## EECS 16A Designing Information Devices and Systems I <br> Spring 2020

## Read the following instructions before the exam.

## Format \& How to Submit Answers

There are 13 problems (4 introductory questions, and 9 exam questions, comprising 34 subparts total) of varying numbers of points. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can. Don't get bogged down in calculations; if you are having trouble with one problem, there may be easier points available later in the exam!

All answers will be submitted to the Gradescope "Midterm 2" Assignment (https: / /www. gradescope. com/courses/83747/assignments/427941). All questions, except introductory questions, are multiple choice and are worth 4 points each. There are 125 points possible on the exam, but your final score will be taken out of $\mathbf{1 0 0}$ points. This means that a score of $75 / 125$, normally $60 \%$, will be bumped up to $75 / 100$, or $75 \%$. You cannot score more than $100 \%$ on this exam.

Partial credit may be given for certain incorrect answer choices for some problems. There is no penalty for incorrect answers.

Post any content or clarifying questions privately on Piazza. There will be no exam clarifications; if we find a bug on the exam, that sub-question will be omitted from grading.

## Timing \& Penalties

You have 120 minutes for the exam, with a 5 minute grace period. After the 5 minute grace period ends, exam scores will be penalized exponentially as follows: an exam that is submitted $N$ minutes after the end of the grace period will lose $2^{N}$ points. The exam will become available at your personalized link at 8:10 pm PT; the grace period will expire at $\mathbf{1 0 : 1 5} \mathbf{~ p m ~ P T}$. If your submission is timestamped at $10: 16 \mathrm{pm}$ PT, you will lose 2 points; if it is timestamped at $10: 18 \mathrm{pm}$ PT, you will lose 8 points.

We will count the latest time at which you submit any question as your exam timestamp. Do NOT edit or resubmit your answers after the deadline. We recommend having all of your answers input and submitted by $10: 10 \mathrm{pm}$; it is your responsibility to submit the exam on time.

If you cannot access your exam at your link by $8: 15 \mathrm{pm}$, please email eecs16a@berkeley.edu. If you are having technical difficulties submitting your exam, you can email your answers (either typed or scanned) to eecs16a@berkeley.edu.

## Academic Honesty

This is an open-note, open-book, open-internet, and closed-neighbor exam. You may use any calculator or calculation software that you wish, including Wolfram-Alpha and Mathematica. No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.

We have zero tolerance against violation of the Berkeley Honor Code. Given supporting evidence of cheating, we reserve the right to automatically fail all students involved and report the instance to the student conduct committee. Feel free to report suspicious activity through this form. (https: //forms.gle/akhBsHVr1WG29Ufg9).

Our advice to you: if you can't solve a particular problem, move on to another, or state and solve a simpler one that captures at least some of its essence. You will perhaps find yourself on a path to the solution.

Good luck!

## 1. Pledge of Academic Integrity (2 points)

By my honor, I affirm that:
(1) this document, which I will produce for the evaluation of my performance, will reflect my original, bona fide work;
(2) as a member of the UC Berkeley community, I have acted and will act with honesty, integrity, and respect for others;
(3) I have not violated-nor aided or abetted anyone else to violate-nor will I-the instructions for this exam given by the course staff, including, but not limited to, those on the cover page of this document; and
(4) I have not committed, nor will I commit, any act that violates-nor aided or abetted anyone else to violate-the UC Berkeley Code of Student Conduct.

Write your name and the current date as an acknowledgement of the above. (See Gradescope)

## 2. Administrivia (1 point)

I know that I will lose $2^{n}$ points for every $n$ minutes I submit after the exam submission grace period is over. For example, if the exam becomes available at my personalized link at 8:10 p.m. PT; the grace period will expire at $10: 15 \mathrm{p} . \mathrm{m}$. PT. If my submission is timestamped at $10: 16 \mathrm{p} . \mathrm{m}$. PT, I will lose 2 points; if it is timestamped at 10:18 p.m. PT, I will lose 8 points.

