Fun with Stacked Caps

Consider a capacitor circuit with switches. Suppose that in Phase 1, the circuit looks like the circuit in Figure 1:

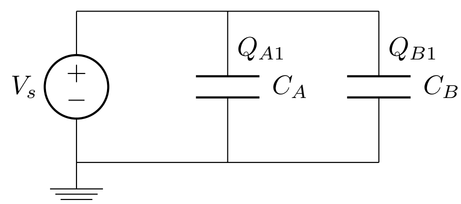

Figure 1: Phase 1

Since the voltage source forces V_s across each capacitor, we do not need the initial state of the capacitors to determine the charges.

$$Q_{A1} = C_A V_s$$
$$Q_{B1} = C_B V_s$$

Suppose we flip switches such that the circuit looks like Figure 2:

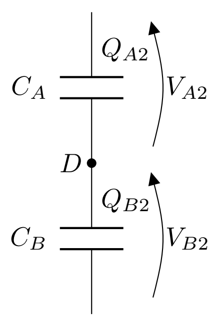

Figure 2: Phase 2

We note that there are no paths for current to flow or charge to move.

Therefore:

$$Q_{A2} = Q_{A1}, \qquad V_{A2} = \frac{Q_{A1}}{C_A} = V_s$$
$$Q_{B2} = Q_{B1}, \qquad V_{B2} = \frac{Q_{B1}}{C_A} = V_s$$

Suppose we now attach a voltage source V_x to this circuit:

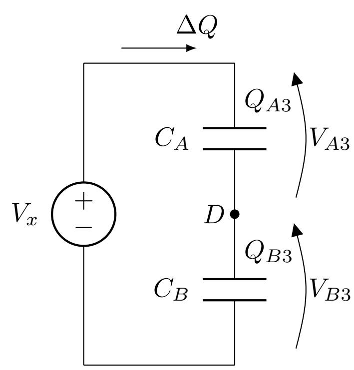

Figure 3: Phase 3

ΔQ denotes the change of charge on C_A. Therefore:

$$\Delta Q = Q_{A3} - Q_{A2}$$

From charge conservation on node D, we obtain:

$$-Q_{A3} + Q_{B3} = -Q_{A2} + Q_{B2}$$

By rearranging the above two equations, we obtain:

$$\Delta Q = Q_{B3} - Q_{B2}$$

We conclude that a change in charge is the same on both capacitors. This change in charge ΔQ is the charge supplied by the new voltage source V_x.

Using KVL:

$$V_x = V_{A3} + V_{B3}$$
$$V_x = \frac{Q_{A3}}{C_A} + \frac{Q_{B3}}{C_B}$$
$$V_x = \frac{Q_{A2} + \Delta Q}{C_A} + \frac{Q_{B2} + \Delta Q}{C_B} \tag{1}$$
$$V_x = \frac{Q_{A1}}{C_A} + \frac{Q_{B1}}{C_B} + \Delta Q \left(\frac{1}{C_A} + \frac{1}{C_B} \right) \tag{2}$$

Note that only if $Q_{A1} = Q_{B1} = 0 \implies$

$$\Delta Q = V_x \cdot \underbrace{(C_A \parallel C_B)}_{C_{eq}}$$

Otherwise, we need to take into account the prior charge on caps (Q_{A1}, Q_{B1}) as in (1). In general:

$$\Delta Q = \left(V_x - \frac{Q_{A1}}{C_A} - \frac{Q_{B1}}{C_B} \right) \cdot (C_A \parallel C_B)$$

For this example:

$$\begin{aligned} \Delta Q &= (V_x - V_s - V_s) \cdot (C_A \parallel C_B) \\ \Delta Q &= (V_x - 2V_s)\,(C_A \parallel C_B) \end{aligned}$$