Solutions: Courtesy of John Noonan.

For each problem, assume the op-amp has a nominal gain of $G = 100$.

1. Op-Amps Without Feedback

For the circuit above, write $H(\omega) = \frac{V_{out}}{V_{in}}$.

Solutions:

(a) $V_{out} = G(V^+ - V^-)$

$b^- = 0$ $V^+ = V_{in}$

Thus, $V_{out} = G(V_{in} - 0) = GV_{in}$

$H(\omega) = \frac{V_{out}}{V_{in}} = G$

(b) If $G$ is 20% lower than its nominal value, what is the percent error in $H(\omega)$ relative to nominal?

Solutions: Since $H(\omega) = G$, the percent error will be 20%.

2. Op-Amps With Feedback

For the circuit above, approximate $\frac{V_{out}}{V_{in}}$ using the op-amp golden rules.

Solutions:

Using the Golden Rules, we know that $V^+ = V^-$. And since $V^-$ is connected to $V_{out}$ in negative feedback, $V_{in} = V_{out}$. Thus, $\frac{V_{out}}{V_{in}} = 1$.

(b) Now write $\frac{V_{out}}{V_{in}}$ without the second op-amp golden rule (you can still assume no current flows into the amplifier inputs). How close is the result to the approximation from (a)? (Give a percentage.)

Solutions:

$V_{out} = G(V_{in} - V_{out})$

$V_{out} + GV_{out} = GV_{in}$

$(1 + G)V_{out} = GV_{in}$

$\frac{V_{out}}{V_{in}} = \frac{G}{1+G}$

Thus, as $G$ approaches $\infty$, $\frac{V_{out}}{V_{in}}$ approaches 1, in which case $V_{out} = V_{in}$.

Percent error in $\frac{V_{out}}{V_{in}}$ relative to the approximation from (a): $(1 - \frac{G}{1+G}) \times 100\% = (1 - \frac{100}{101}) \times 100\% = 1.0\%$. 
(c) Now assume $G$ is 20% lower than its nominal value. What is the percent error in $V_{\text{out}}/V_{\text{in}}$ relative to the approximation from (a)?

\textbf{Solutions:}

Percent error in $\frac{V_{\text{out}}}{V_{\text{in}}}$ relative to the approximation from (a): 

\[ \left(1 - \frac{0.8G}{1 + 0.8G}\right) \times 100\% = \left(1 - \frac{80}{81}\right) \times 100\% = 1.23\%. \]

3. Inverting Amplifier

\begin{center}
\begin{tikzpicture}
  \draw (-1,0) -- (1,0);
  \draw (-1,0) -- (0,1);
  \draw (0,1) -- (0,-1);
  \draw (0,-1) -- (1,0);
  \draw (1,0) -- (0,-1);
  \draw (0,0) -- (0,0) node[above] {$\text{Vin}$} node[below] {$\text{Vout}$} node[above right] {$+$} node[below right] {$-$};
  \draw (0,0) -- (-1,1) node[above] {$R_1$};
  \draw (0,0) -- (1,-1) node[below] {$R_2$};
\end{tikzpicture}
\end{center}

(a) For the circuit above, approximate $V_{\text{out}}/V_{\text{in}}$ using the op-amp golden rules.

\textbf{Solutions:} Using the golden rules, we get that 

\[ \frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_2}{R_1}. \]

(b) Now write $V_{\text{out}}/V_{\text{in}}$ without the second op-amp golden rule (you can still assume no current flows into the amplifier inputs).

\textbf{Solutions:}

\[ V_{\text{out}} = -GV_A \]

\[ V_A = -\frac{V_{\text{out}}}{G} \]

\[ \frac{1}{R_1}V_{\text{in}} - \frac{1}{R_1} \times -\frac{V_{\text{out}}}{G} = -\frac{1}{R_2} \frac{V_{\text{out}}}{G} - \frac{V_{\text{out}}}{R_2} \]

\[ \frac{1}{R_1}V_{\text{in}} = -(\frac{1}{R_1} \frac{1}{G} + \frac{1}{R_2} \frac{1}{G} + \frac{1}{R_1})V_{\text{out}} \]

\[ V_{\text{out}} = \frac{1}{R_1} \frac{1}{1 + \frac{1}{G} + \frac{1}{R_2} \frac{1}{G} + \frac{1}{R_1}} \]

\[ = -\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{G}} \]

(c) If $R_1 = 100\Omega$ and $R_2 = 500\Omega$, find the gain of the circuit using the models from both (a) and (b). What is the percent error?

\textbf{Solutions:}

\[ a: -\frac{R_2}{R_1} = -\frac{500}{100} = -5 \]

\[ b: \frac{1}{\frac{1}{100} + \frac{1}{500} + \frac{1}{100}} = -4.72 \]

About 5.6% error.

(d) Now assume $G$ is 20% lower than its nominal value and recalculate the gain. What is the new percent error compared to (a)?

\textbf{Solutions:}

\[ a: -5 \]

\[ b: -\frac{1}{\frac{1}{100} + \frac{1}{500} + \frac{1}{100}} = -4.65 \]

About 7.0% error.