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Yes
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## 3. What is one of your hobbies? ( 1 point)

4. Tell us about something you're proud of this semester. (1 point)

## 5. Fun with Resistors ( 12 points)

(a) Your partner hands you an old beat-up four-band resistor, $R_{\text {mystery }}$ where the second band has been worn off with time. You can see that the first band is orange, the third band is yellow, and the fourth band is gold. Which of the following ranges does the nominal value of this unknown resistance, $R_{\text {mystery }}$, fall in? Do not consider the range allowed by the tolerance.
There is a resistor color code chart below:


Figure 1: Resistor Color Chart.


Figure 2: $R_{m y s t e r y}$.

For the rest of this problem, assume that you have an infinite supply of $5 \Omega$ and $20 \Omega$ resistors.
(b) What is the minimum number of resistors you need to create a resistor network with an equivalent resistance equal to $4 \Omega$ ?
$\square$
(c) What is the minimum number of resistors you need to create a resistor network with an equivalent resistance equal to $9 \Omega$ ?
$\square$

## 6. (16 points) Analyzing a Resistor Network

Consider the following circuit.

(a) Each branch in the circuit is either labelled with a branch current direction or a branch voltage polarity, but not both. For the branches corresponding to $R_{1}$ and $R_{2}$, label the missing quantity according to passive sign convention.
(b) All resistors have a value of $72 R$. Find the equivalent resistance between terminals $C$ and $E, R_{C E}$, in terms of $R$, and also find the equivalent resistance between terminals $A$ and $B, R_{A B}$, in terms of $R$. Assume that when finding the equivalent resistance between two terminals, all the other terminals are left as is, with nothing attached to them.

The circuit has been duplicated again here for your convenience for parts (c) and (d). The labels (1), (2), and (3) correspond to locations in the circuit schematic.

(c) Write the KVL equation equation for the loop (A)-(1)-(D)-(3)-(B)-A in terms of $V_{1}$ to $V_{7}$ using passive sign convention.
$\square$
(d) Write the KCL equation for the node associated with terminal (E) in terms of the currents $I_{1}$ to $I_{7}$ using passive sign convention.

## 7. (8 points) Linearity of circuits

Consider the following circuit with resistor values $R_{1}=9 \Omega, R_{2}=1 \Omega, R_{3}=6 \Omega$, and $R_{4}=4 \Omega$.

(a) In order to analyze this circuit, Nirmaan wrote down a system of equations for the circuit and then cast it into matrix-vector form. However, some of the entries of the matrix were smudged, and Nirmaan needs your help filling in the missing entries. Fill in the missing values in the matrix below. Hint: Rows (2)-(4) correspond to KCL equations. Rows (5) - (8) correspond to Ohm's Law equations.

$$
\left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & \left(C_{1}\right) & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \left(C_{2}\right) & \left(C_{2}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\left(C_{3}\right) & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & \left(C_{4}\right) & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & \left(C_{5}\right) & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & \left(C_{6}\right) & 0 & 0 & 0 & -1
\end{array}\right]\left[\begin{array}{c}
I_{1} \\
I_{2} \\
I_{3} \\
I_{4} \\
V_{1} \\
V_{2} \\
V_{3} \\
V_{4}
\end{array}\right]=\left[\begin{array}{c}
V_{s} \\
0 \\
I_{s} \\
-I_{s} \\
0 \\
0 \\
0 \\
0
\end{array}\right]
$$

(b) Now consider the same circuit, except with different resistor values.


When $V_{s}=15 V, I_{s}=-8 A$, and $R_{1}=R_{2}=R_{3}=R_{4}=4 \Omega$, we have that the solution to the above system of equations is the vector:
$\left[\begin{array}{l}I_{1} \\ I_{2} \\ I_{3} \\ I_{4} \\ V_{1} \\ V_{2} \\ V_{3} \\ V_{4}\end{array}\right]=\left[\begin{array}{c}-8 \\ 4 \\ 4 \\ 8 \\ 15 \\ -17 \\ -33 \\ 32\end{array}\right]$

What is the power dissipated across the voltage source, $V_{s}$ ? What is the power dissipated across the resistor $R_{3}$ ?

