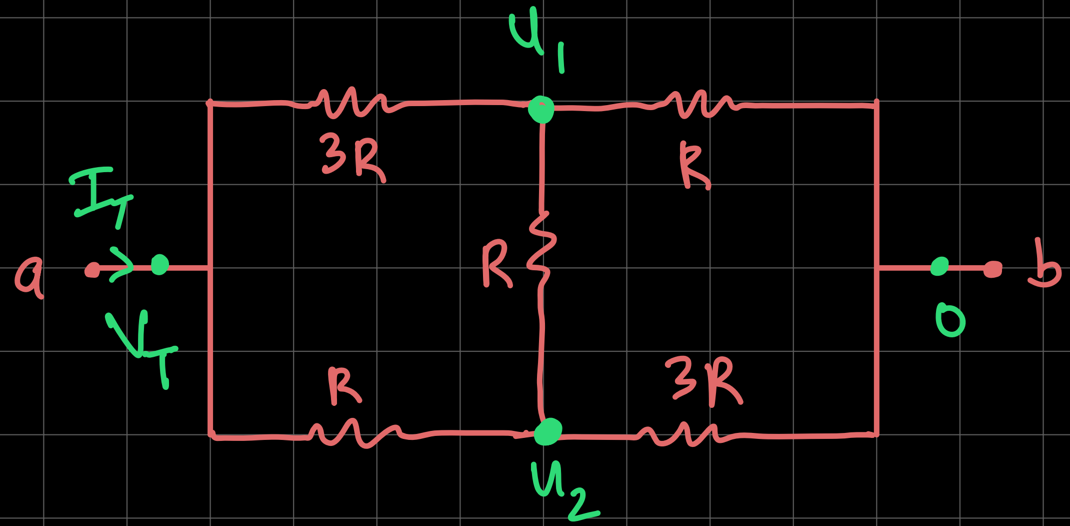
$u_a = \frac{V_s}{2}$ $\quad$ $u_b = \frac{u_a}{2} = \frac{V_s}{4}$

$V_{ab} = V_{th} = \frac{V_s}{4}$



$R_{ab} = R \,\|\, (R + R \| 2R) = \frac{5R}{8}$





$$u_1: \quad \frac{u_1 - V_T}{3R} + \frac{u_1 - u_2}{R} + \frac{u_1 - 0}{R} = 0$$

$$u_2: \quad \frac{u_2 - u_1}{R} + \frac{u_2 - 0}{3R} + \frac{u_2 - V_T}{R} = 0$$

$$\frac{7}{3R} u_1 - \frac{u_2}{R} = \frac{V_T}{3R} \quad — (1)$$

$$\frac{7}{3R} u_2 - \frac{u_1}{R} = \frac{V_T}{R} \quad — (2)$$

$$(1) + \frac{7}{3}(2) \Rightarrow \frac{40}{9}\frac{u_2}{R} = \frac{8}{3}\frac{V_T}{R}$$

$$u_2 = \frac{3V_T}{5} \qquad u_1 = \frac{2V_T}{5}$$

$$I_T = \frac{V_T - u_1}{3R} + \frac{V_T - u_2}{R} = \frac{3V_T}{5R}$$

$$\frac{V_T}{I_T} = \frac{5}{3}R$$

$$\boxed{R_{ab} = \frac{V_T}{I_T} = \frac{5R}{3}}$$