## 8. Superposition ( $\mathbf{1 2}$ points)

Consider the following circuit:


Let $R_{1}=15.0 \Omega, R_{2}=25.0 \Omega, R_{3}=35.0 \Omega, I_{s}=1.0 \mathrm{~A}$, and $V_{s}=4.0 \mathrm{~V}$.
(a) With the current source turned on and the voltage source turned off, find the current $I_{R_{3}}$.
$\square$
(b) With the voltage source turned on and the current source turned off, find the voltage drop across $R_{3}$, $V_{R_{3}}$.
$\square$
(c) With both sources turned on, find the power dissipated across $R_{3}$.

## 9. Thévenin Equivalence ( 12 points)

(a) Find the Thévenin resistance $R_{t h}$ of the circuit shown below, with respect to its terminals $A$ and $B$. Assume that $R_{1}=4 R, R_{2}=R$ and $R_{3}=9 R$.

(b) Now a load resistor, $R_{L}=9 R$, is connected across terminals $A$ and $B$ as shown in the circuit below. Find the supply voltage, $V_{s}$, such that 1 mW is dissipated across the load resistor. Let $R=36 \mathrm{k} \Omega$.

(c) We modify the circuit as shown below:


Find a symbolic expression for $V_{\text {out }}$ as a function of $V_{s}$.

## 10. Capacitive Charge Sharing (16 points)


(a) Consider the circuit above with $C_{1}=6 \mu \mathrm{~F}$ and $C_{2}=9 \mu \mathrm{~F}$. Suppose that initially the $\phi_{1}$ switches are closed and the $\phi_{2}$ switch is open such that $C_{1}$ and $C_{2}$ are charged through the corresponding voltage sources $V_{s 1}=6 \mathrm{~V}$ and $V_{s 2}=17 \mathrm{~V}$. How much charge is on $C_{1}$ and $C_{2}$ ?
$\square$
(b) Now suppose that some time later, the $\phi_{1}$ switches open and the $\phi_{2}$ switch closes. What is the value of voltage $u_{1}$ at steady state?
$\square$
(c) Now let's look at the following circuit. Once again, let $C_{1}=6 \mu \mathrm{~F}$ and $C_{2}=9 \mu \mathrm{~F}$. Let $V_{s}=7 \mathrm{~V}$.


Suppose that initially the $\phi_{1}$ switches are closed and the $\phi_{2}$ switches are open. How much charge is on $C_{1}$ and $C_{2}$ ?
(d) Now suppose that some time later, the $\phi_{1}$ switches open and the $\phi_{2}$ switches close. What is the value of voltage $V_{\text {out }}$ at steady state?

## 11. Flash Memory ( $\mathbf{1 6}$ points)

Solid state drives depend on charge to store information. In several integrated circuit applications, the charge is stored on a floating node of a transistor. A cartoon of such a transistor and its corresponding circuit model are shown below. Note: You do not need to understand transistors in order to do this problem.


Figure 3: Transistor schematic for flash memory.


Figure 4: Corresponding circuit model for flash memory transistor from Figure 3 ,

Depending on the amount of charge on the floating node, the transistor is either storing a " 0 " bit or a " 1 " bit. In order to write a bit, electrons are added or removed from the "floating gate node" (labelled by $V_{F G}$ in Figure 4), which is capacitively coupled to the gate and the source of the transistor as shown in Figure 3. Therefore, during transistor operation, there can be a net charge $Q_{T}$ at the node $V_{F G} \square^{1}$
In each part of the problem, the values of each parameter do not change, and are given in the table below:

| Component Parameters |  |
| :---: | :---: |
| $V_{\text {in }}$ | 2.0 V |
| $C_{G}$ | 35.0 pF |
| $C_{S}$ | 3.0 pF |
| $g_{m}$ | $300.0 \mu \mathrm{AV}^{-1}$ |
| $R_{L}$ | $35.0 \mathrm{k} \Omega$ |

[^0](a) Assuming that the floating gate node $\left(V_{F G}\right)$ has no net charge $\left(Q_{T}=0\right)$. Find $V_{F G}$ in terms of $V_{i n}, C_{G}$, and $C_{S}$, and then plug in values from the table.
$\square$
(b) Now let's assume there is a net charge of $Q_{T}=-20.0 \mathrm{pC}$ on the floating gate node. Find $V_{F G}$ as a function of $C_{G}, C_{S}, V_{i n}$, and $Q_{T}$ and then plug in the numerical values. (Hint: Use conservation of charge.)
$\square$
(c) Now, in order to read which bit the transistor is storing, we measure the value of $V_{\text {out }}$, which is the voltage drop across $R_{L}$, as shown in Figure 4. Assuming that the threshold for reading a " 1 " bit is $V_{\text {out }} \leq$ -1 V , what condition must $V_{F G}$ satisfy? (Note: This part is independent of the previous two subparts.)
$\square$
(d) Regardless of the answer you got in part (c), assume that the threshold for reading a " 1 " is $V_{F G} \geq$ 1 V . How much net charge, $Q_{T}$ do you need on the floating gate node in order to read a " 1 "?

## 12. A Versatile Opamp Circuit ( $\mathbf{1 2}$ points)

The following circuit is a commonly used integrated circuit (IC) that can be configured in a variety of ways to achieve different functionalities. The main advantage of this IC is that it provides precise circuit functions without the need of a high precision resistor network.


In order to see this circuit's versatility, we will configure this circuit in different ways to achieve different functionalities. For each subpart, let $v_{1}=1.2 \mathrm{~V}$ and $v_{2}=1.8 \mathrm{~V}$
(a) Determine the voltage at $O$ for the following configuration.

(b) Determine the voltage at $O$ for the following configuration.

(c) Determine the voltage at $O$ for the following configuration.


## 13. Odometer (16 points)

You are designing a circuit for an car odometer, which measures the distance the car has travelled. The main circuit element that you have is a resistor, whose resistance changes as a function of car velocity, which varies with time:

$$
R_{\mathrm{car}}(t)=\frac{4.0 \alpha R_{0}}{v(t)}
$$

where $R_{0}=100.0 \mathrm{k} \Omega, \alpha=60.0 \frac{\mathrm{~m}}{\mathrm{~s}}$, and $v$ is the velocity of the car in $\frac{\mathrm{m}}{\mathrm{s}}$. Throughout the problem, assume that the capacitor is initially uncharged.
(a) Not knowing where to start, you ask a senior member of the design team, who went to Berkeley, and he gives you the following circuit to get you started.


As a first step, determine $V_{\text {out }}$ in terms of $t, V_{i n}, \alpha, R_{0}, v(t)$, and $C$. (Hint: Apply KCL and use the fact that $I=C \frac{d V}{d t}$.)
$\square$
(b) Regardless of the answer to part (a), the output of the above circuit is given by: $V_{\text {out }}(t)=-\frac{1}{\alpha R_{0} C} \int_{0}^{t} V_{\text {in }} v(\tau) d \tau$. Now, you decide to set $V_{i n}=1 \mathrm{~V}$. Your odometer increases its distance reading by 1 when the voltage, $V_{\text {out }}$, decreases by 0.1 V .

If you want the odometer to change its distance reading by 1 when the car travels 1 kilometer, what should $C$ be? Hint: The distance travelled by the car is given by $\int_{0}^{t} v(\tau) d \tau$.
(c) Your car has the following velocity versus time graph. How much energy is dissipated through the car resistor, $R_{\mathrm{car}}(t)=\frac{4.0 \alpha R_{0}}{v(t)}$ ? Assume that $V_{\text {in }}$ for the circuit given in part (a) is still 1 V .

(d) In order to add functionality, you want to add a comparator that will alert the driver when a certain number of kilometers has passed, using the odometer you designed in part (b). You do this with the following circuit:

$V_{\text {out, ckt1 }}=-\frac{1}{\alpha R_{0} C} \int_{0}^{t} V_{i n} v(\tau) d \tau$ is the output of odometer circuit shown in part (a). If you want the comparator to change states after the car has driven 180.0 km , given that we require $R_{\text {left }}+R_{\text {right }}=$ $70.0 \mathrm{k} \Omega$, what should the value of $R_{\text {left }}$ be?


[^0]:    ${ }^{1}$ The method through which charge is added or removed from the floating gate is known as Fowler-Nordheim tunneling, where electrons are quantum mechanically tunnelled to the floating gate. In this problem, we will not worry about how the net charge is added to the floating gate node at the circuit level.

